Royal
OMA

MEMOIRS

OF

THE ASTRONOMICAL SOCIETY

OF

LONDON.

VOL. I.

LONDON:
PUBLISHED BY BALDWIN, CRADOCK AND JOY,
PATERNOSTER-ROW.

1822.

PRINTED BY R. AND A. TAYLOR,
SHOE-LANE, LONDON.

ADVERTISEMENT.

The Council of the Astronomical Society of London take this opportunity of acquainting the public, that a Committee is, from time to time, appointed to consider and report on the Papers read before the Society: and the Council afterwards select such as they judge most proper for publication in the Memoirs. The grounds of their choice are, and will continue to be, the importance or singularity of the subjects, or the advantageous mode of treating them; without, however, pretending to answer for the certainty of the facts, or the propriety of the reasonings, contained in the several Papers so published; which must still rest on the credit or judgment of their respective authors.

It is therefore necessary to remark that the thanks, which are usually proposed from the Chair, to be given to the authors of such Papers as are read at their accustomed meetings, or to the persons through whose hands they receive them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications.

CONTENTS.

PAPERS.

ADDRESS OF THE SOCIETY,

EXPLANATORY OF

THEIR VIEWS AND OBJECTS.

Circulated prior to their First Public Meeting.

IN a country like Great Britain, in which the sciences in general are diligently cultivated, and ASTRONOMY in particular has made extensive progress and attracted a large share of attention, it must seem strange that no Society should exist peculiarly devoted to the cultivation of this science; and that (while chemistry, mineralogy, geology, natural history, and many other important departments both of science and of art are promoted by associated bodies, which direct, while they stimulate, the highest exertion of individual talent) Astronomy, the sublimest branch of human knowledge, has remained up to the present time unassisted by that most powerful aid; and has relied for its advancement on the labours of insulated and independent individuals.

It may be conceived by some, that astronomy stands less in need of assistance of this kind than any other of the sciences; and that, in the state of perfection which its physical theory has already reached, its ulterior progress may safely be intrusted to individual zeal, and to the great national establishment exclusively appropriated to celestial observations; or, at all events, to those public Institutions and Academies in all civilized nations, whose object is the general cultivation of the mathematical and physical sciences. It may therefore be necessary to state the useful objects which may be accomplished, and the im-

pediments which may be removed, by the formation of a Society devoted solely to the encouragement and promotion of Astronomy.

Owing to the great perfection which the construction of optical instruments has attained in England, and the taste for scientific research universally prevalent, there have arisen in various parts of the kingdom a number of private and public observatories, in which the celestial phænomena are watched, and registered with assiduity and accuracy, by men whose leisure and talents peculiarly adapt them for such pursuits: while others, with a less splendid establishment, but by the sacrifice of more valuable time, pursue the same end with equal zeal and perseverance. Considerable collections of valuable observations have thus originated; by far the greater part of which, however, owing to the expense and difficulty of publication and various other causes, must inevitably perish, or at least remain buried in obscurity, and be lost to all useful purposes; unless collected and brought together by the establishment of a common centre of communication and classification, to which they may respectively be imparted.

This great desideratum, it is presumed, will be attained by a Society founded on the model of other scientific institutions, having for one of its objects the formation of a collection or deposit of manuscript observations, &c. open at all times to inspection; to which the industrious observer may consign the result of his labours, with the certainty of their finding a place, among the materials of knowledge so amassed, exactly proportioned to their intrinsic value. At the same time it will thus be rendered practicable to form a connected series from a mass of detached and incomplete fragments; and the society will render a valuable service to science, by publishing, from time to time, from this collection, such communications or digests as seem calculated, by their nature and accuracy, either to supply deficiencies, or to afford useful materials to the theoretical astronomer.

It will also be an object worthy of the society, to promote an examination of the heavens in minute detail; by parcelling them out, in portions of a very moderate extent, among those members who may find leisure and inclination to direct their attention more peculiarly and constantly to such portions (selection being made as to those which may best accord with the situation of their observatories and their own general convenience); thereby to ascertain the

places, and if possible the proper motions, of all the objects, large or minute, which may fall within their respective limits; and to pass them continually in review, so that no new celestial body of a cometary or planetary nature, traversing their boundaries, may escape detection. For, amongst the vast multitude of similar objects which are scattered over the wide expanse of the heavens, and which equally solicit and distract the attention of the insulated observer, no one of them in particular can be expected to undergo any very rigorous examination, unless distinguished by magnitude or some peculiarity of appearance.

The knowledge of our own peculiar system, and that more extended branch of astronomical science to which the name Cosmology is best adapted, may alike be benefited by this division of labour, and systematic mode of examination. In the planetary system, a wide field of investigation has of late been opened by the discovery of some links in the chain of connexion which no doubt exists between bodies of a cometary and planetary nature. And it is possible that some bodies, of a nature altogether new, and whose discovery may tend in future to disclose important secrets in the system of the universe, may be concealed under the appearance of minute stars, no way distinguishable from others of a less interesting character, but by the test of careful and often repeated observations. Indeed it is worthy of notice, that, of the five small bodies lately ascertained to be permanent members of our system, four were discovered in the short space of seven years, by the partial adoption, on the continent, of this very plan of separate examination; which seems indeed to have been first suggested by the late Rev. F. Wollaston. This diligent astronomer, in a paper published in the Philosophical Transactions for 1784, thus remarks: "The first idea which occurred to me was to make a proposal to astronomers "in general, that each should undertake a strict examination of a certain "district in the heavens; and not only by a re-examination of the catalogues "hitherto published, but by taking the right ascension and declination of "every star in their several allotments, to frame an exact map of it, with a "corresponding catalogue; and to communicate their observations to one "common centre. This is what I should be glad to see begun. Every astro-"nomer must wish it, and therefore every one should be ready to take his "share in it." In fact, Mr. Wollaston not only proposed the plan, but, as far as an individual could do so, put it in execution, by undertaking the examination of the circumpolar regions himself.

Beyond the limits however of our own system, all at present is obscurity. Some vast and general views on the construction of the heavens, and the laws which may regulate the formation and motions of sidereal systems, have, it is true, been struck out; but, like the theories of the earth which have so long occupied the speculations of geologists, they remain to be supported or refuted by the slow accumulation of a mass of facts: and it is here, as in the science just alluded to, that the advantages of associated labour will appear more eminently conspicuous.

One of the first great steps towards an accurate knowledge of the construction of the heavens, is an acquaintance with the individual objects they present: in other words, the formation of a complete catalogue of stars and of other bodies, upon a scale infinitely more extensive than any that has yet been undertaken; and that shall comprehend the most minute objects visible in good astronomical telescopes. To form such a catalogue, however, is an undertaking of such overwhelming labour, as to defy the utmost exertions of individual industry. It is a task which, to be accomplished, *must* be divided among numbers: but so divided as to preserve a perfect unity of design, and prevent the loss of labour which must result from several observers working at once on the same region, while others are left unexamined. The intended foundation of an observatory at the southern extremity of Africa, under the auspices of the Admiralty, may serve to show the general sense entertained of the importance of this subject, and the necessity of giving every possible perfection to our catalogue of the fixed stars. Deeply impressed also with the importance of this task, and fully aware of its difficulty, the Astronomical Society might call upon the observers of Europe and of the world to lend their aid in its prosecution. Should similar institutions be formed in other countries, the Astronomical Society (rejecting all views but that of benefiting science) would be ready and desirous to divide at once the labour and the glory of this Herculean attempt, and to act in concert together in such manner as should be judged most conducive to the end in view.

Another beneficial result to be expected from this institution is the diffusion of a spirit of inquiry in practical astronomy; and, as a necessary consequence, a corresponding diffusion of a general knowledge of the mode of performing and computing astronomical and geodesical observations, and of the use of

instruments; especially such as are likely to be found in the hands of travellers, nautical men, and others who may be placed in interesting situations in remote parts of the world. Widely scattered as Englishmen are, over the surface of the globe, the advantages which might accrue to science from a more general diffusion of such knowledge, are incalculable: yet it is painful to reflect in how few cases, comparatively, among the numbers of our countrymen whose prospects in life lead them to distant climates, the actual use of even the simplest astronomical instruments and tables has formed a part of their education or study. In a national point of view, every thing which may tend to diffuse a knowledge of practical astronomy is obviously of the utmost importance, on account of its application to nautical purposes. Besides which, the difficulty of finding practical observers calculated to fill situations as assistants in observatories, in expeditions of discovery, or on other occasions, at moderate salaries, has been felt in various instances. Were there establishments in our universities and other places of public education, in which young men might be taught the use of astronomical instruments and tables, not only would the theoretical knowledge of astronomy which they are led to acquire in those admirable establishments make a deeper impression, but a greater number of good observers would thus be annually trained up, to the great benefit not only of themselves but of their country. It is understood that at the Royal Military Seminaries some establishment of this kind exists.

It is almost unnecessary to enumerate the advantages likely to accrue from the encouragement which an Astronomical Society may hold out: but among others may be mentioned the perfecting of our knowledge of the latitudes and longitudes of places in every region of the globe; the improvement of the lunar theory, and that of the figure of the earth, by occultations, appulses, and eclipses simultaneously observed in different situations; the advancement of our knowledge of the laws of atmospherical refraction in different climates, by corresponding observations of the fixed stars; the means of determining more correctly the orbits of comets by observations made in the most distant parts of the world: and in general, the frequent opportunities, afforded to a society holding extensive correspondence, of amassing materials which (though separately of small importance) may by their union become not only interesting at the present time, but also valuable as subjects of reference in future.

By means of Corresponding Members, or Associates, in distant countries,

the society may hope to unite the labours of foreign observers with their own; and by thus establishing communications with eminent astronomers and institutions in all parts of the world, to obtain the earliest intelligence of new discoveries or improvements; which it may, perhaps, be desirable to circulate among such of its members as may profess themselves anxious to receive it, without loss of time.

The circulation also of notices of remarkable celestial phænomena about to happen, (with the view of drawing the attention of observers to points which may serve important purposes in the determination of elements or coefficients,) may form another, and perhaps not the least interesting object of the society. To have the same phænomena watched for by many observers, in a climate so uncertain as our own, is the only sure way of having them observed by some: and moreover, the attention of an astronomer may frequently be aroused by a formal notice, especially when accompanied with directions for observing the phænomenon in the most effective way, when probably the mere ordinary mention of it in an ephemeris might fail to attract his observation.

One of the collateral advantages of a society including many practical astronomers among its members, (but which will appear of no small importance to those who possess good instruments,) will be the mutual understanding which will be propagated among amateur astronomers, by frequent meetings and discussion, as to the relative merits of their instruments; and as to the talents and ingenuity of the various artists both of our own and of foreign nations: not to mention the emulation which this must naturally excite to possess the best instruments; and the consequent tendency of such discussion towards a further improvement in their construction, or to the discovery of new ones. Well made instruments will thus unavoidably acquire a reputation, not merely among a few eminently skilful observers in Britain, but throughout the whole astronomical world: and individuals, who have at great expense and trouble amassed a collection of valuable ones, will thus be spared the mortification of knowing that they may at some future time be put up to sale and be disposed of for a half or a third of their value, for want of their merits being known; a consideration which probably has some weight with those who may be collecting instruments at an advanced period of their life.

As the extent of the funds of the society must depend on the number of its

members, it is impossible to conjecture at present how far its views respecting their application may extend. Besides the ordinary expenses attending an institution of this nature, the annual or occasional publication of communicated observations;—the payment of computers employed in the reduction and arrangement of observations, or in computing the orbits of new planets, comets, or other interesting bodies;—the formation of an extensive astronomical library, not only of manuscript but also of printed books;—and perhaps, at some future period, the proposal of prizes for the encouragement of particular departments of the science, either theoretical or practical; or for the improvement of astronomical instruments, or tables,—may be mentioned as worthy objects on which they may be bestowed.

Such are the principal considerations which have actuated a number of individuals to form themselves into a society under the name of the *Astronomical Society of London,* and to give this publicity to their determination, with a view of inviting others to unite in the prosecution of their plans. They have at their very commencement met with the most flattering success, which induces them to hope that, in a short time, every assiduous cultivator of the science will be found to have added his name to the list of members.

The objects of the original members may be sufficiently gathered from what has been already said, and may be thus summed up in few words: *viz.* to encourage and promote their peculiar science by every means in their power, but especially—by collecting, reducing and publishing useful observations and tables—by setting on foot a minute and systematic examination of the heavens—by encouraging a general spirit of inquiry in practical astronomy—by establishing communications with foreign observers—by circulating notices of all remarkable phænomena about to happen, and of discoveries as they arise—by comparing the merits of different artists eminent in the construction of astronomical instruments—by proposing prizes for the improvement of particular departments, and bestowing medals or rewards on successful research in all—and finally, by acting as far as possible, in concert with every institution, both in England and abroad, whose objects have any thing in common with their own; but avoiding all interference with the objects and interests of established scientific bodies.

REGULATIONS OF THE SOCIETY.

Section I.—*Of its Object.*

The Astronomical Society of London is instituted for the encouragement and promotion of Astronomy.

Section II.—*Of its Constitution.*

1. The Astronomical Society shall consist of Members, and Associates : but no person shall be eligible as an Associate, who is a British subject, or whose usual place of residence is in any part of the British dominions.

2. The Officers of the Society shall be chosen out of the Members, and shall consist of a President, four Vice-Presidents, one Treasurer, and three Secretaries; who, with eight other Members, shall constitute a Council for the management and direction of the affairs of the Society.

Section III.—*Of the Election of the Officers and Council.*

1. The President, Vice-Presidents, Treasurer, Secretaries, and the other eight Members of the Council, shall be elected annually by ballot, at the general meeting in February.

2. No Member, who has filled the office of President or Vice-President for two successive years, shall be again eligible to the same situation, until the expiration of one year from the termination of his office.

3. Four only, of the eight Members of the Council who may have served during any one year, shall be re-eligible for the ensuing year.

SECTION IV.—*Of the Election, Admission and Expulsion of Members.*

1. Every person, desirous of admission into the Society as a Member, must be proposed and recommended agreeably to the form No. 1. in the Appendix hereto; wherein must be inserted the Christian name, Surname, Rank, Profession, and usual place of residence of the candidate: and which form must be subscribed by three or more Members, one of whom, at least, must certify his personal knowledge of such candidate.

2. Every recommendation of a proposed Member or Associate must be delivered to one of the Secretaries, and read at one of the ordinary meetings of the Society; after which it shall be hung up in the meeting-rooms of the Society, and there remain until the candidate is balloted for.

3. The ballot shall take place on the second ordinary meeting after that on which the candidate is proposed: except in the case of peers and their eldest sons, and of foreign princes and ambassadors, who shall be balloted for, at the meeting at which the recommendation is delivered to the Secretary, provided a sufficient number of Members be then present.

4. No person shall be considered as elected, unless he have in his favour at least three-fourths of the Members voting.

5. If it should appear upon the ballot that the person proposed is not elected, no notice shall be taken thereof in the minutes.

6. Whenever a person is elected a Member or Associate, one of the Secretaries shall inform him of the same, by letter, as soon as possible.

7. Every person, elected a Member, shall pay his first annual contribution within two calendar months of the day of his election; otherwise his election shall be void: but the Council shall, in particular cases, have the power of extending the period within which such first annual contribution shall be paid.

8. Every *elected* Member, having paid his first annual contribution, and having subscribed the form No. 2 in the Appendix hereto, shall be *admitted* at the first ordinary meeting at which he is present, according to the ensuing form: viz. the President (or other Member in the chair for the time being) addressing him by name shall say, "In the name of the Astronomical Society "of London, I admit you a Member thereof."

9. But when an elected Member, residing at a distance from the metropolis, may not be able to attend the meeting of the Society, he may be admitted by a proxy nominated by himself. In which case, the first annual contribution

being paid and the form No. 2 subscribed, the President, (or other Member in the chair for the time being,) addressing such proxy by name shall say, "In "the name of the Astronomical Society of London, I admit ——— a Mem-"ber thereof."

10. Whenever there may appear cause for the expulsion of any Member, or Associate, from the Society, the Council may call a special general meeting of the Society for that purpose: and if three-fourths of the members then present agree that such Member or Associate be expelled, the President, or other Member in the chair for the time being, shall declare the same accordingly.

Section V.—*Of the Contributions of Members.*

1. The contribution of each Member, residing within fifty miles of the metropolis, shall be four guineas for the first year, payable at the time of his election; and two guineas for every subsequent year: the first of such subsequent contributions to become due and be payable in advance on the 8th day of February then next following, and the succeeding ones, also in advance, on the 8th day of February in every subsequent year.

2. Every Member, not having a residence within fifty miles of the metropolis at the time of his election, shall, on the payment of eight guineas at the time of his election, be exempted from the subsequent annual contributions: but, should he afterwards have a residence within that distance of the metropolis, he shall, from that time, be liable to the annual contributions, like other resident Members.

3. Every Member, admitted as a resident Member, who may afterwards cease to have a residence within fifty miles of the metropolis, shall, on application to the Council and on the payment of the additional sum of four guineas, be exempted from the payment of the annual contributions for the subsequent years during which he may reside beyond fifty miles of the metropolis.

4. Every Member, ceasing to be non-resident, is required to inform the Council that he is become liable to the annual contributions; on failure of doing which he shall cease to be a Member, if the Council shall so determine.

5. Any Member may, at his entrance, compound for his contributions, by the payment of twenty-four guineas; or he may, at any time afterwards, compound for his subsequent annual contributions by the payment of the sum of twenty guineas.

6. Every Member shall be considered as belonging to the Society, and, as

such, liable to the payment of his annual contributions, until he has either forfeited his claim, or signified in writing to the Society his desire to resign; when his name shall be erased from the list of Members.

7. Whenever a Member shall be two years in arrears in the payment of his annual contributions, the name of such Member shall be delivered to the Council; who shall direct one of the Secretaries to write a letter of the form No. 3 in the Appendix hereto, and address and forward the same to such Member, together with a copy of the following regulation.

8. If the arrears shall not be paid within six months after the forwarding of such letter, the name of the Member so offending shall be publicly suspended in the meeting-rooms of the Society, as a defaulter, together with the amount of the contributions due by him to the Society; and such Member shall not have the right to attend any meetings of the Society, nor to enjoy any of the privileges and advantages thereof, until his arrears be paid.

Section VI.—*Of Associates.*

1. Every person, eminent in the science of Astronomy, not being a British subject, nor having a permanent residence in any of the British dominions, is eligible as an Associate.

2. Every such person, proposed for admission as an Associate, must be recommended by four Members; all of whom must certify in writing, agreeably to the form No. 4 in the Appendix hereto, that he is a person eminent in the science of Astronomy, and that they have a personal knowledge of him, or are acquainted with his works.

3. No person shall be considered as an Associate until he has signified his acquiescence in the election.

4. Associates shall have all the rights and privileges of Members, except that of filling any office in the Society.

Section VII.—*Of the Council.*

1. The Council shall have the management and direction of all the affairs of the Society, subject to the control of the general meetings of the Society.

2. The Council shall meet, at the house of the Society, at least once in every month during the session; but any three Members of the Council may, by letter to one of the Secretaries, require an extra meeting to be called.

3. Due notice of each meeting of the Council shall be sent by one of the Secretaries to every Member thereof, residing within the limits of the three-penny post.

4. At any meeting of the Council three Members thereof shall constitute a quorum.

5. All questions shall be decided in the Council by vote, unless a ballot be demanded. The determination of the Council, whether by vote or ballot, shall, at the desire of any two Members present, be deferred to the succeeding meeting.

6. The Council shall draw up a report on the state of the affairs of the Society, to be presented at the annual general meeting. In this report shall be given an abstract of their proceedings during the year.

SECTION VIII.—*Of the Ordinary Meetings.*

1. The ordinary meetings of the Society shall be held on the second Friday in every month, from November to June both inclusive: and five Members shall constitute a quorum, except in the election of Members and Associates, when ten must be present.

2. Business shall commence at eight o'clock in the evening *precisely;* when the minutes of the preceding evening shall be read, and the minutes of the Council laid on the table, to be read if required.

3. The business of the ordinary meetings shall be to propose and ballot for Members and Associates; to announce such donations as may have been made to the Society; to read such communications relating to Astronomy and the subjects connected therewith, as may have been presented to the Society; and to proceed upon any other subject which may have been authorized by the Council.

4. Every Member and Associate shall have the privilege of introducing a visitor at any of the ordinary meetings, on writing his name in a book provided for that purpose.

5. At the ordinary meetings of the Society nothing relating to its regulations or management shall be brought forward.

SECTION IX.—*Of the Annual General Meeting.*

A general meeting of the Society shall be held annually on the second Friday in February, at three o'clock in the afternoon, to receive the report of the

Council on the state of the Society, and to deliberate thereon: to discuss and determine such questions as may be proposed relative to the affairs of the Society: to elect the officers for the ensuing year: and to enact, alter, or repeal regulations, agreeably to the recommendations of the Council.

Section X.—*Of Special General Meetings.*

1. The Council may at any time call a special general meeting of the Society, when it seems to them to be necessary.

2. Any five Members, who are not of the Council, may recommend the adoption or rejection of any measure to the Council, by a letter addressed to one of the Secretaries. Such recommendation the Council shall take into consideration at their next meeting: and if the decision, to which they shall finally come, be not satisfactory to the Members proposing the recommendation, the Council, if required by them or by any five Members, shall bring forward the same for the opinion of the Society at large, at a special general meeting to be held for that purpose; and which they are hereby bound to convene within one month after such requisition.

3. A week's notice shall be given to every Member, residing within the limits of the three-penny post, of the time of such meeting; and of the business for which it is summoned: and no business shall be brought forward at such meeting, except what has been so notified.

Section XI.—*Of altering the Regulations.*

1. Whenever the Council may think it advisable to propose the enactment of any new regulation, or the alteration or repeal of any existing regulation, they shall recommend the same to the Society, at the annual general meeting next ensuing, or at a special general meeting convened for that purpose.

2. Any five Members, who are not of the Council, may recommend any new regulation, or the alteration or repeal of any existing regulation, to the Council, by a letter directed to one of the Secretaries. On the recommendation thus made, the Council shall decide at their next meeting: and if such decision be not satisfactory to the Members proposing the alteration, the Council, if required by them or by any five Members, shall bring forward the same for the opinion of the Society at large, at a meeting especially convened for that purpose.

3. No new regulation, nor alteration or repeal of any existing regulation, shall be brought forward at any meeting of the Society, except in the manner here directed.

Section XII.—*Of Scientific Committees.*

Committees for forwarding specific objects connected with Astronomical science may, from time to time, be appointed by the Council, to whom their Reports shall be submitted for approbation, previously to their being presented to the Society.

Section XIII.—*Of the President and Vice-Presidents.*

1. The duty of the President shall be to take the chair at the meetings, and to regulate and keep order in all the proceedings of the Society: to state and put questions according to the sense and intention of the meeting: and to carry into effect the regulations of the Society.

2. In the absence of the President, the Vice-Presidents in rotation, or in their absence the Treasurer, or one of the Members of the Council, shall take the chair and conduct the business of the meeting: and in case of the absence of all those officers, the meeting may elect amongst themselves any other Member present to take the chair at that meeting.

Section XIV.—*Of the Treasurer.*

1. The Treasurer shall receive, on account and for the use of the Society, all sums of money due to the Society; and, out of those funds, pay all sums due by the Society.

2. He shall keep a regular account of receipts and payments, in the mode which may seem most proper to the Council: who shall have the direction and control of the money in his hands.

3. No sum of money, payable on account of the Society, amounting to five pounds and upwards, shall be paid except by order of the Council, signed by the President or Chairman of the meeting, and registered by one of the Secretaries.

4. The account of the Treasurer shall be audited annually by the Council, who shall report at the annual general meeting the balance in hand, and the general state of the funds of the Society.

5. The Treasurer may, with the approbation of the Council, appoint a proper person to collect the annual contributions of the Members; such collector, however, giving bond, with two good and sufficient sureties thereto, for the faithful discharge of his duty.

Section XV.—*Of the Secretaries.*

1. The duties of the Secretaries shall be to attend all meetings of the Society and of the Council; to take minutes of all their proceedings, and enter them in the proper books provided for that purpose.

2. Their duties at the ordinary meetings shall also be to read the minutes of the preceding meeting; to announce the donations made to the Society since the last meeting; to give notice of any candidate that is proposed for admission, or that is to be balloted for; and to read the letters and papers presented to the Society in the order of time in which they were received, unless the Council shall otherwise direct. But should any person be desirous of reading his own paper, he shall be at liberty so to do, with permission of the President, or Chairman of the meeting.

3. The Secretaries shall have the superintendance of the persons employed by the Society, and the management of the correspondence of the Society and Council: subject however to the direction and control of the latter.

4. The Secretaries shall have the charge, under the direction of the Council, of printing and publishing the memoirs or other papers of the Society.

Section XVI.—*Of the Property of the Society.*

1. The whole of the property and effects of the Society, of what kind soever, shall be vested in four Trustees, for its use: one of which Trustees shall always be the Treasurer for the time being, and the remaining three of which shall be chosen at a general meeting of the Society.

2. Every paper, which may be presented to the Society, shall, in consequence of such presentation, be considered as the property of the Society, unless there shall have been any previous engagement with its author to the contrary; and the Council may publish the same in any way and at any time that they may think proper. But, should the Council refuse, or neglect within a reasonable time, to publish such paper, the author shall have a right to copy the same, and publish it under his own directions. No other person, however,

shall publish any paper belonging to the Society, without the previous consent of the Council.

3. No books, papers, instruments, or other property belonging to the Society, shall be lent out of the Society's house, without leave of the Council: but, every Member has a right, at all seasonable hours, to inspect the books and such papers as the Council may permit, and to take extracts therefrom; and also to inspect the instruments, and take models therefrom, at his own expense.

Section XVII.—*Of Donations and Bequests.*

1. Every person, who shall contribute to the Collection, to the Library, or to the general Funds of the Society, shall be recorded as a Benefactor: his name shall be read at the annual general meeting, and shall be inserted in the next volume of the Memoirs there-after published.

2. Every person, desirous of bequeathing to the Society, any Manuscripts, Books, Instruments or other *personal* property, is requested to make use of the following form in his will; viz.

> "I give and bequeath to the Trustees, for the time being, of the "Society formed in London, under the title of the Astronomical Society "of London [*here enumerate the effects or property intended to be bequeathed*] for the use of the said Society: and I hereby declare that "the receipt of the Treasurer of the said Society for the time being shall "be an effectual discharge to my executors for the said legacy."

3. Every person, desirous of bequeathing to the Society, a sum of money or stock, either for the general purposes of the Society, or to establish a prize for the best production on any particular subject connected with Astronomy, is requested to make use of the following form in his will; viz.

> "I give and bequeath to the Trustees, for the time being, of the "Society formed in London, under the title of the Astronomical Society "of London, the sum of   for the use of the said Society, and for "the purpose of [*here express the particular object in view, if any*]: "the said legacy to be paid out of such part of my personal estate as shall "not consist of chattels *real*. And I hereby declare that the receipt of "the Treasurer of the said Society for the time being shall be an effec-"tual discharge to my executors for the said legacy."

APPENDIX.

FORM, No. 1.

A. B. [*here state the Christian name, Surname, Rank, Profession, and usual place of residence of the candidate*]
being desirous of admission into the Astronomical Society of London, we, the undersigned, propose and recommend him as a proper person to become a Member thereof.

Witness our hands this day of 18

________________ } *from personal knowledge.*

____

____

FORM, No. 2.

I, the undersigned, being elected a Member of the Astronomical Society of London, do hereby promise that I will be governed by the Regulations of the said Society, as they are now formed or as they may be hereafter altered, amended, or enlarged: that I will advance the objects of the said Society as far as shall be in my power: and that I will attend the usual meetings of the Society as often as I conveniently can. Provided that whenever I shall signify in writing to the Society that I am desirous of withdrawing my name therefrom, I shall (after the payment of any annual contribution which may be due by me at that period) be free from this obligation.

Witness my hand, this day of 18

FORM, No. 3.

Sir,

I am directed, by the Council of the Astronomical Society of London, to inform you that it appears by their books, that there were two years of your annual contribution due on the 8th day of February last, amounting to four guineas; the payment of which, to the Treasurer, as early as possible, is earnestly requested.

I am, Sir, your obedient servant,

———————— Secretary.

FORM, No. 4.

We, the undersigned, having a personal knowledge of, or being acquainted with the works of

[*here state the Christian name, Surname, Rank, Profession, usual place of residence, and title of one or more of the works of the person proposed*]

believe him to be a person eminent in the science of Astronomy; and therefore propose and recommend him as a proper person to become an associate of the Astronomical Society of London.

Witness our hands this day of 18

————————

————————

————————

————————

INDEX TO THE REGULATIONS.

REPORT OF THE COUNCIL OF THE SOCIETY

TO THE

FIRST ANNUAL GENERAL MEETING

February 9, 1821.

IN making this first report of their proceedings, the Council cannot but congratulate the Members on the success which has attended the first attempt to establish, in this country, a Society for the promotion of so important a branch of science as Astronomy.

Notwithstanding the difficulties and delays usually attending the establishment of every institution, the efforts of the founders of this society, have been crowned with an accession of strength far beyond their most sanguine expectations; and which seems likely still to increase. Meanwhile, adhering steadily to the principles laid down in the Address, circulated at its first institution, " of avoiding all interference with the objects and interests of other established " scientific bodies," it cannot but be gratifying in the highest degree to have observed a reciprocity of feeling on this subject, recently expressed from the chair of the most eminent scientific institution which this or any other country can boast.

In the infancy of the society much of the time of the Council has been necessarily employed in arranging the usual routine of its business: never-

theless, they venture to hope that, in those subjects which have come before them, they have laid the basis for promoting future improvements and discoveries; which, followed up with zeal and assiduity, must lead to the most beneficial results.

In the science of Astronomy, where the observations of two or three thousand years, and those made in various parts of the world, have not yet led to the rigorous determination of many of the elements of the science, it can scarcely be expected that the efforts of a few individuals, hitherto confined to their own country, can have effected much in the short space of a twelvemonth. But, since more can *now* be done in the compass of a few years than could be formerly effected in a century (owing in a great measure to the superior accuracy of modern instruments, but more especially to those invaluable Principles of Philosophy and that refined analysis first introduced by our illustrious countryman) they indulge the pleasing but not unreasonable hope that, by the active co-operation of a whole scientific body, and the zeal and emulation thereby excited among astronomers of every country, the science will advance constantly and more rapidly towards that state of ultimate perfection, which it is so eminently calculated to attain.

With a view to stimulate such pursuits, the Council have ordered a die to be formed, for the purpose of striking Medals in bronze, silver and gold; to be bestowed, as an honorary distinction, on such persons as may, from time to time, distinguish themselves by any material discovery, or improvement in the science. And, in order to direct the attention of astronomers to those points which appear most worthy of encouragement, they will here state some of the principal subjects on which they have at present decided to bestow such rewards. In the first place, they propose to bestow the medal for the discovery of any new planet, satellite, or comet: or for the re-discovery of any old comet, or of any stars that have disappeared. Considering also the great importance (both in a nautical and in a geographical point of view) of having accurate observations of the eclipses of Jupiter's satellites and of occultations of stars by the moon, they think that the medal should be given for any considerable collection not only of original observations of this kind, but also of well authenticated recorded observations, reduced to the mean time of the meridian of some well known observatory. Observations likewise on the positions of the fixed stars, tending either to the enlargement and perfection

of our present catalogues, or to the more accurate determination of the variable ones in size, colour, or situation;—as well as observations on double stars; tending, in like manner, not only to the enlargement and perfection of the present catalogues, but also to the determination of their angular distance, and of their angle of position;—together with observations on nebulæ—appear proper subjects of such reward. To these may be added, observations on refraction, with a view to the more perfect theory of that phænomenon; particularly at low altitudes, where irregularities take place, when little or no variation has taken place in the barometer or thermometer:—observations on the tides, particularly in situations where the current is not influenced by any contiguous continent, as will be more fully alluded to in the sequel:—observations tending to determine the true figure of the sun, or of the earth:—and, in short, any observations which may be considered likely to advance and improve the science.

But, it is not to observations alone that the Council would wish to confine the bestowing of the Society's medal. The reduction of observations when made is another and oftentimes a more laborious task: and, without the latter, the former would be of little or no service to the astronomer. To this subject, therefore, the Council wish to invite the attention of the computer; as well as to the formation of more simple and easy tables, for the reduction of astronomical observations, than those at present in existence. The formation of new tables for the more recently discovered planets, as well as more accurate tables of the sun, moon and other planets, together with those of Jupiter's satellites, is a subject too important to need the recommendation of this society. The comparison likewise of the places of any of these bodies, observed in the present century at any of the principal observatories, with their places deduced from the most approved tables, but more particularly those of the moon, is an object worthy of encouragement. In the latter case, however, it would be desirable that the numerical value of the arguments of the principal equations should be annexed to each comparison: and that, in all cases, the principles on which the deductions are made should be fully and clearly stated. But, independent of these subjects, there are many other useful tables tending to facilitate astronomical calculations, some of a permanent and others of a temporary or local nature, which would be a great assistance to the practical astronomer, and worthy the patronage of this society. And, without particularizing such subjects, the Council wish it to be under-

stood that these are amongst the objects which they are desirous to reward with the Society's medal.

The Council likewise wish to direct the attention of the diligent inquirer to the recorded observations of preceding astronomers, not only with a view to discover whether any observations are to be there found of any of the more recently discovered planets or comets, but also with a view to the formation of a more complete catalogue of such stars as have, from time to time, disappeared from our sight. These subjects, together with accurate and descriptive accounts of the instruments used by eminent deceased observers, in order to estimate the reliance to be placed on their recorded observations, might fairly claim some mark of distinction.

With respect to instruments, the Council propose to bestow the medal for every improvement which may tend materially to advance the science. They would mention however, as a few amongst the desiderata, an instrument for determining the apparent magnitudes of the stars, or of ascertaining a correct scale whereby astronomers may be enabled to express themselves in one common language on this subject. Likewise a simple but effectual contrivance for enabling an observer to determine the right ascension and declination of small stars, without the necessity of illuminating the field of the telescope. And a method of applying the reflecting telescope to transit or circular instruments, in as convenient and useful a manner as the refracting telescope.

It would be impossible in a report of this kind to enumerate *all* the subjects which may be considered worthy of the Society's medal: neither can the Council establish any general scale for the precise distribution of the three kinds which it is intended to strike. Doubts may frequently arise on subjects of this nature: but, they trust they shall always act with liberality and impartiality, yet at the same time with a due regard to the dignity and character of the society, and the nature of the trust reposed in them. It may indeed appear extraordinary that no mention should yet have been made of the great desiderata of astronomy,—those questions which have exercised the curiosity and employed the time and attention of astronomers ever since the science has assumed its present character—such as the parallax of the fixed stars, their proper motion, the motion or rest of our own system, and its connection with

the rest of the universe. But these and many other points are too obviously suggested by their importance to need any particular notice or encouragement. The man, for whom discoveries of this class are reserved, soars far beyond any distinction which this society can bestow: the applause of the human race attends his labours; and no additional stimulus can be offered to those by which he is impelled.

The subjects of physical astronomy are so various, and of so mixed and complicated a nature, that the Council defer to a future opportunity their observations on this head. In order, however, to show their disposition to encourage the consideration of such subjects (unhappily too much neglected among the geometers of this country) they recommend the proposal of the Society's gold medal and twenty guineas for the solution of the following prize question:

> "For the best paper on the theory of the motions and perturbations "of the satellites of Saturn.—The investigation to be so conducted as to "take expressly into consideration the influence of the rings, and the "figure of the planet as modified by the attraction of the rings, on the "motions of the satellites: to furnish formulæ (adapted to the determi-"nation of the elements of their orbits, and the constant co-efficients of "their periodical and secular equations) from observation: likewise to "point out the observations best adapted to lead to a knowledge of such "determination. The papers to be sent to the Society on or before the "1st day of February 1823."

And, in order to conceal the name of any unsuccessful competitor, the Council propose that each memoir should bear a motto; and that a sealed paper, bearing the same motto, contain the name of the author. In such case, the name of the successful candidate only will be divulged, and the sealed papers of the rest will be destroyed, unopened, in the presence of the Council. The successful paper must be left with the Society, to be published as they may direct; and the rest will be returned on proper application of the authors.

The pecuniary resources of the society, although of course not large, are sufficient to answer every expense which may be incurred by the adoption of the plans recommended by the Council. The number of resident members is 82; out of which, 12 have compounded for their annual payments; and there

are 37 non resident members: so that the total assets of the Society arising from this source are 907*l.* 4*s.* together with a present annual income of about 140*l.* subject to a further increase from the acquisition of new members. The report of the Auditors will state the sums actually received, and the sums paid by the society, together with the balance remaining*. It may be proper however here to mention that the whole of the compositions are, by a resolution of the Council, to be invested from time to time in the Navy 5 per cents. in the joint names of the trustees; and that the payments of the non resident members are at present invested in East India Bonds with a view to a similar permanent investment: it being conceived that these two sources of income should be kept distinct from the annual payments.

The expenses of the Society, up to the present time, have been very trifling, principally owing to the very liberal spirit exhibited by the Geological Society, in granting the use of their commodious apartments for the meetings of the Council and the Members of this Society, in the very infancy of its existence: and for which, this Society is bound to retain a due and grateful remembrance.

---

* INCOME.		£.	s.	d.
64 Subscriptions of 4 guineas each	=	268	16	0
29 do. of 8 do.	=	243	12	0
12 compositions of 24 do..................	=	302	8	0
Div. on £141 . 10 . 4 Navy 5 per cent. July 1820	=	3	10	9
Div. on £215 . 5 . 5 do........... Jan. 1821	=	5	7	7
		823	14	4
Cash advanced by the treasurer		2	15	0
		£826	9	4
EXPENDITURE.				
5 India Bonds of £100 each................	=	513	12	5
£239 . 0 . 11 Navy 5 per cent.	=	247	16	0
Printing and Stationery	=	34	19	2
Bookcase	=	16	16	0
Sundry expenses	=	13	5	9
		£826	9	4

Besides the above 5 India Bonds of £100 each and the £239 . 0 . 11 Navy 5 per cent. in the joint names of the trustees, there are subscriptions, due to the Society, not yet collected, amounting to £92 . 8 . 0

At the close of the last session, the Council received a communication from Capt. *Basil Hall,* expressing his readiness to attend to any instructions on subjects wherein he might be of service to the science of Astronomy, in his intended voyage to the South Seas. They availed themselves of the offer of this intelligent and enterprizing officer, and requested his attention to the following principal points.

To observe, as frequently as possible, the conjunctions of the moon and planets with the fixed stars; measuring with a micrometer their differences of right ascension and declination, or taking the measure of their distance in a straight line: the time and place of observation being correctly noted.

To look out for occultations of the fixed stars by the moon; and particularly for those which may be presumed to be of short duration, with a view to the illustration of the theory of Cagnoli respecting this mode of determining the figure of the earth. And it was remarked to him that, as the moon was now, and would be for some few years, in such a position with respect to her nodes as to pass over the *Pleiades* every lunation, it would be particularly desirable to look out for the occultations of those stars.

To make frequent sweeps of the heavens, with a telescope having a large field of view and small magnifying power, for the purpose of discovering any comets; and to note the progress and circumstances of the same; making sketches of their appearance. The Council at that time were not in possession of the calculated place of the comet which is expected to return in 1822. But, having since received an ephemeris of its apparent positions, computed by M. Encke, they will endeavour to forward it to Capt. Hall, and request him to look out for the same: a circumstance the more to be desired, since it is expected to assume a different appearance in the southern hemisphere to that which it will present in Europe.

It appeared unnecessary to remind Capt. Hall of the several eclipses of the sun and moon, together with the eclipses of Jupiter's satellites, the transit of Mercury over the sun's disc on Nov. 4, 1822, and the several phænomena noted in the various ephemerides; and which would of course be the object of his attention without any particular suggestions from this Society. But the Council took the liberty of directing his attention to certain points, should he

be favourably situated for observing any of the solar eclipses, or the transit of Mercury.

They also requested him to make observations on the position of Mars, at the time of his opposition in February 1822, with respect to the three following fixed stars, which are situated near the path of his orbit, and whose mean places for the day of opposition are here stated: viz.

Star.	Æ. in time.	N. Dec.
42 Leonis	h ′ ″ 10 . 12 . 15,9	° ′ ″ 15 . 52 . 11,9
446 Mayer	19 . 17,6	15. 1,0
i 46 Leonis	22 . 41,6	2 . 53,3

When Mars approaches either of these stars, the differences in right ascension and declination, between the planet and the star, must be taken as accurately as possible for several successive days, with a micrometer; or their distances measured, in a straight line: the time and place being correctly noted down. Such observations, compared with corresponding ones made in England, will serve to determine the parallax of Mars.

With the same view it was proposed to Capt. Hall, to make observations on Venus, at the time of her inferior conjunction in March 1822, by comparing her with α Ceti, on the parallel of which she will be a day or two before and after the conjunction: a simple and easy method being at the same time suggested, whereby Venus might be readily found in the day time, notwithstanding her proximity to the sun.

The attention of Capt. Hall was also directed to the subject of refraction, in order to determine whether the quantity of refraction varies (cæteris paribus) in different parts of the globe; or whether any new light can be thrown on this uncertain phænomenon, in the various places he might visit.

Upon most of these subjects, it must be evident to the members of the society, that there is a necessity of having corresponding and simultaneous observations

in this country: otherwise the labours of Capt. Hall (should he be favourably situated for observation) will be in a great measure lost to the public. Those members, therefore, who possess the requisite instruments, are called upon to co-operate on these points, and to register their observations, in order that they may be compared at a future opportunity. Without this assistance the efforts of the Council will have been exerted in vain, and the time of an active observer employed to little or no advantage. And here the Council cannot avoid suggesting, to those astronomers who possess the requisite instruments, the propriety of observing and recording the position (in right ascension and *declination*) of those stars which are situated near to, and on the same parallel with, any of the planets near the time of their opposition; since such observations would serve as a mode of comparison for those observers who, with less powerful instruments, might be more favourably situated for making observations at this important point of the orbit of the planets.

The Council further represented to Capt. Hall that it was scarcely to be expected that a traveller, who is frequently changing his situation, can make many *fundamental* observations in astronomy; such as may serve as a basis for future researches. But, that he might do much in *comparative* astronomy; by taking those elements as correct which have been determined by observations made in fixed observatories, and comparing the objects of research therewith: some examples of which have been already alluded to; and others were suggested for his consideration.

But there was still another, and a very important point, to which the attention of Capt. Hall was requested (and the same cannot be too strongly pressed on any future voyager, or settler in distant climates, favourably situated for such inquiries): namely, to make regular observations on the tides, in favourable situations for determining their theory. It is well known that the tides, adjoining large continents and their contiguous islands, are so affected with the various sources of error arising from the situation of the harbour and the nature of the bottom of the ocean for a considerable distance around it, that not only a very long series of observations is required to destroy or compensate those errors, and allow the true co-efficients of the formulæ, for determining their value, to appear with any tolerable exactness; but also the co-efficients themselves, so determined, are essentially affected by such local peculiarities; and consequently incapable of affording any thing beyond relative results. In

order to obtain results unaffected with these inappreciable causes of error, the places of observation should, if possible, be chosen on small islands shooting up abruptly from an unfathomable depth in the midst of a wide ocean, extending 30 or 40 degrees, at least, in all directions; or, at all events, a very great distance from any large continent. The islands in the Pacific and South Atlantic oceans, which are bedded on coral banks or the effect of volcanic eruptions, are precisely of this nature. If we may trust the accounts of voyagers, many of these are mere vertical shafts, or insulated columns, shooting at once from the very bottom of the ocean, without shoals, or any gradual declivity. Round these, the tides must rise and fall with perfect uniformity: and it is exceedingly probable that, in these cases, a much shorter series of observations would be requisite for framing accurate results; and that even those of a single month, in moderately calm weather, might have considerable value in the present improved state of the theory. The situation of the Gallapagos islands, on which Capt. Hall will probably spend some time (it being one of the stations at which he proposes to swing the invariable pendulum), possesses peculiarities which entitle it to notice, although it does not satisfy all the conditions. It is immediately under the equator: and should he be there about the time of the equinox, the very vertex of the aqueous spheroid, which will then pass over the spot, may be made the subject of his observations. These islands likewise present another remarkable peculiarity of situation. For, according to the results obtained from a theorem given by M. Biot, they stand within a very few degrees of the point where the magnetic intersects the terrestrial equator. It is therefore desirable that observations should be made with a view to ascertain the accuracy of this conclusion. It may also be remarked, that it is near this spot that the magnetic equator is supposed to deviate into the serpentine form, as mentioned by the same eminent writer.

The formation of an Astronomical Library being one of the objects of the Society, the attention of the Council has been directed thereto. At present, however, they have not thought it advisable to expend any money on this department. Nevertheless they are happy in announcing that the foundation has been laid, by the donation of many valuable works on the science by several members of the society.

It having been represented to the Council, that the observations made at

the observatory of the East India Company at Madras were preserved in the library of the East India Company, application was made for them to the Court of Directors. With a truly laudable zeal for the cause of science, the Court immediately assented to this request; and ordered not only that the present observations for a series of years (commencing with the year 1793, and continued with some slight interruptions to the present time) should be deposited with this society, but that the future observations should be forwarded in like manner. Those observations, accompanied with many other valuable papers on astronomical subjects from the same quarter, are now in the library of the society.

Since the last general meeting of the Society the Council have considered the expediency of recommending a few alterations in the Regulations. And they have unanimously agreed to propose: First, that no person, who has filled the office of President for two successive years, should be again eligible to the same situation, until the expiration of one year from the termination of his office; similar to the present regulation respecting the Vice-presidents: Secondly, that the six members of council alluded to in Section 2. Regulation 2. should be extended to eight, and consequently that four should be re-eligible in every ensuing year, instead of three only: Thirdly, that any person who may be desirous of reading his own paper to the society, may be at liberty so to do: Lastly, that the mode of calling special general meetings should be so far altered, as to direct that the subject to be discussed at such special meetings shall be previously laid before the Council; and, if the decision of the Council be not satisfactory to the members proposing the subject, to make it imperative on the Council to call a special general meeting within a given period. Distinct resolutions, on each of these points, will be submitted for your consideration*.

During the last year the Council have considered the propriety of appointing Committees for various purposes: amongst others, one for determining on a set of questions to be proposed to persons possessing astronomical instruments, in order to ascertain the merits of the same; and another to determine on the expediency of procuring tables of the apparent places of the 46 Greenwich

* These resolutions were carried unanimously; and the Regulations have been altered accordingly in the present edition.

stars for every day in the year. But, at present, nothing decisive has been effected on these points.

One of the objects of this Society being an examination of the heavens in minute detail, the Council have likewise frequently discussed this subject, but without being able to agree on a plan, proper to be recommended for the adoption of the members. They consider it, however, a subject of so much importance, that they will early resume it: for, until every remarkable star in the heavens is recorded, and its place assigned in the catalogue, it is vain to pretend to an accurate knowledge of the true system of the universe.

It is gratifying to observe that the advantages, likely to acrue to science from the establishment of a society like this, appear to be duly appreciated by the continental astronomers. From many of them (whose names, among the most illustrious in modern science, now stand on our list as associates, or are suspended in our meeting room as candidates for admission) letters have been received by the foreign secretary, expressive not merely of the most unqualified approbation of our objects, but of a desire to co-operate actively in their accomplishment: and which has been evinced in more than one instance by the communication of interesting notices, and astronomical works.

On the whole, the Council cannot view this new impulse which appears to have been given to astronomy in all parts of the world, without anticipating the most beneficial results to the science. The establishment of several new observatories on the continent of Europe (one of them above the 60th degree of north latitude) under the direction of men eminent in science, and vieing with each other in the most honourable branch of emulation—the rising efforts of our countrymen in the East Indies—the zeal of our brethren on the American continent—the foundation of a public observatory at Cambridge and another at the Cape of Good Hope (both so honourable to our own country)—must ensure the good wishes of every friend to science, and excite the admiration of every reflecting mind.

PAPERS.

I. *An Account of the Repeating Circle, and of the Altitude and Azimuth Instrument; describing their different constructions, the manner of performing their principal adjustments, and how to make observations with them; together with a comparison of their respective advantages.* By EDWARD TROUGHTON, ESQ., *F.R.S., and Member of the* AMERICAN PHILOSOPHICAL SOCIETY.

Read January 12, and March 9, 1821.

OF all astronomical instruments, those fixed in national observatories must be considered of the first importance to science; and in a commercial country, like our own, perhaps those subservient to nautical astronomy ought to be regarded as the next in point of utility. Those which I would call the third class are numerous; they are such as are used in the small observatories of the amateur, to which they are in general equally adapted, as to the service of the gentleman who may travel to foreign parts. Of those, the two I have named in the title, are the most approved of for these purposes; and to draw up a comparison of their respective constructions and merits, is what I have chosen for the subject of this communication. Were I able to treat it as it deserves, I should entertain no doubt of its coming within the views of this

society, nor of its usefulness; particularly in assisting those, who may not already have become acquainted with the different kinds of instruments, in the selection of such as may be best suited to their purposes.

The repeating circle, till within these few years, has been very little used in this country, and in truth its merit but ill appreciated; facts however are not wanting, although dispersed and insulated, sufficient to remove all prejudice; particularly experiments recently made, with a small instrument of this kind, at the principal stations of our grand national survey. On the continent of Europe, where the art of graduation is not so successfully cultivated as it is with us, an instrument, which of all others depends the least upon accuracy of division, could hardly escape being too much commended: be this as it may, observations lately made on the other side of the British channel, simultaneously with those used in the survey mentioned above, have I believe given the best informed of all parties a more correct idea of what may be expected from this instrument.

The altitude and azimuth instrument has I think been almost exclusively made in this country: many of them have been sent abroad, but from their not having been used in great national operations, the advantage of them has seldom been made known to the world. Nearly the same may be said of those which remain at home; for although some of them have been much and skilfully used, yet owing to their having been only in the hands of private individuals, who had no common medium of communication, the labours of those who possessed them have hitherto been almost lost to astronomy. From this general remark I must however except the observations of the 36 brightest fixed stars, which Mr. Pond made at Westbury with a 30 inch circle of this kind, and which appeared in the *Phil. Trans.* for 1806. This indeed was the first thing (notwithstanding some doubts and surmises from abroad) that unequivocally demonstrated a change of figure in the Greenwich quadrant, and subsequently led to the procuration of new instruments for our national establishment.

The repeating circle has by no means failed for want of publicity; on the continent, astronomers and others have written a great deal about it, and the results of thousands of observations have been published; the greater part of which were made on *Polaris*; a star, to which, on account of its slow motion, this instrument is peculiarly adapted. Although the altitude and azimuth instrument, as a portable one, was produced about the year 1792, we find no description of it in print, until the article Circle appeared in Rees's and

BREWSTER'S Encyclopedias*; the latter of which is referred to for the use of those who may wish to see a more detailed account of both the instruments under consideration, than can be given in the following brief descriptions.

Description of the Repeating Circle.

The lowest part of this instrument is a strong tripod, having at its extremities three steady foot-screws; one of which, at least, should stand upon a well known apparatus, for the purpose of supplying a slower and finer motion to the upper part, than can be given by the screw itself. This apparatus should support that particular foot which during observation is directed to the meridian, or is opposite to the object observed. In the centre of the tripod is fixed a strong vertical axis, of a height sufficient for allowing head-room for observing conveniently when the telescope is pointed towards the zenith. A pillar of the same height with the axis, is nicely fitted at both ends upon the latter, and both together, when the axis is vertical, produce a steady azimuthal motion. To the lower end of the pillar is fixed an azimuth circle: and to the higher end, a cross piece; on the two extremities of which stand, about five inches apart, two upright bars for supporting a cross axis, to which the principal circle by its centre-work is attached, and round which axis the circle may be turned into any position from one side of the pillar to the other. A semicircle is fastened to one end of the cross axis, which, together with a clamp attached to one of the upright bars, affords the means of securing the circle in any position. The principal circle, or that of repetition, has (affixed to the middle of its plane, and opposite to the one divided) centre-work, the length of which is equal to about two-thirds of the diameter of the circle; the outer part of which, being perforated from end to end, becomes the socket for an axis of the same length, to which the index of the circle and telescope is attached in front. The index has four branches placed at half right angles to the telescope, each of which subdivides the divisions of the circle into spaces of 10″. To the middle of the cross axis is fixed a socket, which receives about two-thirds of the length

* It is true that the late Rev. Francis Wollaston, in the *Phil. Trans.* for 1793, gave a description of a two-feet Circle which had an azimuth. That instrument, however, was solely designed for a meridian one, and was in fact quite unfit for any other purpose. The same gentleman, in the Appendix to his *Fasciculus*, points out the best means of using an altitude and azimuth circle (properly so called), but without giving any description. The Westbury circle described in the *Phil. Trans.* for 1806, although well constructed for observing azimuths, was not designed for taking transits, and besides was not a portable instrument.

of the centre-work: and the exterior surface of the remaining third of that work becomes the axis for another telescope and a level to revolve contiguous to the back of the circle. This is a complicated matter, difficult to be described or understood without a figure: it will however be sufficient, if it is conceived that there are three concentric motions in planes parallel to that of the circle: namely, a general one within the socket of the cross axis, which carries round together, the circle, level and two telescopes; another, by which, upon the exterior part of the centre-work the level and back telescope revolve; and a third, that gives motion to the fore telescope and the verniers, so as to make them advance upon the circle, which is produced from the interior axis. These motions are independent of each other, and are all furnished with clamping and tangent screws. A counterpoise is placed upon the exterior end of the centre-work, which, by balancing the circle, telescopes, and level, keeps them stationary in any position. The greatest part of these instruments, which have been constructed in London, have the back telescope on one side of the axis, and the level parallel to it on the other side, which latter, being made heavier than would be otherwise required, becomes a counterpoise for the former, a thing not attended to in the earlier constructions of the repeating circle. The azimuth circle of this instrument, only just named above, was in the first construction small, and of no other use than to point out roughly when the upper circle had been turned half round; but, in most of those made in London, to that circle has been given the same radius, and the same attention paid to its execution as to the upper one. In the best construction of this part the circle is attached to the tripod, and three indices fastened to the vertical pillar revolve round it; thus may a horizontal angle be taken on three equidistant parts of the circle, and, what is of equal importance, by simply reversing the position of the telescope and turning half round in azimuth, a similar observation may be made, in which the readings will fall at 60° distance from the former ones. By this double operation simple errors of division may be considered as very much diminished, each sight having been read off on six places; and in both parts of the operation the error arising from eccentricity is, as to any sensible quantity, totally done away.

Description of the Altitude and Azimuth Instrument.

The lower part of this instrument, like the other, consists of a tripod and feet-screws; which latter, being a recent contrivance, and hitherto unde-

scribed, may in this place deserve particular notice. Each of the three screws is double; that is, a screw within a screw: the exterior one, as usual, has its female in the end of the tripod, and the female of the interior screw is within the exterior. The interior one is longer than the other, its flat end rests on a small cup on the top of the support, and its milled head is a little above the other. Now by this arrangement we gain three distinct motions; for, by turning both screws together, an effect is produced equal to the natural range of the exterior screw: by turning the interior one alone, the effect produced is what is due to this screw: and by turning the exterior one alone (which may be done, because the friction of the interior screw in the cup is greater than that which exists between the two screws) an effect is produced, equal to the difference of the ranges of the two screws. Thus, were the exterior one to have 30 turns in an inch, and the interior 40, the effect last described will be exactly equal to what would be produced by a simple screw of 120 threads in an inch. This is an improvement applicable to all instruments that are supported on screws, and of course to the repeating circle. A few of the last made in London possess this advantage. The vertical axis of the altitude and azimuth instrument is fixed in the centre of the tripod, of a length equal to about the radius of the circle. At the lower end is centred upon it the azimuth circle, in close contact with the tripod; to the three branches of which it is fastened, but in such a manner as to admit of a circular motion of about 3°, which motion is governed altogether by a slow moving screw. The intention of this motion is, in geodetic operations, to bring the zero of the circle to the point of commencement; and, in astronomy, to place that point exactly in the meridian. In an instrument for my own use, however, I could dispense with this adjustment, because I know that it is easier and more accurate to *read off* than it is to *set:* and from what point I begin to reckon, is a thing quite indifferent to me. Just above the circle the axis is embraced by a cone, which is also well centred upon the upper end. To the lower end of the cone is fixed an entire circular plate, formed in the strongest manner; which not only bears the two or three microscopes that read and subdivide the divisions of the circle, but also supports the whole of the upper works. On opposite sides of the cone, and distant from it about half the radius of the plate, are erected two columns, of a height to support a transit axis, so as to allow the telescope to pass the upper end of the vertical axis, when it is pointed towards the zenith. The transit axis is one-third longer than the distance between the columns, upon which *out-riggers* are placed, having Y's or

angles on their extremities, that support the axis: and each of the angles is acted upon by an adjusting screw, not only for making the transit axis horizontal, but also for placing the centre of the circle of the same height with the horizontal microscopes. The horizontal axis is crossed, as in an ordinary transit, by a telescope; the length of which exceeds the diameter of the circle by about one-third. The circle framed upon the axis is double; the two parts being placed at a distance from each other to allow the telescope a lodging between them: and they are connected with each other by pillars inserted perpendicularly between them. The front portion of the circle (or that which bears the division) is of a less radius than the other by about one-eighth part of an inch: the longer radius of the portion behind is what is required for the clamp and screw for slow motion to act upon, while the shorter radius of the one in front keeps it clear of that apparatus when the axis is reversed for collimation. Many of these instruments have been constructed with vernier readings; but as I consider those by the microscopic micrometer preferable, I shall confine this description to the latter. My preference to one of those excellent contrivances for minute subdivision is mainly grounded on the circumstance that, in the employment of the more ancient method, the indices rub against the divisions which they subdivide; whereas in the modern, which is detached, the motion is free and unembarrassed.

In the description of the repeating circle, the advantage of three readings was stated; but that contrivance originated with the instrument I am now describing: and if it be a real improvement (which I believe no one will doubt), the repeating circle owes the advantage solely to the latter. Three readings are not only better than two, but also better than four: for, with four, when the objects are in the horizon, or near it, on reversion the opposite indices only change places; a circumstance clearly in favour of the odd number. But, in astronomy, where the upper circle is chiefly concerned, the same advantage does not occur: for, at the zenith, on reversion, the telescope changes place in azimuth only; therefore, as the indices have no change of place, more readings than two could be of no use. It is true that in proceeding downwards we gradually come to the horizon, where the same effect, that was stated respecting the azimuthal angle, takes place: but here the uncertainty of refraction destroys all confidence. I may also state that three readings to a vertical circle cannot be all equally well illuminated in the night time; nor at any time are they to be read with equal convenience, as is the case where two readings are placed horizontally. However, as microscopic read-

ings are expensive, and as astronomy is generally the chief object of those who procure this instrument, two microscopes to each circle may be sufficient. But were I to have for my own use an instrument of this kind with verniers, the lower circle should have three, and the upper one four.

When the vertical circle has two readings, and these microscopic, they are affixed to the ends of two horizontal tubes fastened to one of the columns; which also support a good hanging spirit level. Another level of the best quality is occasionally applied to the pivots of the transit axis, in order, independently of every thing else, to verify its horizontality.

Adjustment of the Repeating Circle.

The vertical axis is made perpendicular by means of the feet-screws of the tripod, and the spirit level, in the same manner as is required for other instruments; an operation so easy and well known, that to mention it is all that seems necessary. To adjust the collimation of the telescope parallel to the plane of the circle, an object should be chosen as nearly in the horizon as can be estimated: the middle wire of the telescope under adjustment, being correctly pointed to the object, what is shown on the indices of the azimuth circle must be carefully noted. Reverse the telescope both vertically and horizontally, bisecting again the same object with the same wire, and again read off what the indices give. Take of these readings the mean, or middle point, and set with great care the indices so as to show that mean. Now, by the screws, which act upon the wire-plate, move the wire so as to make it bisect the object: this being well done, the other telescope, to be adjusted, wants only its vertical wire moved in the same manner till it bisects the object. The above, however, is true only when the object is very distant; for, as both telescopes are eccentric, as respects the vertical axis, and unequally so, it becomes necessary, when no remote object can be seen, to put up marks, say two circles, the radii of which are equal to the eccentricities of the respective telescopes. The next essential adjustment, is to place the plane of the upper circle vertical; or its axis horizontal. The best practical method of doing this, and which is quite equal to the purpose, is to look with the front telescope at any elevated object, whether remote or near, and having made the middle vertical wire bisect it, look at the same object when reflected from the surface of a fluid. If the wire does not cut the reflected image, the circle must be turned round the cross axis, to bring the wire as nearly as can be

estimated half way towards that image; now by turning the instrument in azimuth, make the bisection, then elevate the telescope to the object, and if the bisection is not perfect, the operation of estimating and turning in azimuth must be repeated. A level, which is placed parallel to the axis of the circle, must now be adjusted so that the bubble may stand in the middle of its tube; which afterwards becomes the index for the vertical position of the circle. Another adjustment, which is not however of so much importance as either of the former, is to make the cross axis at right angles to the vertical one; which is indeed the business of the maker. If, when the vertical axis is adjusted, he brings the upper circle horizontal by means of a pocket-level, which is to be placed upon the face of the circle at right angles to this axis, then, by placing the level parallel to the axis, he will see which of the supports wants to be shortened by the file.

Adjustments of the Altitude and Azimuth Instrument.

The axis of azimuth is rendered vertical by means of the level and feet-screws; exactly in the manner that was required in the other instrument; and it may be stated that either or both of the levels belonging to it may be used for this purpose. That adjustment, which answers to the second for the repeating circle, or setting the line of collimation perpendicular to the axis, is no other than the usual way practised for doing the same thing in a plain transit; namely, by moving in azimuth bring the middle vertical wire to any object, then reverse the horizontal axis end for end upon its supports, and if in this position the wire does not cut the same object, alter one half the error by turning in azimuth, and the other by means of the screws which act upon the wire-plate. The transit axis is brought to the horizontal position simply by placing the level upon the pivots of the axis, and observing if the air-bulb changes its place on turning the level end for end. If it does, nothing more is wanted to effect the adjustment, than with the screw below either of the pivots, to bring the bubble, according to the indication of the divided ivory scales, just half way towards the place which it occupied in the first position.

Both the circles under consideration require many more adjustments: but as those belong to the minor parts, and are common to many instruments, even to enumerate them in a paper like this, could hardly answer any useful purpose.

Manner of using the Repeating Circle.

In geodetical observations this instrument gives the angular distance between two observed objects, whatever be their elevation above, or their depression below the horizon. The horizontal angle is always the thing wanted; to obtain which, it is necessary to find by observation how the objects are situated respecting the horizon; these give the requisite data for trigonometrical computation. Previous however to this, the observed angle itself has to be corrected for the eccentricity of the telescopes; which correction varies according to the quantity of eccentricity, and the measured, or estimated, distance of the observed objects. To place the plane of the repeating circle parallel to the line that joins two objects, the angular distance of which was to be observed, had been no easy task, until about thirty years ago from my little gazebo I attempted to take the angular distance of two spires. Their distance was by no means my object; it was simply to acquire the habit of observing by repetition, and putting to trial an instrument that I thought well of. After having made three attempts, without effect, to obtain the thing wanted, and a fourth placing me still further from the point, I quitted my instrument, disgusted at my own unskilfulness, and retired to consider whether the instrument had not within itself some principle from which a precise rule might be made out. This inquiry proved successful, for I saw that by pointing one foot of the tripod, the cross axis and the back telescope towards one of the objects, the fore telescope by turning round the cross axis and by its own proper motion might be brought to the other object without altering the angular direction of the back telescope. The rule is this. Set one foot of the tripod as nearly as you can guess in a line with that object of the two, which you judge to have the least elevation or depression; and with the plane of the circle vertical, and the back telescope horizontal (both to the exactness of two or three minutes), bring the back telescope to the object, partly by turning in azimuth, and partly by turning or propping the foot-screw. Next turn the circle round on the cross axis, until it seems to the eye to occupy the proper position; then a second time bring the back telescope to the object by the foot-screw, and turning in azimuth; lastly, complete the operation by bringing the upper telescope to the other object by its own proper motion in conjunction with that of turning round the cross axis. The above operation being performed (which it is necessary to repeat at every angle that is taken, even

at the same station,) the business of observing by repetition may be commenced as follows. Set the fore telescope to zero, as is usual; or what is better, as was said before, read off what the indices (being clamped) happen to show; and, by turning round by the general motion, place the intersection of the middle wires exactly on the object to the left. Then, by its own motion in the same manner set the back telescope to the object on the right; and examine if the angle between the objects be accurately comprehended between the two telescopes. Now by the general motion, without touching any thing else, move the back telescope until its wires coincide with the object on the left. To complete the first operation unclamp the fore telescope, and carry it round to the object to the right; when its indices will have advanced upon the graduated limb through an arc equal to double the angular distance of the objects. To read off this double result would be rather detrimental than useful; instead of which, with the fore telescope fixed at this position, the three steps of another operation, as described above, should be taken in order to obtain a second double result. A third, fourth, &c. course of operations must succeed, until it is judged that sufficient has been done to produce the accuracy required. At last the indices must be read, and the total number of degrees, minutes and seconds, that have been passed over by the indices, taken and divided by double the number of operations; when the simple angle between the objects will appear. If all the results had been read, the intermediate errors of division would have come into the account, and produced an effect that has been avoided by the process described: for, except at the beginning and end, the observations were carried on as if there had been no divisions. It is in this solely, that this instrument claims an advantage over others, and justly; for they have a beginning and end to every double result; but this, as far as graduation is concerned, has only a beginning and an end to a whole course of observations. In geodesy, the levels are of no use, except in the operation of bringing the plane of the circle into that of the two objects: and it may be observed here, that in astronomy the back telescope is altogether unnecessary.

To observe zenith distances of the heavenly bodies by repetition, is a process so similar to what has been described, that a shorter course may be taken to explain it. The instrument being adjusted, and the indices set or read, by the general motion (the level being horizontal) bisect the star and examine that both are correct at the same time: now turn the instrument half round in azimuth, correcting with the foot-screw the position of the level if required, and move the telescope by its own proper motion to the star again, which will

cause the indices to pass over an arc equal to twice the zenith distance. Again turn the instrument half round in azimuth, and with the telescope fixed in the last position, by the general motion again bisect the star, and again by its proper motion make the level horizontal: now turn half round in azimuth, correct the position of the level as before, and in order to come at another double zenith distance, carry round the telescope to the star again. This process having been continued until enough has been done, the total arc passed over by the indices, divided by double the number of complete operations, gives the zenith distance of the star. There is indeed another way of observing by repetition with this instrument. For the same effect will be produced, if, instead of turning half round in azimuth, the circle be turned to the other side of the pillar, on the motion of the cross axis. But, in this case, there must either be a stop to prop the circle on the other side when its plane is vertical, or else the level must be a hanging one, which will give the circle its vertical position whether it is above or below the axis. It would be altogether unnecessary to describe the process of repetition in this case; for, except in what has just been stated, it differs not from the former one. A nominal difference indeed takes place; for the former method proceeded by stops of double zenith distance, and this proceeds by stops of double altitude.

Manner of using the Altitude and Azimuth Circle.

In geodesy this instrument, being adjusted, measures without further trouble angles between objects upon the horizontal plane, whatever may be the number required to be taken at one station. The parts of the instrument are all concentric, and therefore whatever it gives, whether the objects are remote or near, requires no correction. The angles having been read off individually will be vitiated by the errors of division; and even in graduation that may be deemed good, those errors may be too great in some cases that occur. Such cases are well known to the judicious surveyor, and may be obviated by simply turning the whole instrument upon the stand, and setting the axis vertical; again and again taking the angles which he wishes to ascertain with the utmost accuracy: for, by this expedient, he will get any angle upon as many different parts of the circle as he pleases or thinks necessary. This may at first view appear to be a principle derived from the other instrument; but, let it be remembered that it is no more than taking *means*; a thing practised and well understood before the repeating instruments were brought into existence.

In astronomy the azimuth circle is of little use, except in furnishing ready means of bringing the upper circle into the plane of the meridian; it has however been used for making out, in conjunction with the other circle, the quantity of refraction at different altitudes: but as, perhaps, the times for this purpose furnish better data than azimuths, to mention the matter is sufficient. Observing altitudes of the celestial bodies is a thing so familiar to every one who is in the least acquainted with these subjects, that to describe it would only be to lengthen this paper. I may however remark that in meridian altitudes, there is time to get an observation with the face of the circle to the east, and another with its face to the west; which together give a collimated double result, before the diminution in altitude becomes sensible. Yet I feel somewhat diffident about recommending this mode of observing, although I have practised it myself with success. An observer, before he attempts it, should be expert both in managing the instrument and reading off the angles. Respecting stars near the equator, there is only about one minute of time on each side of the meridian, that the star would continue to be bisected by the wire of a telescope that magnifies sixty times; therefore, before the double observation is attempted, the time should be accurately known. In truth, all hurried operations with this instrument may be avoided and left to the repeating circle.

Comparison of the two Instruments.

I come now to compare the two instruments with each other in their application to the different purposes for which they are designed; and to prevent the frequent recurrence of long names, I shall call the repeating circle R, and the altitude and azimuth instrument A; an expedient which, perhaps, if sooner adopted, might have improved this paper.

To find the difference of latitude between distant places is a most important problem in extensive geodetic operations, and for this purpose R has often been represented as equal or even superior to the zenith sector: but as the latter instrument has always been constructed with a powerful telescope, and is in its nature the most simple possible, I beg leave to dissent from that opinion. For, however an instrument may be constructed, or in whatever manner it may be used, I have no faith that it can give results nearer the truth, than a quantity that is visible in the telescope. To do this, an instrument, with respect to any error that is not corrected by reversion, must be perfect; a thing of which I have no idea. Yet every one is not of my opinion. As an

instance of this, a celebrated astronomer, a few years ago, in the south of Europe, made observations for finding the attraction of a mountain with a small instrument of the construction R; and obtained a deflection of the level equal to two seconds; and, although his telescope could not have been more than 15 inches long, from this experiment brought out a density of the earth nearly coinciding with the Schehallien experiment, and with the more recent one which Cavendish obtained by direct attraction. Yet has that same astronomer, from later experiments, found the *fixed error* of the same or a similar instrument (made by a foreign artist by no means unknown to fame) to amount to from five to ten seconds. Should this error, however, which is called fixed, turn out to be at all fortuitous, it is possible that, taking five for its amount (to say nothing about ten), a result might have been obtained of an equal quantity contrary to attraction. But on the other hand, were the error of such an instrument absolutely fixed, although it gave the altitudes incorrectly, yet might it give the differences correctly; and consequently the above dwarfish experiment might be permitted to stand on its own little base. It is a circumstance little suspected, and not very easy to explain, how an instrument, that is capable of reversion, can have errors that are not correct in the double result. But although this, like the above statement, and the remarks I have made upon it, may appear digressive, yet will it not be altogether useless to inquire into the nature of two of the most obvious sources of error, to which this instrument in particular, and others in a less degree are subject.

It is not perhaps so well known as it ought to be, either to observers or artists, that the air-bulb of the spirit level changes its position with a difference of temperature. This, it is probable, is wholly occasioned by the glass tube (I mean from its expansion) being larger at one end than at the other: for I have observed that as the temperature increases, the bubble always deviates towards the larger end, and as it diminishes, the deviation takes place the contrary way. The error occasioned by this cause, whatever it may amount to at the end of a series of repetitions, will be divided, and affect the result by no more than the mean deviation belonging to a pair of observations. This kind of error is fixed, as far as a single course of repetition is concerned; but, when the change of temperature is reversed, an error equally fixed will affect observations in the opposite direction.

Another and still more fatal source of error to which R is peculiarly liable, arises from the resistance of the centre-work to the action of the tangent-screw. This will be more or less, according to the care and judgement that have been

employed in the construction of the instrument: but to bring it to nothing must for ever be beyond the skill of the artist, because it lies in some measure in the nature of the materials. I will now try to explain how this source of inaccuracy affects observation, and to this end will suppose that an observer, when he bisects a star, moves the index with the tangent screw, so as to advance it on the graduated limb according to the numbering of the degrees; the telescope indeed will be properly pointed, but the index will show too much. On reversing the position of the instrument, the telescope will naturally relieve itself from the friction at the centre, and take the position due to the index, instead of retaining that which it had when the star was bisected; therefore the following motion of the telescope will begin from a wrong point; and if the screw is always turned in the same direction, will produce an error in excess at every pair of observations. As much may be said respecting the motion of the level, which, on account of its socket embracing the exterior of the axis, will meet with a greater resistance than the telescope did. As far as these two parts of repetition are concerned, it is clear that the habits of an observer may make this source of error either fixed or accidental, and by constantly turning the screws in a particular way, he may have for a fixed error either their sum or their difference. There is another motion equally connected with the operation of repetition, and quite as liable to this kind of error; namely, the general one, which carries round together the circle, the telescope and level. To follow up these three sources of error, and show how in their different combinations a series of repetitions would be affected, is a task which I dare not attempt. Instead of which I will content myself with remarking, that the complicated centre-work of this instrument, and the different motions having a tendency to drag each other, together with the change of position that takes place without the different steps being registered, subject it to greater errors, when the graduation is tolerable, than those which it professes to correct. To examine whether the screws, which govern the three motions here treated of, give immediate motion to the respective parts which they act upon, is a good criterion, but I think incapable of detecting a quantity of error less than two or three seconds; which quantity, if it recurs as fixed, will not be divided in obtaining the general result, because it affects all the operations, and it is the accumulated error only that is reduced.

In the instrument A, the resistance of the centre-work is extremely small; the pivots of the transit axis merely resting on their angular supports, while the screw for slow motion acts immediately upon the circle, carrying also the tele-

scope which makes one piece with it. This is the simplest action that an instrument can possess; the forces in opposition to each other are quite inconsiderable; there is no rubbing of indices; and the angles are read off immediately after observation without any thing having changed place. The error of the level, mentioned before, will also affect this instrument: but, by judicious management, as will be shown presently, may be almost wholly counteracted. The method indeed would apply to R, but at an expense of time that could not be admitted. It is this: reversing the position of the circle in azimuth, by the level, place the axis truly vertical, which then becomes a substitute for the level during the short time a star is bisected; after this observation is read off and the telescope set for another star, I would note the position of the level, and again reverse the instrument: when, if any deviation is seen, I would bring the level by the foot screw half-way to the point where it stood before. This operation places the axis vertical again, whether a deviation of the level or of the whole instrument has taken place. To do this in the easiest manner, is to make a series of observations with the divided side of the circle to the east and west alternately; this way saves half the trouble of frequent reversion, renders observations independent of each other, and makes it resemble in its use the zenith sector, as nearly as one instrument with a level can do a better with a plumb-line.

When A is used for ascertaining the value of a celestial arc, corresponding to a measured distance on the surface of the earth, many stars should be marked for observation, none of which ought to be more than 60° from the zenith; and to avoid hurried observation, they should be picked from a catalogue at nearly equal distances of right ascension. In doing this there can be no difficulty, because those of the fourth or fifth magnitude are for this purpose as good as any others. As to the time required for observing a star, different observers will want more or less, but surely the space of five minutes is enough for any one. If he has previously written down, from his catalogue, the degree and first figure of the minutes, he has no more to do in reading off than, when the star is bisected, to put down the last figure of the minutes and the seconds as he takes them from the micrometers. For the next star he should set to what his catalogue gives, then reverse the position of the instrument by the divisions of the azimuth circle, afterwards correct for any deviation of the level and verticality of the axis, and then wait for the appulse of the star to the meridian. In every pair of observations, the place of a star is read off on four points of the circle; and, where those marked for observation extend to 60° of zenith distance both to the north and south, 240° of the circle are employed.

To extend it further could produce no good effect, for the uncertainty of refraction would counteract the advantage arising from a greater range. In using A, if an observation is hurt by a passing cloud, or any other circumstance that may render it doubtful, it had better not be written down, even should it agree with the general result; and, on the other hand, if the observer is satisfied with an observation, it should not be omitted for being somewhat discordant. With R, if any one of the steps of a series of repetition is not well performed, the whole will be vitiated should the observer go on. He may perhaps be able to save what has gone before, by reading off what has been done, and beginning a new set; but this cannot be done in all cases. I remember an instance, where, in a series of twenty double observations on *polaris* (that had been made with extraordinary care), by the mistake of the assistant, in the very last of them, by his turning the screw of the indices instead of that of the level, the whole labour was irretrievably lost.

When an observer is so circumstanced as not to be able to stay at a place more than a short time, R seems to have the advantage; for, in a few hours a series of repetitions may be made on *polaris* to answer his purpose; since the polar distance of that star is now so well known, that little will be lost by not having opportunity of observing it both above and below the pole. A is not, however, without its resources in this respect; for in stars that have not more than 25° of polar distance, there will be quite time enough for reversing the position of the instrument, and obtaining collimated results. I must here remark that this can only be done in high latitudes, because near the equator, the slow moving stars are too much affected by variable refraction to be relied on in nice matters. This last remark must be placed very much to the discredit of R, which in its nature is adapted to the slow moving stars alone.

Thus it has been shown that, respecting its most effective operations, the repeating circle is local with regard to both the heavens and the earth. It must be granted that at every step, even when a quick moving star is observed, if the time has been noted, the whole may be reduced to the meridian: and were it wanted, which it is not, I should claim the same concession in favour of A. But let it be remembered, that at any considerable distance from the meridian, the time becomes so important a datum, that it requires an exactness, scarcely to be found, except in a well regulated observatory. Therefore, in observing stars near the equator with R, the observer has the choice of two difficulties; he may either extend his observations to a distance on both sides of the meridian to obtain the requisite number of repetitions, incurring the error arising from uncertainty of time; or keeping close to the meridian, put up with a limited

range, and render his work equally unsatisfactory through that circumstance. It was before stated that, to avoid hurried observation in the use of A, stars at nearly equal distances of right ascension should be chosen; and it should be remarked, that it is almost equally important to have them also at nearly equal distances of declination; because the more regularly they are dispersed over the whole range of arc employed, the more perfect will be the correction of erroneous dividing: for two stars having the same declination will evidently be affected by the same error, and if they differ only a little, will probably partake of the same. When A is used in the observation of many stars, the errors both of observation and graduation are reduced by taking the mean. With R, the errors of many observations on one star are diminished precisely in the same degree, but as the errors of the whole intermediate arc are passed over and do not enter at all into the account, only those of the zero reading and the final one are charged upon the general mean. This distinction, certainly in favour of R, may be gathered from what has been said before, and is the *little all* of R. It is here brought forward again, for the use of those numerous observers who have not seen through the blind process of repetition, and who have attributed to this instrument powers of approximating towards the inaccessible point of truth little short of the miraculous.

I will now subjoin in detail the observations of some zenith distances of a fixed mark made about ten years ago with an 18 inch repeating circle at St. John's College Cambridge, by a gentleman who ranks high in science at that distinguished university, and who is a most worthy member of this society.

Double Obs.		Zenith dist. by repetition.			Zenith dist. by successive pairs.
		° ′	″		″
1	=	92. 45.	3,2	—	3,2
2	=	—	4,8	—	6,3
3	=	—	6,3	—	9,4
4	=	—	6,8	—	8,3
5	=	—	7,5	—	10,4
6	=	—	6,9	—	4,0
7	=	—	7,2	—	8,7
8	=	—	6,9	—	5,2
9	=	—	7,1	—	8,7
10	=	—	7,0	—	5,9
11	=	—	7,2	—	8,7
12	=	—	6,8	—	3,1

Since writing the above, I have put my hand upon another experiment of the same kind, which was made to determine the angular distance between two land objects. It was made at Glasgow by a gentleman whose science is universally acknowledged, and who was eminently qualified to do justice to the subject. The diameter of the instrument was 12 inches.

Double Obs.		Angle by repetition.				Angle by successive pairs.
		° ′		″		″
1	=	63. 33.		52,5	—	52,5
2	=	—		51,0	—	49,5
3	=	—		50,0	—	48,0
4	=	—		48,7	—	45,0
5	=	—		49,3	—	50,0
6	=	—		49,1	—	50,0
7	=	—		48,6	—	45,0
8	=	—		48,5	—	45,0
9, 10	=	—		48,8	—	51,5

The two series of observations with R, inserted above, are all I can find where the collimated angles have been read off. They serve to show, in such instruments as were used, the amount of the errors of observation, division and reading off, that one collimated result leaves to be corrected by repetition or taking means. Thus, in the first example, the greatest differences from the general mean are $+ 3'',4$ and $- 3'',7$, and in the second, $+ 3'',7$ and $- 3'',8$. The examples also exhibit the irregular manner in which the successive pairs differ from each other; while the columns of repetition with more regularity approximate towards a constant quantity, as the divisor of the total arc becomes greater. It may be useful sometimes, to exhibit a series of observations in this manner, for it shows what may be expected from an instrument after a certain number of repetitions. It is curious that, in both of the examples, the same result is obtained at the fourth pair as was gained at last, a circumstance, no doubt, purely accidental, and which could not have been known, had the ordinary means of reduction been employed.

With respect to the accuracy of the angles obtained in the two series of observations, it might be presumed, that in neither case, could they differ from the truth by a quantity much greater than half a second: but this presumption depends entirely upon the instrument being perfect, or free from fixed error; a thing, for the detection of which, an observer is not furnished with any direct means.

The instrument R has been equally praised by those who have used it for taking angular distances, as it has been in astronomy; and I have in this place but little to add to my former remarks. The observer is not here, as in astronomy, compelled to perform his operations in a short time: he may take as much as he pleases, and in truth he will want not a little. Even by the improved method, it requires much time to place the circle in the plane of the two objects: then follows the process of repetition; then that of observing the elevation or depression of the objects; and lastly, measuring, if they are near, or estimating, if remote, their distance from the station, to enable him to correct the angle for the effect of eccentricity of the telescopes. All this must be done before he has data for computing the horizontal angle; and all this is requisite for every angle that he takes; even from the same station. A gives the horizontal angle at once; its parts are perfectly concentric, and therefore the distance of the objects is not concerned: the motion of the telescope, like that of a transit, is truly perpendicular, therefore the elevation or depression of the objects comes not into the account. When the adjusted instrument is placed with the axis of azimuth vertical, the observer may proceed through all the angles of his station as fast as he can observe and read them off. They will, however, be affected by the errors of graduation; but it has been shown how these may be diminished; namely, where there are three readings, by reversion; and where there are only two, in cases where extreme accuracy is required, by turning again and again different parts of the circle towards the same object. And although a fresh adjustment of the vertical axis thereby becomes necessary at every step, this is an expedient attended with far less trouble and loss of time than can be brought about by repetition and its requisite accompaniments.

Were it urged in favour of R, that by repetition, the errors of division being almost annihilated, it becomes more fit for ascertaining the position of a star in the heavens; because, in observing one with A, the readings always take place on the same divisions, and consequently the errors of those divisions are charged upon the place of the star: this I should readily grant, were the two instruments equal in other respects. But I have to observe that, in the present state of practical astronomy, neither one instrument nor the other is at all fit for such a purpose. It need hardly be remarked, that the resistance and dragging of the centre-work, which I explained when I endeavoured to trace to its sources the cause of fixed error, will produce the same injurious effect on terrestrial measurement as they do in astronomical observations.

As a transit instrument I believe R was never thought of: indeed, the im-

perfect manner in which its adjustment is performed, together with its weak frame, renders it altogether unfit for that purpose. A is a transit, and as perfect as the telescope which it bears will allow. This instrument, in the observatory, is certainly the most important of all, and as a portable one, the best, and attended with the least trouble for keeping the rate of a chronometer. Add to this, that by taking the difference of right ascension between the moon and stars, it affords perhaps the very best means of ascertaining the difference of longitude between distant places*.

For observing equal altitudes, both instruments are very good; and I would only give the preference to A as being more likely to preserve the position of the telescope unvaried, by keeping in adjustments better during the requisite length of time.

For taking altitudes at a distance from the meridian for finding the time, as much may be said in favour of A: yet for this purpose, upon the whole, R seems to have the advantage; because these variable altitudes may be taken by repetition, and the errors of division, to which A is liable, almost done away. Ready means of placing an instrument in the plane of the meridian are of the utmost importance to the travelling astronomer, and that by corresponding azimuths is perhaps the best. A is not only peculiarly adapted to this method, but also furnishes the best means of doing the same thing by all the other ways. When R has a good azimuth circle, its plane may be brought into the meridian by the same means, at least near enough for its own purpose, for it cannot with propriety be regarded as a meridian instrument at all.

Respecting the dimensions of the two circles under comparison, I have, considering every thing, thought that 18 inches should be the greatest diameter for R: this admits of a two feet telescope, and gives sufficient room for the screws for clamping and slow motion to pass each other, and to be conveniently handled. The pressure of heavy telescopes upon the centre-work is certainly detrimental, however desirable good ones may be; and were it not for want of finger room in a 12 inch one, notwithstanding the diminutive telescope which it bears, I would give the preference to the latter dimension. The instrument A may safely be extended to a diameter of two feet, beyond which it could not justly be called portable; but besides this, there are other reasons for not carrying it much further than this limit: and I admire the courage more than the sound judgement of those, who have constructed very large instruments on this principle. With regard to the other limit, I would remark that as to instru-

* Many writers have given an erroneous rule for this purpose: see *Phil. Mag.* Vol. XV. p. 97.

ments smaller than one foot diameter, however useful they may be for surveying of land and other inferior purposes, I should consider them for astronomy as little better than playthings.

As to its form and general appearance, R is, of all the instruments subservient to astronomy and geodesy, the most uncouth and unsightly. The whole of the effective parts are placed on one side of its single supporting pillar; and on the other a weight, almost equal to the instrument, is placed for the purpose of keeping it in equilibrio. But ugliness is not the worst thing that attends this unavoidable combination; for it renders the instrument top-heavy, tottering, and weak. In these respects A is certainly very much superior. The whole of its fabric is regular and self-balanced; the upper circle, being supported like a transit on two columns, is thus rendered firm and steady. Respecting sightliness, I think the man of taste would, in the different forms it has appeared under, pronounce it agreeable, I dare not say beautiful; and here I may be allowed to remark, that the art of instrument making, as a matter of taste, is far behind many others. In this country, indeed, at the beginning of the art, instruments were adorned with the flourishes of the engraver, chaser, and carver (now long out of fashion): but these are not the beauties which I mean; those of uniformity of figure, and just proportions are alone what I have in view; and I cannot for a moment think that these are at all inconsistent with either strength or accuracy.

Through the whole of this paper every reader will have seen that I am an advocate for A, and I have made no endeavour to conceal it; yet, if I have said more for it than it deserves, or given to R less than its due, it is a thing I am quite unconscious of. Having now finished what I had to say by way of comparison, a concluding remark or two only remain to be added. One of our artists, through a course of twenty-five years, has made the repeating circle under various forms; some of them have repeated horizontal angles, others vertical angles, and more of them have done both; which last have obtained the name of repeating theodolites. All these are of a firmer fabric than that treated of in this communication; yet after having, as he fancies, gone through all the changes of repeating instruments, he owns that he has never satisfied himself in a single instance. I am informed that some of our instrument-makers are at this time endeavouring to improve the repeating circle; but I would submit it to their serious consideration, whether their time and talents might not be better employed in perfecting the art of graduation, and in the construction of instruments of better promise. As it was the rudeness and in-

accuracy of dividing which brought this instrument into existence, one would think that as the art becomes cultivated, it will fall into disuse. The art in this country is certainly sufficiently advanced to set repeating instruments aside: and, if I am rightly informed, several foreign artists are at this time pursuing the course of its improvement, in which for many years they had been impeded by circumstances which science could not control. It is therefore my opinion, that as the division of instruments becomes generally improved, so will the repeating circle hasten to its dissolution; and perhaps, on account of the great services, which, in its time, it has rendered to astronomy and geodesy, some future age may be induced to chaunt its *requiem*.

II. *The Description of a Repeating Instrument upon a new construction. By* G. Dollond, Esq. F.R.S.

Read April 13, 1821.

In describing this instrument, it is not my intention to make any comparisons between the properties it possesses, and those of other instruments of similar construction. I shall therefore briefly describe the various parts of the instrument, with their applications; and then particularize their novelties: but shall not enter so minutely into their description, as to render letters of reference requisite. The general plan of the instrument will be easily comprehended from a view of fig. 1. Plate I.

It is constructed upon the repeating principle, which is applied both to the vertical and horizontal circles. The vertical circle is 15 inches in diameter, and is made to revolve upon an axis to which the telescope is attached, upon the principle of the transit instrument; and which carries three verniers with a very secure clamping adjustment, each vernier subdividing the circle into ten seconds. At the back of the circle there is a very accurate level, with a divided scale, showing seconds; which may be considered to represent the horizon, or that plane to which all the vertical angles must be referred. This level is made to revolve upon the axis of the circle, in the same plane as that in which the circle revolves, upon the axis to which the telescope is affixed; it is furnished with a clamp and adjusting screw. Beyond this level and close to the pivot of the transit axis, is a circle of six inches in diameter, very firmly attached to the axis of the principal circle, and may be considered as a part of it. For the purpose of giving motion to this, as well as to the principal circle, there is a clamp with an adjustment, fixed to two of the pillars; of which there are four, for supporting the transit axis: each pair carrying a triangular bearing for the pivots of that axis. These bearings are furnished with the requisite adjustments for bringing the axis truly horizontal; the horizontality of which is proved to be correct, by a very accurate hanging level, that has its bearings

upon the upper part of the pivots, exactly opposite their bearings in the triangles. This level is made to reverse, and may be continued in its place during the observation, or removed at pleasure. The axis is perforated, and has a diagonal reflector in the centre. A small lantern also is applied to the pillars, for the purpose of illuminating the cobwebs ; the communication of heat, from the lantern to the instrument, being cut off by the introduction of a non-conducting substance. Near the opposite end of the transit axis, to that which has been previously described, are applied two small levels, which are attached to two contrate wheels, and are thereby moved with the same pinion, through equal spaces in *contrary* directions ; their use being to place the telescope to the altitude, or zenith distance, each time the instrument is *reversed* ; or, in other words, each level alternately becomes a finder to the telescope. There is also a circle of six inches in diameter, attached to the transit axis, between the two levels and the pivot. This circle is divided, and is furnished with two verniers that are attached to the pillars, for the purpose of setting the telescope to the zenith distance, when it is used as a simple transit instrument. The telescope is 17 inches in focal length, and has an aperture of two inches, which produces an image sufficiently brilliant to enable the observer to see the pole-star in day-light. In the principal focus there are placed five vertical, and three horizontal lines. The horizontal line of collimation is proved to be correct by reversion. The vertical line of collimation does not require any adjustment, on account of the repeating principle of the instrument. The magnifying powers of the telescope are 20, 30, 50, and 100 ; and are furnished with the usual prisms and reflectors. The four pillars, upon which the vertical circle &c. rest, are supported by a strong plate ; in the middle of which is placed the principal horizontal centre : and to which plate are also affixed three verniers, one of them having a clamp and adjusting screw, and each of them subdividing the circle into ten seconds. This circle is 12 inches in diameter, and has a centre, which turns upon the principal horizontal centre, with its exterior fitted into the socket, that forms the middle piece of a triangular frame work, which supports the whole of the instrument. This frame-work is furnished with three socket adjusting screws, that serve for feet. The flanch of the centre, belonging to the horizontal circle, is formed so as to answer the purpose of a clamping circle, when the verniers and upper part of the instrument are used for repeating the observation ; and a very strong clamp, and adjusting screw, are applied to the triangular frame for this operation. And to prove that no motion of the circle takes place during the repeating of the observation, there is a very

accurate level (showing seconds) which is capable of being clamped to the circle; and which possesses the various motions requisite to render it completely subservient to the horizontal motion of the circle, so as to detect the least possible movement of that circle in azimuth. Every part of the instrument is correctly counterpoised, and the verniers are successively read off by attached microscopes.

Having briefly described the general construction of the instrument, I shall now point out its various novelties, and their advantages.

The repeating principle upon which the instrument is founded, is too well understood to require any explanation; I shall therefore only remark, that I consider it to be of very great advantage to *portable* instruments; particularly as they cannot be prepared (on account of their price and dimensions) with such accurate divisions, or such powerful telescopes, as those larger instruments possess, that are furnished for fixed observatories. It may be here remarked, that this instrument can be used as a simple altitude and azimuth instrument: for when the circles are clamped, the verniers set to zero, and the levels adjusted, an altitude or horizontal angle may be taken with as much accuracy, as with any instrument of the same dimensions, constructed *without* the repeating properties.

The first novelty in this instrument (as a repeating instrument) is the transverse or transit axis. By this construction, the telescope is rendered independent of the plane of the circle, and by the length of the axis is compelled to move in a truly vertical plane: and by reversing the axis, the line of collimation may be perfectly verified. This construction, therefore, renders it also a good instrument for observations in right ascension.

The second novelty is the application of the two small levels, or finders, which afford a very great convenience, when repeating zenith distances: as by this application the telescope can be readily placed to those distances, each time the instrument is reversed, *without* the aid of a divided circle.

There is also a novelty applied to the lantern which will be found extremely convenient. It consists of two plates of brass, having a square hole in each; these plates are moved in *contrary* directions by rack and pinion; and by this contrivance the observer is enabled to regulate the light in any proportion that may be required.

There is also an entirely new application, which will be extremely advantageous when taking horizontal angles. This is the level, which is applied to the principal horizontal circle; and which in every respect answers the pur-

pose of a second telescope, while it is much more convenient; as the observer can instantly perceive the least possible motion of the circle, without the necessity of his changing his position: and if it should be required to take horizontal angles at night, the advantage will be very considerable.

There is lastly a new appendage, which will be found very useful, when repeating the vertical angles. It consists of two arms, fitted to the lower end of the centre that belongs to the horizontal circle, and has a motion sufficiently tight to keep it at the place to which it is set. When the telescope is presented to the object for observation, one of these arms is brought to coincide with a projecting piece in the triangular frame, and when the instrument is turned half round, by bringing the other, or opposite arm, to coincide with the same projecting piece, the object will be again in the field of view of the telescope.

The whole of the instrument is differently framed from any that has been previously made, and I have no doubt will be found to possess many conveniences that have not been described. It was finished in January 1819, and is applicable to all the uses where vertical and horizontal angles are required to be taken.

III. *On a Method of fixing a Transit Instrument exactly in the Meridian.* *By* F. BAILY, Esq. F.R.S. *and* L.S.

Read June 9, 1820.

THE transit instrument is so essential a part of the apparatus of the practical astronomer, that every attempt to facilitate the use of it will doubtless be received with indulgence. When this instrument has been brought *nearly* in the plane of the meridian (which may be done by any of the methods pointed out in the several works on practical astronomy) it may be adjusted *accurately* by either of the following modes: 1°. by observation of the pole-star, at the time of its upper or lower culmination: 2°. by observing any of the circumpolar stars at the time of their upper and lower culmination: and 3°. by observing the culmination of any two stars differing from each other considerably in declination. The two former methods (independently of their requiring a building peculiarly constructed so as to command an uninterrupted view of the meridian, from the northern to the southern horizon) are liable to some objections, to which it is not my intention at present to advert: but the latter method may be practised in every situation in which a transit instrument may be placed, and as the results are extremely correct, I shall confine my remarks to this mode only of adjusting the instrument. Moreover, there are many persons, fond of practical astronomy, who have not the convenience, or who do not wish to incur the expense, of constructing a building of the kind above mentioned; and who are therefore compelled to fix their transit instruments on the sill of one of their windows, or in some other similar situation: many, again, who are travelling, with a view to improve the connected sciences of astronomy and geography, are obliged to fix their transit instruments in the most convenient and safe situation, where their prospect may be confined to a southern aspect:—to all such persons the method here alluded to, is the only one which can be adopted. Portable transit instruments, adapted to such purposes, are now made with great neatness and accuracy, and of various

sizes; and are a valuable addition to every œconomical observatory, and to every person travelling for the purposes above mentioned. When placed on the *inner* sill of a window, they have a range of above 70° in altitude; and when placed on the *outer* sill, they may be pointed even to the zenith.

I shall therefore suppose that an instrument of this sort has been brought *nearly* in the plane of the meridian, by any of the known methods for that purpose: after which it may be *accurately* adjusted by determining its deviation from the meridian by the method, above mentioned, of observing the transit of two stars, differing considerably from each other in declination, and whose right ascensions are well ascertained. The principles of this method have been treated on by M. Lalande in his *Astronomie*, vol. 2. page 715; by M. Delambre in his *Astronomie*, vol. 1. page 421; and by M. Biot in his *Traité d'Astronomie*, vol. 3. *Additions*, page 130.: with one or other of which I shall presume the reader to be previously acquainted.

The stars which should be chosen for the purpose, are those which differ at least 50 degrees from each other in declination: but the nearer that difference approaches to 90 degrees, the more correct will be the results. Their right ascensions, on the contrary, must be as near as possible to each other; a circumstance which will moreover prevent the possibility of any error arising from a variation in the rate of the clock during the interval of the observations. And here it may be proper to remark that the time, used in these computations, is *sidereal* time: if therefore a clock or watch, which marks *solar* time, be made use of, it must be corrected in the manner hereafter mentioned.

This being premised, it will be readily seen that, in this parallel of latitude, one of the stars will have *north* declination, and the other *south* declination: and, in order to avoid repetition, I shall call the former the *northern* star, and the latter the *southern* star. Their declinations I shall denote by N and S respectively: and it may be useful to know that they may be taken out to the nearest *minute* only, as great accuracy is not required in this respect. The right ascensions, however, of the two stars (which must be expressed in *time*) should be taken out from the most approved tables, and corrected for aberration and nutation*; in order that their apparent positions in right ascension may be

* When the two stars are at *equal* distances from the equator, and differing but little from each other in right ascension, their *mean* places on the given day may be taken; as they will be nearly equally affected by aberration and nutation. Many *pairs* of stars, situated in this manner, may be mentioned: such as β *Geminorum* and τ *Navis*, γ *Coronæ Borealis* and α *Regulæ*, &c.

exactly stated: on which indeed the accuracy of the method depends. The apparent right ascension of the northern star I shall denote by $Æ^n$, and the time of its observed passage, as shown by the clock, I shall denote by T^n: the apparent right ascension of the southern star I shall denote by $Æ^s$, and the time of its observed passage by T^s. The latitude of the place I shall denote by L; and the quantity sought (or the deviation of the instrument in azimuth) by A.

Now, in order to determine A, we must first take the difference of the apparent right ascensions of the two stars, and also the difference of the time of their observed transits; that is, we must make $(Æ^n - Æ^s) = d\,Æ$ and $(T^n - T^s) = d\,T$; and the formula for finding A will, agreeably to the principles laid down by MM. DELAMBRE and BIOT, be

$$A = (d\,T - d\,Æ) \times \frac{\cos N . \cos S}{\sin (N+S) \cos L}.$$

If the quantity $(d\,T - d\,Æ)$ be *positive*, the deviation of the transit instrument will be to the *east* of the meridian: on the contrary, if it be *negative*, the deviation will be to the *west*. When it is $=0$, the instrument is exactly in the plane of the meridian, and consequently does not require any correction.

By the help of a table expressing, for any given latitude, the value of $\frac{\cos N . \cos S}{\sin (N+S) \cos L}$ in numbers, according to the sum of the declinations (or the difference of the polar distances) of the two stars observed, we may, almost by inspection, obtain, in every case, the value of A, or the deviation of the transit instrument required; and consequently bring it afterwards exactly in the meridian, so as to be enabled to adjust it at any time to a meridian mark. The table, which I have here given, is calculated for the latitude of Greenwich ($=51°. 28'. 40''$): but since it is not necessary to be very exact in the declination of the star, it will suit any other place not very distant from that parallel of latitude. I might have constructed the table so as to have been *general*, for all latitudes, by merely taking the value of $\frac{\cos N . \cos S}{\sin (N+S)}$; which value must then have been divided by the cosine of the latitude of the place where the observer might be situated. But, I have preferred, in the present instance, confining the table to the latitude of Greenwich; subjoining, however, a correction for the use of it in any other part of England.

The first perpendicular column of the table denotes the sum of the declinations (or the difference of the polar distances) of the two stars, for every degree

from 42° to 72°: and opposite thereto is set down, in separate columns, the value for finding the deviation of the instrument in azimuth, according to the value of N, or the northern star, from 24° to 40°; those limits being sufficient for the purposes alluded to in the preceding part of this paper. The proportional part for any intermediate difference may be readily seen, on inspection. These values, multiplied by ($d\,T—d\,\text{Æ}$) or the difference between *the difference of the apparent right ascensions of the two stars, and the difference of their observed transits,* will show the value of A, or the total deviation of the instrument in *time*; which, multiplied by 15, will give the deviation in *arc:* and when the deviation of the instrument has been thus determined, it may be corrected in the usual manner. An example or two will best explain the use and application of the table, and the mode of operating in such cases.

On July 1, 1819, I placed my transit instrument nearly in the meridian; and in order to ascertain how much it deviated from the true meridian I observed the two stars γ *Lyræ* and τ *Sagittarii.* The passage of the former was observed at $18^h.\ 52'.\ 37'',3$, and of the latter at $18^h.\ 56'.\ 4'',5$ sidereal time. The apparent right ascensions of those stars, on that day, were $18^h.\ 52'.\ 9'',8$ and $18^h.\ 55'.\ 39'',7$ respectively: and their declinations were $32°.\ 27'$ north, and $27°.\ 55'$ south. Consequently the operation will stand thus

$$\begin{array}{ll} \text{Æ}^n = 18^h.\ 52'.\ \ 9'',8 & \qquad T^n = 18^h.\ 52'.\ 37'',3 \\ \text{Æ}^s = 18.\ \ 55.\ \ 39\,,7 & \qquad T^s = 18.\ \ 56.\ \ \ 9\,,9 \\ \hline d\,\text{Æ} = -\quad 3.\ \ 29\,,9 & \qquad d\,T = -\quad 3.\ \ 32\,,6 \end{array}$$

whence ($d\,T—d\,\text{Æ}$)$=-2'',7$. This value, being negative, shows that the deviation is to the west: and in order to determine the quantity of the deviation, we must take the sum of the declinations (or the difference of the polar distances) of the two stars; which in this case is equal to 60°. 22′; or, for the sake of round numbers, equal to 60°: and the declination of N (or the northern star) is about 32°. Consequently against the number 60 and under the column headed 32° we shall find 1·39; which being multiplied by $-2'',7$ will give $-3'',75$ for the deviation of the instrument in *time*: and this multiplied by 15 will give $-56'',3$ for the deviation in *arc* westerly.

Again, on Jan. 1, 1820, having reason to suspect that the transit instrument (from some motion which had been given to it) deviated from the plane of the meridian, I observed the passage of ε *Canis majoris,* and of *Castor:* the former at $6^h.\ 52'.\ 45'',6$, and the latter at $7^h.\ 24'.\ 28'',4$. The apparent right ascension

of those stars on that day was $6^h. 51'. 34'',3$ and $7^h. 23'. 7'',3$ respectively; and their declinations were $28°. 44'$ south and $32°. 16'$ north. Consequently the operation will stand thus

$$\begin{array}{rl} Ꭱ^n = & 7^h.\ 23'.\ \ 7'',3 \\ Ꭱ^s = & 6.\ \ 51.\ \ 34\ ,3 \\ \hline dᎡ = & +\ 31.\ \ 33\ ,0 \end{array} \qquad \begin{array}{rl} T^n = & 7^h.\ 24'.\ 28'',4 \\ T^s = & 6.\ \ 52.\ \ 45\ ,6 \\ \hline dT = & +\ 31.\ \ 42\ ,8 \end{array}$$

whence $(dT - dᎡ) = +9'',8$. The sum of the declinations (or difference of polar distances) being in this case 61°, we shall find that the value to be adopted is 1·36; which being multiplied by $+9'',8$ will give $+13'',33$ for the value of A in *time*, or (multiplying this by 15) $+3'. 20''$ for the value of A in *arc*. And this quantity being positive shows that the deviation was to the east.

If observations of this kind be made about sunrise or sunset, and after the passage of the stars, the telescope be pointed to the horizon and compared with some object there, a meridian mark may be set up, which may be corrected from time to time by subsequent observations on various stars, similarly situated.

I have already stated that in all cases of this kind, the time employed is supposed to be sidereal time, and that if a clock or watch be used which marks mean solar time, the interval between the observations must be corrected accordingly. This correction is made by converting the value of d T (which is expressed in sidereal time) into mean solar time, in the usual manner, by adding the acceleration of the fixed stars for that interval. Thus, in the case last stated, suppose that the passage of ϵ *Canis majoris* had been observed at $12^h. 10'. 31'',6$, and the passage of *Castor* at $12^h. 42'. 9'',2$ mean solar time: the difference between these two (or d T) would be $31'. 37'',6$, to which the acceleration of the fixed stars for that interval $(=5'',2)$ must be added; whence the difference will be, as before, $=31'. 42'',8$. So that, by means of this correction, it will be indifferent whether the clock shows sidereal or mean solar time.

Before I close this paper I shall point out another important use to which these observations may be applied; namely to correcting the error of the clock at the time of observation. For after the quantity of the deviation is found, as above explained, the error of the clock may be determined by means of the transit of either of the stars employed; that is, of either N or S; but, for the sake of uniformity in the investigations, I shall confine my remarks to N, or the northern star. Let the observed time of the passage of N be denoted as before by T^n; and the apparent right ascension of N by $Ꭱ^n$, and let the error of the clock at

the time of observation be denoted by E. Then, from the principles laid down by M. Biot, we shall have

$$E = (T^n - Æ^n) + A.\frac{\sin (L-N)}{\cos N}.$$

The value of $\frac{\sin (L-N)}{\cos N}$ for the latitude of Greenwich I have thrown into numbers, and placed in the last line of the table at the end of this paper, so as to be ready for immediate use when required. It is denoted by *c*, since it serves to denote the correction of the clock. The application of the formula is very simple: the rule being as follows. From the observed time of the transit of the northern star deduct the apparent right ascension of the same star; to the difference add the product A*c*: the sum is the error of the clock; which, when it is negative, shows that the clock is too slow.

For example; in the first case mentioned in this paper, the difference between the observed and apparent time of the transit of γ *Lyræ* is $(18^h.\ 52'.\ 37'',3 - 18^h.\ 52'.\ 9'',8) = +27'',5$: the deviation of the transit instrument has been found to be $-3'',75$ in time, and the number in the table, against N ($= 32°$) is ·39: the product of these two is $-1'',5$: so that $27'',5 - 1'',5 = 26'',0$ is the error of the clock at the time of observation, which being positive shows that the clock was too fast. I shall here repeat that the observed *time*, here alluded to, is supposed to be *sidereal* time: and therefore if *mean solar* time be employed in the observation, it must be converted into sidereal time, by any of the methods laid down for that purpose. It may be useful to remark that, in all observations of this kind, it is presumed that the proper adjustments of the transit instrument are made previously to observation: and particularly that the axis of the telescope is rendered perfectly level: otherwise the observation will partake of the error arising from this source, and render a further correction necessary.

I shall conclude by observing that M. Delambre prefers this mode of adjusting a transit instrument to that of observing the passage of the circumpolar stars, which requires an interval of at least 12 hours, during which time considerable alteration may have taken place in the rate of the clock; and therefore cannot be conveniently practised except when the days are very short, and in a building constructed peculiarly for meridional observations. Whereas the observations, here alluded to, may frequently be completed in a few minutes; at all times of the year; and often by daylight. The tables are very easily computed, and therefore every practical astronomer who requires greater

accuracy should calculate them for the latitude of his own observatory. In which case, the labour will be very considerably abridged if he confines the table to the declination of those stars which are most frequently used by him for such comparisons.

It may be proper to state, that the values in this table (except those in the last line) must be multiplied by the following numbers, for any other parallel of latitude to the southward or northward of Greenwich: viz. if

south 1°,	by	·979
north 1°,	by	1.023
—— 2°,	by	1.047
—— 3°,	by	1.072
—— 4°,	by	1.099

so that in no part of England will the correction amount to $\frac{1}{10}$th, nor if within 2 degrees of the latitude of Greenwich, will it amount to $\frac{1}{20}$th of the whole value. The last line, for the correction of the clock, is adapted to the latitude of Greenwich *only*.

Sum of Dec.	*Declination of the Northern Star* N.																
	24°	25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	37°	38°	39°	40°
42°	2.08	2.08	2.07	2.06	2.06	2.04	2.03	2.02	2.00	1.99	1.97	1.95	1.93	1.91	1.89	1.86	1.84
43	2.03	2.03	2.02	2.02	2.01	2.00	1.99	1.97	1.96	1.94	1.93	1.91	1.89	1.87	1.85	1.82	1.80
44	1.98	1.98	1.98	1.97	1.96	1.95	1.94	1.93	1.92	1.90	1.89	1.87	1.85	1.83	1.81	1.79	1.77
45	1.94	1.93	1.93	1.92	1.92	1.91	1.90	1.89	1.88	1.86	1.85	1.83	1.81	1.79	1.78	1.75	1.73
46	1.89	1.89	1.88	1.88	1.87	1.87	1.86	1.85	1.84	1.82	1.81	1.79	1.78	1.76	1.74	1.72	1.70
47	1.85	1.84	1.84	1.84	1.83	1.83	1.82	1.81	1.80	1.79	1.77	1.76	1.74	1.73	1.71	1.69	1.67
48	1.80	1.80	1.80	1.80	1.79	1.79	1.78	1.77	1.76	1.75	1.74	1.72	1.71	1.69	1.68	1.66	1.64
49	1.76	1.76	1.76	1.76	1.75	1.75	1.74	1.73	1.73	1.71	1.70	1.69	1.68	1.66	1.65	1.63	1.61
50	1.72	1.72	1.72	1.72	1.72	1.71	1.71	1.70	1.69	1.68	1.67	1.66	1.64	1.63	1.62	1.60	1.58
51	1.68	1.68	1.68	1.68	1.68	1.67	1.67	1.66	1.66	1.65	1.64	1.63	1.61	1.60	1.59	1.57	1.55
52	1.64	1.64	1.65	1.64	1.64	1.64	1.64	1.63	1.62	1.61	1.61	1.59	1.58	1.57	1.56	1.54	1.53
53	1.61	1.61	1.61	1.61	1.61	1.61	1.60	1.60	1.59	1.58	1.58	1.56	1.56	1.54	1.53	1.51	1.50
54	1.57	1.57	1.57	1.58	1.57	1.57	1.57	1.56	1.56	1.55	1.55	1.54	1.53	1.51	1.50	1.49	1.47
55	1.53	1.54	1.54	1.54	1.54	1.54	1.54	1.53	1.53	1.52	1.52	1.51	1.50	1.49	1.48	1.46	1.45
56	1.50	1.50	1.51	1.51	1.51	1.51	1.51	1.50	1.50	1.49	1.49	1.48	1.47	1.46	1.45	1.44	1.43
57	1.47	1.47	1.48	1.48	1.48	1.48	1.48	1.47	1.47	1.47	1.46	1.45	1.45	1.44	1.43	1.41	1.40
58	1.43	1.44	1.44	1.44	1.45	1.45	1.45	1.44	1.44	1.44	1.43	1.43	1.42	1.41	1.40	1.39	1.38
59	1.40	1.41	1.41	1.41	1.42	1.42	1.42	1.42	1.41	1.41	1.41	1.40	1.39	1.39	1.38	1.37	1.36
60	1.37	1.37	1.38	1.38	1.39	1.39	1.39	1.39	1.39	1.38	1.38	1.37	1.37	1.36	1.35	1.34	1.33
61	1.34	1.34	1.35	1.35	1.36	1.36	1.36	1.36	1.36	1.36	1.36	1.35	1.35	1.34	1.33	1.32	1.31
62	1.31	1.31	1.32	1.33	1.33	1.33	1.34	1.34	1.34	1.33	1.33	1.33	1.32	1.31	1.31	1.30	1.29
63		1.29	1.29	1.30	1.30	1.31	1.31	1.31	1.31	1.31	1.31	1.30	1.30	1.29	1.29	1.28	1.27
64			1.26	1.27	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.28	1.27	1.26	1.26	1.25
65				1.24	1.25	1.25	1.26	1.26	1.26	1.26	1.26	1.26	1.25	1.25	1.24	1.24	1.23
66					1.22	1.23	1.23	1.23	1.24	1.24	1.24	1.23	1.23	1.23	1.22	1.22	1.21
67						1.20	1.21	1.21	1.21	1.21	1.21	1.21	1.21	1.21	1.20	1.20	1.19
68							1.18	1.18	1.19	1.19	1.19	1.19	1.19	1.18	1.18	1.18	1.17
69								1.16	1.16	1.17	1.17	1.17	1.17	1.16	1.16	1.16	1.15
70									1.14	1.14	1.15	1.15	1.15	1.14	1.14	1.14	1.13
71										1.12	1.12	1.12	1.12	1.12	1.12	1.12	1.11
72											1.10	1.10	1.10	1.10	1.10	1.10	1.10
$c =$	·50	·49	48	·46	·45	·44	·42	·41	·39	·38	·36	·35	·33	·31	·30	·28	·26

IV. *On the doubly-refracting property of Rock Crystal, considered as a principle of Micrometrical measurements, when applied to a telescope. By the* Rev. *W.* Pearson, *LL.D. F.R.S. and Treasurer of this Society.*

Read March 10, 1820.

As the improvement of astronomical instruments is one of the professed objects of the Astronomical Society of London, I shall not make any apology for taking an early opportunity of laying before its members a method of measuring small angles, that has for its basis that singular property of several crystallized bodies, the phenomenon of *double refraction.* Of these bodies, rock crystal (on account of its hardness, transparency, and capability of receiving a high polish) seems to be peculiarly adapted, to answer the purpose of constituting that species of micrometer, which is most convenient for measuring the diameter of a body in motion, or the angular distance between two bodies in motion; viz. the *double-image* micrometer.

Although the phenomenon of double refraction of the rays of light, passing in a particular direction through some crystallized diaphanous substances, but particularly the Iceland spar, has been known to men of science ever since the time of Bartholinus and Huygens, yet few probably are acquainted with the precise means, that have been successfully employed, to render this curious principle useful in micrometrical measurements. The Abbé Rochon, a late member of the Royal Institute of Paris, discovered, and made known about the year 1783, a method of compounding two prisms of rock crystal in such a manner, that any small object seen through them appeared *double*; and the constant angular distance between the two images, thus formed, was ingeniously made the ground-work of a micrometrical telescope. A pamphlet, giving a particular description of this telescope, was published by the inventor, in the year 1807, at Paris, under the title of "*Mémoire sur le Micrometre de Crystal de Roche:*" which pamphlet is become scarce; and indeed cannot now be obtained from any of the booksellers in Paris. It may be proper therefore to

give a brief account of the construction and principal defects of this original instrument (of which I believe there is no description in the English language) before I proceed to consider a different and more recent application of the same principle of double refraction, to form a micrometer in conjunction with an astronomical telescope.

When Abbé Rochon availed himself of the *ordinary* and *extraordinary* refractive powers of a piece of transparent rock crystal, he was aware that it was necessary for the rays of light to be transmitted through a section made in one particular direction in order to produce two well defined images: but this direction he could not readily ascertain, unless the form of the crystal remained perfect, or so nearly so, that its axis could be known. Hence much labour was probably lost, and several pieces of fine crystal were destroyed, in forming experimentally a prism, with the plane, facing the incident ray, cut in such a direction, as would give the most distinct images. This practical difficulty, no doubt, enhanced the value of the original instrument, and limited the number that were made under such a disadvantage: but when the ingenious Malus, some few years ago, investigated what he denominated the *polarization* of light, his instrument greatly facilitated the determination of the sections, most suitable for forming the faces of the doubly-refracting prism, and also of a second prism, which, without altering the constant angular measure of the first, renders it *achromatic*, and therewith constitutes a regular body with parallel faces; which may be called the *prismatic* solid. When a pencil of rays of light is transmitted through a prism of rock crystal in a direction parallel to its *axis*, it suffers only the ordinary refraction: therefore the achromatizing prism must have that face, which receives the incident rays, cut at right angles to the axis, in order to prevent double refraction in it. But, in the other prism, the hypothenusal face must be made parallel to the axis; and then the rays, falling obliquely on it, after having passed the first prism, will be divided, and will form double images free from colour. When the two prisms, thus cut, are polished, and brought into contact at their hypothenusal faces, the quantity of the constant angular measure, or distance between the centres of the two images, will be directly in proportion to their refracting angles, which must be alike in both prisms; and as the thickness of the prismatic solid will, in all cases, depend on the measures of the refracting angles of each separate prism, this thickness, when the other faces are of similar dimensions in different prisms, will be a *prima facie* criterion of the distance at which the two images will be formed from each other.

In constructing an achromatic prismatic solid two remarkable circumstances have occurred: One is, that whatever may be the measure of the refracting angle, in *degrees*, of any prism of double refraction, its constant angular measure, of the distance between its two images, will be very nearly the same number of *minutes*. And from this extraordinary coincidence the French opticians are accustomed to measure degrees of the refracting angle, in forming a prism, that shall measure just so many minutes. The other circumstance is, that a larger measure can be obtained by a prismatic solid, when the contiguous faces of the prisms are moistened, or smeared with some gummy substance, than when they are in contact in a dry and clean state. The incident rays will be entirely transmitted at a greater angle of obliquity in the former case, than in the latter; and on this account Canada balsam, or mastic en larmes, is employed to unite the two prisms together; which either of them will do, without affecting the transparency: and as the refractive powers of these substances are nearly the same, as that of the crystal itself, the value of the measure is not affected by their interposition.

The method of applying the double solid, as a micrometer in the original contrivance, was thus: the solid, being fixed into a piece of brass tube, was made to slide within the tube of the telescope, from the object-glass all the way to its focus, where the image was formed; while an index, connected with this tube, penetrated a longitudinal slit made in the principal tube, and passed along it, serving, at the same time, to give motion to the attached solid, and to indicate the divisions engraven along the tube, in a line of graduations running parallel to the slit, and co-extensive with it. On this longitudinal scale there were just as many divisions, as the constant angle of the doubly-refracting prism contained minutes; so that the same prism always necessarily applied to the same focal distance of the object-glass; and as the telescope seldom exceeded 22 or 24 inches in length, it was not expected to measure an angle to the accuracy of a *few seconds*. The chief use of the instrument seems to have been, to ascertain distances, either absolutely, or comparatively, as a coming-up glass; and it was therefore chiefly employed in the naval and military services; which is the reason, why the French distances corresponding to any measured angle, (subtended by a scale of given or supposed dimensions) were engraven, at the places of the angular measures, in a regular series of numbers. A table of distances corresponding to any number of minutes and decimal parts measured by the instrument, was also printed, and made a companion thereof; in which were directions for the application of the micro-

meter to the determination of the distance of a regiment; in which a soldier is proposed to be taken at five feet six inches and a half, French measure. For such a purpose the instrument was both convenient and competent; but it appears doubtful, whether it was ever usefully employed in *astronomy*. For, in the first place, the magnifying power of the telescope was not sufficient, to determine any measure of a small angle to the accuracy of a few seconds; and secondly, as the rays of light were transmitted through the solid crystal, before they arrived at the focus of the object-glass, and formed the image of any object, the vision was disturbed, and the images were seldom well defined at the extremities. Indeed M. ARAGO informed me lately, that he never could measure the diameter of the sun accurately with a ROCHON'S micrometer, as it has hitherto been constructed. With respect to the principle on which the micrometer is constructed, it has been already said, that when a pencil of rays of light fall on the exterior face of the first prism, they will pass through it without being divided, by reason of the passage being in a direction parallel to the axis of the crystal, of which the first or achromatizing prism is formed. But when this pencil, after suffering only the ordinary refraction, reaches the sloping face of the second prism, at the place of union with the first (as this face is cut in the direction of the axis), the pencil cannot *cross* it, without being subject to both the ordinary and extraordinary refractions; and is therefore divided into two pencils, each of which produces an image with one half of the light only, that a single image, formed by ordinary refraction alone, would have had, if the pencil had not been divided. Let us suppose now that the prismatic solid is placed contiguous to the object glass; that the point, where the pencil begins to divide itself, is the angular point, where the two straight lines meet, in which the two divided pencils proceed after division; and that they continue to diverge, after emergence from the doubly-refracting prism, until they arrive at the focus of the object-glass, where they form two images: then if a piece of unpolished glass, by way of screen, were situated in the focal point, so as to intercept the two diverging pencils, it is obvious, that the distance between the images of any distant luminous point, thus seen, will be the subtense of the constant angle; and will be the greatest possible while the prism remains at the greatest distance from the screen. But remove the prism nearer, and the subtense will apparently diminish, in proportion to the diminished distance, on the principle of similar triangles. And lastly, when the prism occupies the place of the image, in the focus, the subtense will be nothing; or, in other words,

the two images will coincide. Hence the total length of the tube of the micrometrical telescope, as made by ROCHON, or under his direction, constituted the scale of measurement; and it is evident, that either increasing the focal length of the object-glass, or diminishing the constant angular measure of the prism, will give an extent to each minute of the scale, which, if the vision were good, would admit of subdivisions even to single seconds. But it is understood to be extremely difficult, to obtain a piece of rock crystal, of the size of a good object-glass, that is both perfectly transparent, and of uniform density; and in a conversation with Mons. LENOIR, last summer, I learnt, that however choice the crystal may be, it requires the object-glass also to be *very good*, before good images can be obtained through the doubly-refracting prism; a desideratum more easily obtained in London than in Paris.

Having never seen a ROCHON'S micrometer before I had this interview with Mons. LENOIR, and having been previously engaged in contriving new arrangements of micrometers, I felt considerable interest in inspecting the micrometer in question, as constructed by this eminent optician: and felt quite satisfied, from the trials made on his premises, of the accuracy of his information. On inquiring whether any allowance was made in the angle for measuring distances, for want of parallelism of the incident rays of light, Mons. LENOIR showed me a small scale, sometimes applied to the eye-end of the long tube of his telescopes, to indicate the excess of focal length, beyond the solar focus, at short distances, and to give in this manner the requisite correction. On examining an object with the face of the prism in the focal point (to satisfy myself, that the image, then formed, is a single well defined image), I had occasion to adjust nicely for distinct vision: and in doing this, I discovered that, when the prism was out of the focal point, a pair of images would be formed at the *anterior* side of this point, as well as at the posterior, as they have reference to the eye looking through the telescope. And it appeared probable that equal distances on each side of the exact focal point gave equal measures, as far as the small space, left near the eye-end of the tube, would admit the prism to go. From this circumstance I immediately concluded, that two images might also probably be formed by a prism, *after the rays had proceeded through the lenses* constituting the eye-piece; but did not perceive, at the time, what the measure of an angle would be, that might probably be so obtained. I desired Mons. LENOIR to adapt me a double prism of clear crystal to a cell, that I might apply it at *the eye-end of a telescope,* for the purpose of making experiments on this mode of application: on which he laughed at the

idea, and assured me, that, as the angle of every prism is *constant*, there is no other way, but that adopted by M. ROCHON, by which a *variety* of measures could be taken; when I endeavoured to explain to him how Dr. BREWSTER had *varied* the *measure* in his patent telescope, by a variation in the *power* (a term which, it appeared, he did not understand, until I applied the word *amplification*). At length Mons. LENOIR undertook to comply with my wish of his fitting a prism into a cell; but he had no idea that a small prism, somewhat larger than the pupil of the eye, would have been sufficient for my purpose, and therefore he mounted one large enough to be used on ROCHON'S principle. While this prism was preparing I visited M. ARAGO at the Royal Observatory, and on informing him what I had in hand, he appeared surprised, and fetched from a private cup-board, or drawer, a celestial eye-piece, with a small prism actually applied to it in a cell, in the way I had ordered LENOIR to fit up his. I then learnt, that the objection, as to indistinctness of vision, which applies to ROCHON'S construction, is obviated by this new application of the prism; by means of which the image, regularly formed without previous transmission of the rays through the crystal, is viewed double; and I have since found, that the eye-lenses only modify the measure of the angle of the two images, as seen through them and the prism conjointly. What M. ARAGO had determined this modification to be, he did not inform me, except that he tabulated, from experiment, the angular measures which resulted from different arrangements of his lenses, that produced different amplifications: and that the angles so measured are very small. But the objects appeared, he said, much more distinct, than when the images are formed by ROCHON'S micrometrical telescope.

On my return from the Royal Observatory, where I learnt that SOLEIL made the *small* prisms, I applied to this optician to make me about half a dozen, similar to those which he had made for M. ARAGO; to be adapted to a similar eye-piece. But I was informed, that he knew nothing of the construction of M. ARAGO'S eye-piece, nor to what purpose he applied his prisms. As the subject of eye-pieces was familiar to me, and as I had previously succeeded in giving *variable powers* to a terrestrial eye-tube, I had no difficulty in contriving a celestial one, that has the same property, and that I found was essential to give a value to the micrometrical measure with each successive power that was to be used, to modify the operation of each prism (or rather pair of prisms), that I had ordered. But before I proceeded to construct an eye-piece with variable powers, it was necessary to satisfy myself in what way, and to what

degree, the modification of the measure of a small angle was affected by an eye-piece of *known* power. The probability was, that as the angle of the prism remains constant with the same power, and diminishes apparently as the power increases, the ratio of the diminution of the angle might be inversely as the increase of the power. And I soon ascertained (from applying a prism to my pocket telescope with variable powers, that I had with me in Paris, and from examining with it a graduated scale, with the prism first at the object-end and then at the eye-end) that this is the ratio, as nearly as experiment can determine. I then contrived such an arrangement of the lenses of an eye-piece, which SOLEIL constructed under my direction, as completely answers the purpose for which I contrived it; and I have since ascertained, both from theory and practice, that universally *the constant angle of a prism of double refraction, divided by the magnifying power of any telescope, is the measure of the small angle,* as viewed through the same prism, nearly in contact with the eye, in that telescope. For instance, if the constant angle of the prism be 30′, and the power of the telescope 60, the small angle, measured in this way, will be $\frac{30'}{60} = 30''$.

But before the doubly refracting prism can be rendered useful in measuring small angles, the constant angle which it measures, as viewed by the unassisted eye, must be accurately known; and also the magnifying power of the telescope, as used with it. For on these data the accuracy of the measure, taken by this method, entirely depends. I shall therefore devote the remainder of this memoir to a consideration of these two necessary objects. The first practical method that occurs, of ascertaining the constant angle of a prism, is that which calls in the aid of trigonometry. If a disc of writing paper is placed on a dark post, at such distance, that the two images of it, seen through the prism, just come in contact, the diameter of the disc, and the distance of the post from the eye, accurately measured, will supply data for calculating the angle subtended by the disc; or (which is the same thing) the constant angle of the prism in question. For, if the diameter of the disc be taken as radius, and the distance as tangent, in the same denomination of measure, the former divided by the latter will give the natural tangent of the angle required; the corresponding angular value of which may be had by inspection of a good table of natural tangents, to the required degree of accuracy. This method supposes the eye competent to judge of exact contacts at some distance, which cannot be done with sufficient precision without the aid of a telescope. But as no apparent alteration takes place in the constant angle of a prism, when

placed before, or *beyond* the object-glass of such instrument, the determination may be made with considerable correctness, when all the light is excluded from the telescope, except what is allowed to pass through the prism, before it enters the telescope. A disc of one inch, placed at the distance of 95.49 yards from the eye, will subtend exactly one *minute* of angular measure: and therefore a yard at the same distance will subtend 36′. And when the two images of such a scale come in contact, as seen through any prism at that distance, its constant angle will be 36′; but more or less, according to the number of inches that the deviation from exact contact is in excess or defect: and the contact may be made exact by varying the distance accordingly. M. Biot in his *Précis Elémentaire de Physique,* vol. 2. page 273, has explained M. Arago's method of determining the constant angles of his prisms on this principle; but to avoid the trouble of varying the distance for different prisms, this eminent astronomer made concentric circles on a distant plane, and selected out of the number such as would come exactly into contact at the same distance, when viewed successively through different prisms; and thence calculated the corresponding angles. M. Arago's prisms are very thin, and consequently have small constant angles, requiring but little power of magnifying. But when M. Biot says, that his friend determines the magnifying powers of his telescopes by a prism not much exceeding a *millimetre* (or about $\frac{1}{25}$ of an English inch) in thickness, he must probably have made a mistake: such prism might be used to measure an extremely small angle; but it will be seen presently, that it would not answer so well, for the purpose that is here mentioned, as one considerably thicker, or having a larger refracting angle.

That I might the better exclude the superfluous light from the object-glass of a telescope of 45.75 inches focal distance, when I determined the constant angles of my prisms, a small hole was made in its large cap with a screw in it to receive the cells of nine different prisms, that have corresponding male screws, which are also adapted to the cap of my variable eye-piece, when wanted at the eye-end for making observations. Now as I had a very nice spider's-line micrometer by Troughton fitted to this telescope, it occurred to me, that the distance between two small discs, or luminous points, might be very accurately measured by this micrometer, when the two images made by a prism, screwed into the cap of the object-glass, were taken as those points: and it was found on trial, that, though the distance, at which a small white disc required to be placed, in order to be distinctly visible, was not considerable, yet much trouble was avoided, and great accuracy obtained, by adopting the

scale of this micrometer, with its tabular value corrected for distance, in preference to the more obvious method of determining the visual angle of a triangle, subtended by a larger disc, where the data were more difficult to ascertain with precision. When the cells, containing the small prismatic solids, were screwed into the large cap of my telescope, successively, on a clear day, the two images of the disc were better defined in some of them than in others. But in all of them the image produced by the extraordinary refraction, was not so well defined, as that produced by the ordinary refraction : it had also some colouration, or indication of polarized light ; and I perceived that this method of viewing a disc, with the considerable power of a good telescope, affords an excellent criterion of the comparative goodness of any prism, selected out of a number, having nearly the same constant angle. When the telescope was adjusted for distinct vision, and the disc was seen clearly through one of the prisms, applied to the cap of the object-glass, the distance between its corresponding limbs was measured with great care, by means of the spider's lines, in the usual way ; and the revolutions and parts were noted, with their proper angular measure and correction for distance, from which the true angle was deduced. The same was done for all the other prisms in succession, and the subjoined table contains the results. The distance at which the disc was placed was only $64\frac{1}{4}$ yards, and the table from which the corrections were taken, was one which I had previously had occasion to calculate.

TABLE I.

Prisms used.	Revolutions of Micrometer.	Apparent Measures.	Corrections for Distance.	True Measures of the Constant Angles.
N° 1	4.20	3′. 2″,7	− 5″	2′. 57″,7
2	7.80	5 . 39 ,3	− 9,5	5 . 29 ,8
3	13.85	10 . 2 ,5	−17	9 . 45 ,5
4	21.43	15 . 32 ,2	−26	15 . 6 ,2
5	31.00	22 . 28 ,6	−38	21 . 50 ,5
6	34.00	24 . 39 ,0	−42	23 . 57 ,0
7	34.05	24 . 41 ,2	−42	23 . 59 ,2
8	55.37	40 . 8 ,6	−68	39 . 0 ,6
9	61.34	44 . 28 ,3	−72	43 . 16 ,3

Though I had no reason to doubt the accuracy of the constant angles contained in the last column of this table, yet I was desirous of corroborating these determinations by some other method, of which the magnifying power of the telescope used might enter into the elements of calculation. From a habit of using the same micrometer with telescopes of various focal lengths, as well as from theory, it was previously known, that the measure of one revolution, of any micrometer's screw, varies inversely as the focal lengths of the object-glasses with which it is used; or (which is the same thing) inversely as the magnifying powers of the telescopes vary, when the same eye-piece is used. I considered therefore that the power of any one of my telescopes, multiplied by the corresponding value of one revolution of my micrometer's screw, would always give the same product, as the power of any other telescope multiplied by the corresponding value of a revolution of the same screw applied to it. That is, if P be the power of one telescope, and V the value of a revolution of the micrometer's screw, when used with it; and p and v be respectively the power and value with another telescope; then there will be this ratio, $P : p :: v : V$. But it is known, that in such ratio the product of the two extremes is always equal to the product of the two mean quantities; consequently we have $P \times V = p \times v$. Now as I had previously determined the values of my micrometer with several telescopes of $2\frac{1}{2}$, $3\frac{1}{2}$, and $6\frac{1}{2}$ feet focal length, respectively, and had also ascertained their powers, I took a mean of all the separate products, and found it 2040″; the product of the $6\frac{1}{2}$ feet telescope being $P \times V = 79.65 \times 25''.62 = 2040'',6$, and the rest differing but a few seconds each. Having thus the average of $P \times V$ of all my telescopes, I took this constant number as the basis of my second method of determining the constant angles of the prisms thus; the prisms were now applied, in succession, to the small cap at the *eye-end* of the telescope of 45.75 inches, with the view of measuring the distance between the centres of the same disc that had been used with the prisms in the cap of the *object-end*. In the first position, all the three spider's lines were doubled: viz. the horizontal one, and the two vertical ones. But, turning the cap, which held the prism, round a little, brought the two images of the horizontal line into one, while it opened the other images, or lines, wider apart: a little motion given to the screw, however, soon brought the second and third lines into one black strong line, and left the first and fourth more faint, at equal distances to the right and left. In this situation I found I had obtained the measure of the angle wanted, for the second line of the first image was become co-incident with the first line of

the second image, and the distance of either of the extreme lines from the strong black one, in the middle, was the quantity of the measured angle, as indicated by the micrometer: the same thing was done at the other side of the micrometer's zero, and a mean of the two measures gave the true one, without any index error. The circular disc was disregarded in this operation, and the measures of all the angles were taken by the spider's lines only, in the way that has been described: the various prisms having been successively applied to the cap at the eye-end of the telescope. This process is as simple as accurate. When any prism is screwed into its place, the two images of the horizontal line must first be brought into one *strong* line, and then the two or four vertical images of the co-incident, or separated lines (as the case may be), must be brought nicely into *three*; of which the middle one will be always much *darker* than either of the others, by reason of there being then two images occupying the place of one. It is indeed astonishing with what degree of precision the small angle of any prism may be taken in this way; and, what at first was not suspected, the micrometer indicated the same quantity, to whichever telescope it was thus applied with any prism, or even when it was detached from the telescopes altogether. Hence it was ascertained, that the modification of the small angles, now measured, had no reference to the length of the telescopes, but only to the lenses of the eye-piece, through which the images were viewed: and that the constant product 2040″ (which has also reference to the same eye-piece in all the telescopes) was the number to be separately multiplied, by these micrometrical measures, to produce the respective constant angles of the prisms. An inspection of the annexed table, which contains the results, will show how correctly the constant angles of each prism, unaltered by the power of the telescope, have been deduced by an inverted operation, giving the greater quantity from the smaller, but by a method quite independent of the former, and in no way connected with either distance, or adjustment for vision, except as to the distinctness of the spider's lines, that remained the same in all the measurements.

TABLE II.

Prisms used.	Const. Prod. × Revolutions.	Measure of Const. Angles.
N° I	2040″× .09	3′. 3″,6
2	× .16	5 . 26 ,4
3	× .29	9 . 51 ,6
4	× .44	14 . 57 ,6
5	× .64	21 . 45 ,6
6	× .70	23 . 48 ,0
7	× .71	24 . 8 ,0
8	×1.15	39 . 6 ,0
9	×1.27	43 . 11 ,0

The measures, contained in this little table, depend on a very careful examination of the co-incidence of the second and third images of the spider's lines in the micrometer; in which a very small error will be multiplied many times by the constant product 2040″, here used as a factor; and therefore the constant angles contained in Table I. must be preferred; the whole angle having been obtained at one measurement. But as the latter method of determining the constant angle of any prism, requires not a telescope, and may be taken by the micrometer alone, by the help of the constant factor 2040″, peculiar to the micrometer in question (and to all micrometers having the same thread on its screw, and the same magnifying power in the eye-piece), it may often be convenient to try the angle of a new prism thereby, in selecting one, of a required constant angle, out of many that may be offered for such selection: and in this way the micrometer becomes a new instrument; and may, not inaptly, be called a *prismometer*, when applied to this particular purpose.

We have now before us, in these two tables, data sufficient for determining, with considerable accuracy, the *power* that was used with the telescope of 45.75 inches focus, when the measures of the micrometer contained in the first table were taken. It has already been said, that if the constant angle of any

prism be divided by the power of the telescope, to which it is applied as a double-image micrometer at the eye-end, the quotient will be the measure of the angle, subtended by a line joining the centres of the two images of the object observed. Therefore, if the natural constant angle of any prism be divided by the measure obtained with any given power, the quotient will be that power: the constant angle being a quantity always equal to the product of any power by its corresponding measure. Thus, if we take the micrometrical measures of the nine prisms, contained in the second column of the first table, as taken with the prisms before the object-glass, and divide them by the small measures taken with the same prisms at the eye-piece, as contained in the second column of the second table, we shall have nine different determinations of the telescope's power: and a comparison of each with an average of the whole will show which determination, or which measure of the constant angle, has been most, and which least, accurately taken in the second table. The results are tabulated in the third, or following, table; and afford the easy means of detecting the nature and extent of the small errors existing in the second table.

TABLE III.

Prisms used.	Measures.	Powers.
N° 1	$\frac{4.20}{.09}$	46.6
2	$\frac{7.8}{.16}$	48.7
3	$\frac{13.85}{.29}$	47.7
4	$\frac{21.43}{.44}$	48.7
5	$\frac{31}{.64}$	48.4
6	$\frac{34}{.7}$	48.6
7	$\frac{34.05}{.71}$	47.9
8	$\frac{55.37}{1.15}$	48.1
9	$\frac{61.34}{1.27}$	48.3

If an average be taken of all the powers in this table, it will be found to be 48.11; and a comparison of the constant angles of the respective prisms, as

given in the two preceding tables, will show, that generally, where the power now determined by any prism is in *defect*, as compared with the average, the constant angle in the second table, when compared with the same in the first, is in *exeess*, as it ought to be; and the contrary. Which circumstance proves, that the constant angles determined by the first method alone may be relied on. But the powers which have been now determined, though relatively correct, are all absolutely too great, by reason of their having been taken at a short distance, when the drawer of the telescope was necessarily elongated, to produce distinct vision of the disc observed. The proper correction may be thus applied. If f be put for the solar focus of the object-glass of any telescope, and d for the distance of the object observed (which in this instance was $64\frac{1}{4}$ yards), then, by a theorem in dioptrics, we shall have $\frac{f^2}{d-f} = e$ for the excess of the elongated over the solar focus of the object-glass. And as the powers are directly as the focal lengths, where the eye-piece remains the same, we shall have the following ratio, as $f + e : f : :$ increased power : solar power. Thus, since $f = 45.75$ inches, or 1.27 yard, and $d = 64\frac{1}{4}$, the calculation of e will be $\frac{1.27 \times 1.27}{64.25 - 1.27} = .0256$ of a yard, or .922 of an inch for the elongation; and consequently the proportion will be as $\overline{45.75 + .922} : 45.75 : : 48.11 : 47.16$. Hence we find that, though the power of the telescope was 48.11, when viewing an object at $64\frac{1}{4}$ yards distance only, yet its real power, when adjusted to a celestial body, or with parallel rays, will be only 47.16. And accordingly on marking with a pencil the two positions of the drawer, or sliding tube, of the telescope, when adjusted successively to the disc, and to the sun, the distance between the two pencil marks measured full nine-tenths of an inch; which practical measurement corroborates the calculation arising from the theorem. After these very satisfactory results had been obtained, it occurred, that a still further confirmation of them might be had, by taking a measure of the power, already determined, with a very good double-image dynameter by Mr. DOLLOND, which was accordingly done in the usual way, but with unusual care. And this comparison, by a method totally different from, and independent of, the preceding operations, has put the question of accuracy beyond a doubt. A cap, with an aperture of 3.01 inches, was applied before the object-glass of the said telescope, having the same eye-piece; and when the adjustment was made for distinct vision of a celestial body (a precaution always to be taken in taking celestial powers), the small image of this aperture was 3.20 revolutions

of the dynameter's screw, on an average taken at both sides of zero of the scale: and as each revolution is equal to $\frac{1}{50}$ of an inch, this quantity, expressed decimally, will be $3.20 \times .02 = .064$; so that $\frac{3.01}{.064} = 47.03$ is the solar power thus ascertained; which differs almost insensibly from the celestial average of the powers determined by the prisms and micrometer in conjunction. Now if this determination of the power be correct, its multiplication by the tabular value of the micrometer, 43″.5, as formerly determined by measures of the sun, ought to give the constant product 2040″, or nearly so: and accordingly the product, $47.03 \times 43''.5 = 2045''.8$, differs less than six seconds, in this large quantity, from an average taken from several telescopes, to which the same micrometer had been successively applied. And hence arises another method, which may be called the *comparative* method, of determining either the power of a telescope, or the value of a revolution of its micrometer, when the other is known; provided the constant product, ascertained by the same micrometer, applied previously to another telescope, be also known, which, in the cases under our consideration, we have seen is 2040″. For, if we divide this constant quantity by the known power, the quotient will be the value of a revolution of the micrometer's screw; and, on the contrary, if we divide the same constant product by the value of the micrometer's revolution, the quotient will be the power of the telescope, so long as the same eye-piece is used.

Thus we have several methods of obtaining the magnifying power of a telescope, which will apply to either the refracting or reflecting constructions; and also of determining micrometrical measures, as they are connected with the power; both when the prism of double refraction is used as a micrometer, or dynameter; and also when the spider's line micrometer and double-image dynameter with the divided eye-lens, are used. And the deductions, derived from different and independent sources, that corroborate one another, will be considered as a decisive proof, that there is no error in any of the principles, on which the instruments are respectively constructed. When I come to show the practical mode of applying an union of variable power with the constant angle of an achromatic prismatic solid, by means of a micrometrical eye-piece constructed for this purpose (which I propose to make the subject of a subsequent communication), I trust that its adoption in certain delicate measurements, in practical astronomy, may prove advantageous.

V. *On the construction and use of a Micrometrical Eye-piece of a Telescope. By the* Rev. W. Pearson, *LL.D. F.R.S. and Treasurer of this Society.*

Read April 14, 1820.

AT the conclusion of my paper on the doubly-refracting power of rock crystal considered as the basis of a new micrometer, I proposed to offer another communication explanatory of the application of this principle to some of the most delicate measurements in practical astronomy. But before I proceed to describe the micrometrical eye-piece, that has lately been constructed for the purpose of rendering the constant angle of a prismatic solid, giving double images, of practical utility, it may be proper to advert to the *rationale* of the contrivance. When two lines are required to measure, or to be made equal to, one another, the equality must be effected by either lengthening the one or shortening the other; and, generally speaking, in the ordinary intercourse of social life, the scale of measurement adapts itself to the object to be measured, and is therefore divided and subdivided into many parts. But when the object is capable of enlargement to any extent within certain limits, it may be made to adapt itself to a constant undivided scale, so as to be exactly measured thereby; provided the total length of the scale fall within those limits. This mode of measuring may not at first be obvious, because not practicable, while the object to be measured retains its natural dimensions. If, however, any small object has its dimensions apparently enlarged by a microscope or telescope, until its length is commensurate with a given constant scale; or (which is the same thing) until it becomes the subtense of a constant visual angle, the measure of the enlarged object, or image thereof, thus obtained, will exceed the natural size of the said object, as much as the magnifying power of the instrument exceeds the power of natural vision. If therefore the measure of the enlarged object be diminished, as much as is equal to the known power of magnifying, the resulting measure will be that of the object's natural dimensions. The following simple experiment will practically illustrate this

mode of measurement. Take a slip of writing-paper, about one inch long by half an inch wide, and stick it, when a little moistened, on some pane of a window that faces the sky; and, receding across the room, place a prismatic solid, that gives double images, of a constant angle of from 20′ to 30′, or more, close before one eye, while the other is closed, and two oblong patches will appear, either side by side, or in some other relative position. Turn the prism round its centre, with its face parallel to the window, till the two patches are situated lengthwise exactly on the same vertical line; and holding the prism in this position approach the window very slowly, when the following appearances will take place, in consequence of the change of the visual angle, subtended by the patch, with a change of distance. In the first place, by a gradual approach of the eye the patches will come into contact; then they will form a black line by overlapping. This line will grow broader by degrees, until the patch is equally divided between the two halves, or squares, one of which will be much darker than the other. A further approach towards the window will enlarge the black portion, and diminish the pale one, until the prismatic body comes into contact with the patch, when the second image will entirely vanish. In this experiment the distance between the centres of the two images of the patch, depends on the size of the visual angle, and this again on the distance of the eye from the patch, which itself remains unaltered, though *apparently* enlarged by a diminution of distance.

The analogy between the effects thus produced in one experiment, and the effects produced on the visual angle by a telescope, without reference to the object's change of distance, may be explained visibly, in a striking manner, by another experiment made by means of the telescope itself. Direct a telescope, of 30 inches focal length, for instance, to a distant post, or trunk of a tree, and adjust for distinct vision; then remove the eye-piece, and recede 30 inches from the place of the image in the telescope, and hold the face so, that one eye, directed through the telescope, may view the image of the post, or trunk, while the other sees the object itself: then a juxta-position or superposition of the image, as it regards the object, seen with the other eye, will show that the telescope has no magnifying power, when the eye is situated at the same distance from the image, that the image is from the object-glass. Let the eye now approach the telescope gradually, and the image will appear to be as gradually enlarged, when compared with the object, which it represents, by reason of the corresponding enlargement of the visual angle. There is, however, a limit to the distance, at which the human eye unassisted can view the

image distinctly. As the rays proceeding from the image become more and more divergent, by the proximity of the eye, but few of them enter the pupil, and these few do not come to a focus on the retina, from want of sufficient refractive power in the substance of the eye, to shorten the focus of diverging rays: the consequence is, that at a certain distance from the eye the image becomes misty and disappears. Take now a lens of one foot focus, and interpose it at the distance of its own focus from the place of the image; and the rays, which emerge from it, will become parallel, and the eye will become again capable of viewing the image at a shortened distance. Here the magnifying power, or ratio of the image to its object, is $\frac{30}{12}$: namely, the length of the focus of the object-glass divided by the distance at which the image is distinctly visible. Take away the lens of one foot focus, and substitute another of six inches, at the distance of its own focal length from the image, and the visual angle under which this image will now be distinctly visible, with parallel rays entering the eye, will be increased in the ratio of 30 : 6, or $\frac{30}{6} = 5$ will be the magnifying power. In our former experiment the approach was to the object itself; in this, it is to the *image* of the object, as formed in the telescope; but the principle is the same: the visual angle varies with the distance of the eye from the object, or image respectively, and the measure of the subtense of this varying visual angle is effected in the same manner in both cases. Again substitute a lens of just one inch focus, or an equivalent compound eye-piece, and the visible image will apparently exceed its object in the ratio of 30 : 1. Let now a prismatic solid, that exactly measures any distant object, considered as the subtense of its natural constant angle, be applied before and close to this lens of one inch, or its equivalent eye-piece, and this same constant angle will measure, or be subtended by, just one-thirtieth part of the image of the said object. And in all other cases, the quantity of the measure will vary with the visual angle, but inversely: or, in other words (as has been before asserted), "the constant angle divided by the power of the telescope will universally give the exact measure."

From this consideration of the effect produced by a telescope, it may be clearly understood, that the office of an object-glass, or great speculum, is merely to form a good *image* of any distant object; which image will be greater or smaller, in proportion to the focal distance of that object-glass, or speculum: and that the office of the eye-piece is, to allow the eye to *approach*

that image, without altering the parallelism of the rays proceeding from it, after emergence from the eye-lens. And the nearer that approach is effected, the larger is the visual angle, under which the image is seen; or, in common language, the greater is the magnifying power of the telescope.

I come now to consider the theory, and to describe the construction, of a celestial eye-piece, that affords the ready means of varying the power of any telescope at pleasure, and that consequently is peculiarly adapted, to render the mode of measuring small angles, which has been described, subservient to use. If we take two plano-convex lenses of any diameters, but of unequal radii, and place them at a given distance d from each other, then, by the principles of optics, the focal distance of the two lenses so separated, when measured from the lens next to the focal point, will be expressed by $\frac{(F-d)\,f}{F+f-d}$, F and f being the respective focal lengths of the two lenses. Whence it will be manifest, that the focal length of a lens, of the same diameter as the lens receiving the incident rays, and having the same visual angle as the two lenses so compounded, must be expressed by $\frac{F\,f}{F+f-d} = \phi$; that is, the product of the focal distances of the two lenses, divided by their sum, minus the distance between them, will give ϕ, the focal distance of the single lens required. If now s be put = the solar focal distance of the object-glass of any telescope, $\frac{s}{\phi}$ will express the magnifying power, P, of that telescope; therefore $\frac{s\,(F+f-d)}{F\,f} = P$; varies as $F+f-d$: which shows, that the increase or decrease of magnifying power depends upon the decrease or increase of the distance interposed between two lenses, that form a celestial eye-piece consisting of two such lenses; and that consequently a variation of distance will always be accompanied by a corresponding variation of power in the telescope, to which an eye-piece, giving variable distances, is applied. In the practical application of the theorem in question, the variable quantity d has a limit, with respect to distinctness of vision of an eye-piece to be thus composed, for, when it becomes equal to $F+f$, the eye-piece will become an inverting opera-glass, or small celestial telescope, provided that the inner lens, as is usual, have the longer focal distance of the two. On considering what would be a desirable proportion of $F:f$, that should *double* the power of a four feet telescope, with a scale of distances not too long, I found that the proportion of two to one would produce this effect, provided the lenses, of two inches, and one inch fo-

cal length respectively, were removed an inch and half from each other. Accordingly while in Paris I gave Mons. SOLEIL directions to construct an eye-piece of these dimensions, in which the inner lens, of two inches focus, was to have a motion given to it, by a screw lying parallel to the length of the tube; by which it might be made to approach, or recede from, the eye-lens of one inch focus at pleasure; while a vernier indicated the distance. The eye-piece thus constructed fully answered my expectations. In the mean time I calculated the equivalent single lenses and corresponding powers for each tenth of an inch, to satisfy myself that the differences of the powers would always numerically correspond with the differences of distance, according to the theory; and these calculations proved so satisfactory, that I shall subjoin them in the form of a little table.

Table of variable Powers with a Telescope of 4 feet.

Distances.	Equivalent Lenses.	Powers.	Differences.
.0	.66	72.0	
			2.4
.1	.69	69.6	
			2.4
.2	.714	67.2	
			2.4
.3	.74	64.8	
			2.4
.4	.77	62.4	
			2.4
.5	.80	60.0	
			2.4
.6	.83	57.6	
			2.4
.7	.87	55.2	
			2.4
.8	.909	52.8	
			2.4
.9	.954	50.4	
			2.4
1.0	1.00	48.0	
			2.4
1.1	1.05	45.6	
			2.4
1.2	1.11	43.2	
			2.4
1.3	1.177	40.8	
			2.4
1.4	1.25	38.4	
			2.4
1.5	1.33	36.0	

The titles of the four columns in this Table explain their respective contents; from an inspection of which it will appear, that when the two lenses are removed one inch and a half from contact, the power will be diminished from 72 to 36, which numbers are respectively the maximum and minimum quantities of the scale. As the powers of all telescopes vary with the focal distances of their object-glasses, or specula, when the same eye-piece is used, it is easy to perceive, that a table of powers for any other telescope may be derived, by direct analogy, from the numbers contained in this table; and also that the tenths in this table may readily be extended to hundredths of an inch distance, by the constant addition of .24, in ascending from the lower extremity of the third column, or column of powers. It is also obvious, that the difference between the greatest and smallest powers, divided by the number of intervals, will give the common difference at once, without reference to the intermediate equivalent single lenses.

Having now determined a succession of powers from 72 down to 36 by an eye-lens of one inch focus, I calculated what must be the focus of a second eye-lens, that will give a minimum power of 72 with the same inner or field lens; and found that, at a distance of one inch and a half, it must be exactly a quarter of an inch: and also that the maximum power with such an eye-lens will be 216, or triple the minimum power of 72. Hence $216-72=144$ will give the aggregate of the common differences, and $\frac{144}{150}=\cdot 96$ will be the quantity to be continually added to 72, to give a complete table of 150 distances, of $\frac{1}{100}$ of an inch each, without further trouble. From this explanation and exemplification of the theory of a compound eye-piece, having a variable distance between its two lenses, it is evident, that, when the extreme powers are once determined by a dynameter, carefully applied to any telescope, having such eye-piece, they will be sufficient data for constructing an appropriate table; even though the lenses should not come at all into contact, at the place where the determined power is a maximum. If therefore a diaphragm is considered necessary, to cut off the straggling rays, and to limit the field of view with the higher powers, the scale will no otherwise be affected thereby, than as it will be shortened by the distance reaching from the eye-lens to the diaphragm so introduced. When a diaphragm is used, the construction of the eye-piece will be that of Huygens or Campani, according to the distance between the two lenses; but when the diaphragm is removed, and the distance is reduced within the focus of the eye-lens, it becomes the construction of

RAMSDEN'S eye-piece, which is usually applied to astronomical instruments, that give right ascensions and declinations, as well as to micrometers in general.

The next consideration was to ascertain how far the practical application of an eye-piece, constructed on this plan, would accord with the theory. But before this desirable object could be properly effected, it was requisite to determine the respective focal lengths of all the lenses to be used, with great precision. This was done by the following, and, as far as I know, new method, which takes no cognizance of the thickness of the lenses, but which notwithstanding is capable of the utmost accuracy. It has already been said that when s is the solar focus of an object-glass, and ϕ that of a single eye-lens, $\frac{s}{\phi}$ will give P, or the magnifying power invariably; and this will be true, whether the lens be a double convex, or plano-convex. But the solar focus of an object-glass is easily determinable from the scale of a micrometer used with it, as well as the power from a dynameter, and we have therefore by transposition $\frac{s}{P}=\phi$. For this purpose a telescope of 45.75 inches focal length was chosen, and a disc of exactly one inch diameter was stuck on the face of its object-glass; then a single lens, ordered to be made of one inch focus, was used at the eye-end, and when the telescope was adjusted for distinct vision to the sun, the image of this disc was measured by a good dynameter, at the place of the eye, and found to be .0258 with parallel rays. Whence $\frac{1}{.0258}=38.76$ was the power thus determined; and $\frac{45.75}{38.76}=1.180=\phi$ the required solar focus of the lens. Again a second lens, supposed to have its solar focus at two inches, was applied to the same telescope, under the same circumstances, and the image of the disc was now found by measurement $=\cdot0486$; hence $\frac{1}{\cdot0486}=20.6$ gave the power, and $\frac{45.75}{20.6}=2.22=\phi$ was determined to be the true focus. These two lenses were both plano-convex, and had their convex faces turned towards the incident rays, when their foci were determined: they were then formed into a compound eye-piece, capable of adjustment for distance, or space between them, as will be hereafter described more particularly; and when they were in contact, or at zero of the scale, the image of the disc was .0172 or .0173, and the maximum power consequently $\frac{1}{\cdot0173}=57.97$. Now, to ascertain how far this determination accords with the theorem, when

the thickness of the eye-lens is taken at .08 of an inch, we have $\frac{2.22 \times 1.18}{2.22 + 1.18 - .08} = 0.789 = \phi$, the focus of a single lens equivalent to the compound eye-piece: and $\frac{45.75}{0.789} = 57.98$ for the power, which is almost the same as before determined. The two lenses were then removed to the distance of an inch and half from each other, in which situation, when distinct vision of the Sun was obtained again, the image of the paper disc measured .0308; therefore $\frac{1}{.0308} = 32.37$ was the minimum power. Also, by our theorem, the focal length of the equivalent single lens, ϕ, in this position, will be $\frac{2.22 \times 1.18}{2.22 + 1.18 - 1.5 - .08} = 1.439$; and the corresponding power $\frac{45.75}{1.439} = 31.8$.

This coincidence of theory and practice at both ends of the scale, is a result highly satisfactory, and arises partly out of the accurate determination of the foci of the lenses, partly from the goodness of the dynameter, and partly, probably, out of the mean refractive quality of the glass of which the lenses are composed. Taking now the two extreme powers of this eye-piece at 58 and 32.38, very nearly as determined practically, we shall have 25.62 (their difference) for the aggregate of the common differences throughout the whole scale of 1.5 inches; and $\frac{25.62}{15} = 1.708$ will be the common difference for each tenth of an inch, or .1708 for each hundredth of the inch through the scale of distances, from 0 to 1.5. If therefore we increase the smallest power 32.38 by the constant addition of .1708, 150 times, we shall arrive at 58, the maximum power at the zero of the scale. But as the powers above determined are too low for many purposes, particularly in astronomy, the eye-lens of 1.18 focus was screwed out, and two others of $\frac{1}{2}$ and $\frac{1}{4}$ of an inch focus respectively, were adapted to cells that would screw into the same opening, as successive eye-lenses, to carry the scale of powers as high as could be wanted for any useful purpose. On trial with the telescope of 45.75 inches and the dynameter, as before, the foci of these two additional lenses turned out to be .444 and .188; the former giving, by the dynameter, a scale of powers extending from 50 to 117.1, and the latter from 88.9 to 251.6. But before these results were tabulated, it was deemed advisable to make other determinations of the focal distances of all the four lenses, not only with different discs of the same telescope, but with another telescope of 77.25 inches focal length also: and a comparison of the two determinations by different telescopes, as exhibited in

the subjoined small table, will prove with what remarkable accuracy the focal distances of small lenses may be determined by the simple method above described and exemplified.

	Teles. 77.25.	Teles. 45.75.
Focus of inner lens .	2.215	2.220
Do. of No. 1 eye-lens	1.188	1.180
Do. of No. 2 Do. . .	0.446	0.444
Do. of No. 3 Do. . .	0.181	0.180

With respect to the construction of an eye-piece that will admit of being conveniently used in making observations, by means of the variable powers, a little experience has shown, that the screw, or rackwork, may be dispensed with, and that two tubes sliding over one another in such way, as to allow the two lenses to come into contact, will afford a very convenient scale. For when the lenses are respectively fixed at the outer ends of their tubes, the open end of the larger tube will become the index to the divisions marked on the exterior face of the inner one; and when those divisions are fiftieths of an inch, their halves may be read as hundredths, and numbered accordingly. An eye-piece of this construction was made by Mr. Thomas Jones of Charing-Cross, and is represented by figures 2, 3, and 4 of plate I.; fig. 2 giving the external appearance of the eye-piece, fig. 3 being the section of the same, and fig. 4 a section of the eye-cap containing the prismatic solid. In figs. 2 and 3, *a d* is the outer or sliding tube, and *c b* the inner one, that is attached to the drawer of the telescope by the screw at *c*. The two lenses are seen in fig. 3 at *a* and *d* respectively; and when the cap *d*, in fig. 4, is screwed on at *d*, in figs. 2 and 3, the prismatic solid comes nearly into contact with the outer lens, which may at any time be displaced, or changed for another, by removing the cap from its place. An advantage, attending this adaptation of two tubes, is, that, while the inner one is screwed fast to the tube bearing the rackwork for distinct vision, the outer one is at liberty, not only to slide backwards and forwards, as the quantity of the measure may require; but will turn round also freely, in making the requisite adjustment for the line of position of any two stars, or in making one of the

images of any measurable disc revolve round the other ad libitum, to ascertain if there be any sensible difference in their diameters, at right angles to each other.

It will be convenient to have a succession of about half a dozen prismatic solids, giving double images, with different constant angles, such as were described in my last memoir, each having its separate cell screwing into the cap that covers the eye-piece, in order to obtain various measures, as the observations may require: but the final adjustment of the measure, taken with any particular prism, will depend on the place, on the scale, to which the index points, for giving the distance between the two lenses. In measuring the diameter of a small body of sensible dimensions, the sliding tube *a* containing the prism, must be steadily and gradually moved by the finger and thumb, backwards or forwards, until, after adjustment for good vision, the two images of the object come exactly into contact, edge to edge: in this situation the *distance* indicated, will be the argument for entering the table of *powers*; and the power, there seen by inspection, will be the proper argument for entering the table of *measures*, which will give at sight the apparent diameter, in seconds and parts of a second, without further correction. Should the power consist of a fractional part besides the integer, a corresponding fractional part of a second must of course be taken into the account where extreme accuracy is wanted.

When the angular distance between two stars, satellites, or other luminous points is required to be measured, there will be two pairs of images formed, and these may fall in any direction with respect to each other: but turning the moveable tube *a* with the prism round, more or less, will bring the four luminous images into one straight line; in which position, if the second and third images coincide exactly, the measure will at once be correct; but if not, the distance between the lenses must be varied, until this coincidence takes place. Should the prism used be found to have too great or too small an angle, at any of the distances marked on the scale, it must be changed for another having a more suitable angle, and must be adjusted as before directed. In all cases where one of two contiguous stars is much smaller, in appearance, than the other, and is yet visible through the prism, the small one will be lost by super-position on the larger, and must therefore be made to pass over its centre by a slow motion given by rotation of the tube, when an estimate may be made of the exactness of the central transit: or otherwise, the four visible images may be formed into an exact *square*, when it will appear,

whether or not the bounding sides of the figure are equal to each other; and if they are, the proper distance will be indicated in that position. The only objection that is likely to apply to this mode of measuring small angles, is that which will arise sometimes out of a want of light, particularly when one of the stars is too small to be visible with one half of the light of the telescope; and on this account, a telescope of large aperture will always be preferred; and as small a power must be used, as is just sufficient to produce a good separation of the double stars to be measured, or to give a measurable disc of the body, that is to have its diameter ascertained, whenever there would otherwise be a deficiency of light. But generally speaking, the light coming from the heavenly bodies will admit of powers sufficiently high; and a diminution of light, in stars of the first and second magnitudes, will be found advantageous, as well as pleasant to the observer. Indeed the ease and accuracy with which the measure can be effected will, it is presumed, prove a high recommendation to the general adoption of this eye-piece, in all celestial observations, to which it is applicable. It is hardly necessary to add, that in high altitudes, this eye-piece may have a diagonal reflector, or prism of glass, applied at either end of it, without otherwise interfering with the observation, than by diminishing the light a little; provided that the reflecting plane is ground and polished in a perfect manner, so as not to distort the images. It will be convenient to have the book, in which the observations are to be registered, ruled into nine columns, seven of which must be filled up at the time of observing; but the two columns of *powers* and of *measures*, may be filled up from the tables calculated for this purpose, such as those which conclude this memoir, at any subsequent period, when most convenient. I beg leave to subjoin here a collection of observations actually made during the current year, with the identical eye-piece, which I have here described; and, to avoid the detail of a number of particular adjustments, I have put them into the form of a table, which will at once specify the proposed form of a convenient register, and exemplify the use of the micrometrical eye-piece, in its application to various celestial measurements.

Observations made with a doubly-refracting Eye-piece.

Date.	Object.	Teles.	Eye-lens.	Dist.	Prism.	Power.	Meas.	Remarks.
1820. Jan. 16	Venus	77.25	3	0.42	4	85.3	10″.6	Near the horizon.
—	Jupiter	Do.	3	1.64	6	50.5	29.4	Above Venus, but too low.
—	Do.	Do.	3	1.56	7	52.7	28.3	Average = 29″.5 taken both
—	Do.	Do.	3	0.72	8	76.7	30.7	horizontally and vertically.
—	Saturn	Do.	3	0.575	5	80.85	16.2	in the direction -∞-.
—	Do.	Do.	3	0.35	5	87.3	15.0	in the direction =8=.
—	Rg of Do.	Do.	3	1.15	8	64.4	36.5	thus seen -o-o-.
—	Mars	Do.	3	0.30	5	88.7	14.8	on a mean of 3 trials. Mars in oppos.
22	Do.	45.75	2	0.64	5	88.5	14.8	as before.
—	Do.	Newt.	3	0.18	5	85.2	15.37	average 15″.26 with reflecting telescope.
—	Do.	Do.	2	0.40	8	155.0	15.15	
Mar. 5	Do.	45.75	2	0.59	4	90.8	9.96	average now 10″.08.
—	Do.	Do.	1	1.14	5	127.9	10.2	
16	Do.	Do.	2	.40	4	99.2	9.13	average now 9″.14.
—	Do.	Do.	1	1.00	5	143.1	9.15	
—	Castor	Do.	2	0.40	2	99.2	3.33	between the centres of the two.
20	Do.	Do.	2	0.40	2	99.2	3.33	well defined.
—	Mars	Do.	2	0.40	4	99.2	9.13	Eye-lens touching the diaph., and ∞ not quite in *perfect* contact.
—	Castor	Do.	1	1.40	2	99.7	3.31	nearly as with lens 2, before.
31	Mars	Do.	2	1.20	3	63.4	9.24	Long. diam. } fine evening and good observation.
—	Do.	Do.	2	.98	3	73.2	8.05	Short. diam.
—	Do.	Newt.	2	1.22	4	97.6	9.28	Long. diam. } good observation.
			2	1.00	4	113.0	8.02	Short. diam.

The observations recorded in this register were not made with a view to publication, Venus and Jupiter being in unfavourable altitudes at the time: but those which follow, may be considered as carefully taken under more favourable circumstances, and are probably pretty correct. A little examination of the resulting measures will develope several interesting circumstances, and authorize the most sanguine expectations of important deductions that may arise out of a careful employment of the simple apparatus that forms the sub-

ject of this communication. For instance, we see that Saturn's diameter, taken in the direction of the edge of his ring, then beautifully seen as a straight line, was 16″.2 on the 16th of last January (1820), while that in the transverse direction was only 15″. This difference has been confirmed by subsequent, as well as preceding measures, taken with other telescopes, both refracting and reflecting; and the result proves that the two contrary diameters of this planet are exactly as 27 : 25 with respect to each other. At the same time the length of the line representing Saturn's ring measured 36″.5; or 20″.5 more than the diameter of the body of the planet: thus leaving a projecting ansa, at each side, of 10″.25, when seen at the distance of the earth from Saturn on the evening of the observation. With respect to Mars, it is remarkable, that, in his opposition on the 16th of January last, and for six days after, his apparent diameter was about 9″ less than it was on the 25th of October 1815, when he was eight days after opposition, and when by a good micrometer by Troughton, I measured his diameter (on an average of several repetitions) at 23″.987. This difference shows, in a striking manner, that the orbits of either Mars or the Earth, or both, must be eccentric; since there is a considerable variation of distance between these bodies, at the times of different oppositions. From the 22d of January down to the 20th of March the diameter of Mars has been gradually diminishing from 15″ to 9″.1, notwithstanding the apparent geocentric motion of the planet has been successively retrograde, stationary, and progressive, during this interval.

The angular distance between the centres of the two stars composing Castor, to which the telescope was accidentally pointed on the 15th of March, is given in the catalogues at 5″; but the measures, which have been obtained with two different powers, and as many different prisms, under favourable circumstances, on two different evenings, concur in giving this angular distance only $3''\frac{1}{3}$: and, what will appear remarkable, this appears to be also the exact measure of the real (or spurious) diameter of Pollux; which star on our best globes is given of the second magnitude, while his brother Castor is given of the first. How far an eye-piece of this construction may be advantageously used as a measuring standard of the magnitudes of stars in general; or to determine the question of annual parallax of any star, must be seen from future practice. Lastly, on the 31st of March, on measuring the diameter of Mars with both a refracting and reflecting telescope, there appeared an evident difference between the horizontal and vertical diameters of this planet, on account of his increased distance from opposition, and on ascer-

taining this difference (on an average of all the measures, which agree in a wonderful manner) it turns out, that the darkened portion of the planet is 1″.185; and it may hence be inferred, as the distance of this body from the Earth continues to increase, that the diameters ascertained on the 16th and 20th of March were measures of the *diminished* diameter, since they were smaller than the longer diameter was found on the 31st, as registered at the conclusion of the table of observations.

In the following *table of powers*, column I, headed *Distance*, must be entered with the numbers indicated, on the face of the inner tube, by the extreme end of the outer one, considered as an index; and then on the same horizontal line, under the column No. 1, No. 2, or No. 3 of the lenses (as the case may be), the numbers will be found, that express the *power* of the telescope. For instance, when the distance is 0.60 or .60, the power with No. 1 will be 186.5, with No. 2, 90.3, and with No. 3, 47.8. The powers however will be a little different with different eyes; and therefore every observer should construct his own tables.

In the *table of measures*, the power determined in the last-mentioned table must be found in the first vertical column (or column of *powers*), and in the same horizontal line, under the proper number of the prism used, as seen in the uppermost horizontal line, will be found the corresponding measure required. For example, If the prism numbered 5 has been used, at such a distance between any two of the eye lenses, as indicate a power in the first table of 100, the measure taken under such circumstances will be found here 13″.10; which simple mode of obtaining the results has been fully exemplified in the register of observations, already examined.

Table of Powers.

Telescope 45.75 Inches.

Dist.	Lens No. 1.	Lens No. 2.	Lens No. 3.	Dist.	Lens No. 1.	Lens No. 2.	Lens No. 3.	Dist.	Lens No. 1.	Lens No. 2.	Lens No. 3.
.00	251.6	117.1	58.0	.30	219.1	103.7	52.9	.60	186.5	90.3	47.8
.01	250.5	116.7	57.8	.31	218.0	103.3	52.7	.61	185.5	89.9	47.6
.02	249.4	116.2	57.7	.32	216.9	102.8	52.5	.62	184.4	89.4	47.4
.03	248.4	115.8	57.5	.33	215.8	102.4	52.4	.63	183.3	88.9	47.2
.04	247.3	115.3	57.3	.34	214.7	101.9	52.2	.64	182.2	88.5	47.1
.05	246.2	114.9	57.1	.35	213.6	101.5	52.0	.65	181.1	88.0	46.9
.06	245.1	114.4	57.0	.36	212.6	101.0	51.9	.66	180.0	87.6	46.7
.07	244.1	114.0	56.8	.37	211.5	100.6	51.7	.67	178.9	87.1	46.6
.08	243.0	113.5	56.6	.38	210.4	100.1	51.5	.68	177.9	86.7	46.4
.09	241.9	113.1	56.5	.39	209.3	99.7	51.4	.69	176.8	86.2	46.2
.10	240.8	112.6	56.3	.40	208.2	99.2	51.2	.70	175.7	85.8	46.0
.11	239.7	112.2	56.1	.41	207.1	98.8	51.0	.71	174.6	85.3	45.9
.12	238.6	111.8	56.0	.42	206.1	98.3	50.8	.72	173.5	84.9	45.7
.13	237.5	111.4	55.8	.43	205.0	97.9	50.7	.73	172.4	84.4	45.5
.14	236.4	110.9	55.6	.44	203.9	97.4	50.5	.74	171.3	84.0	45.4
.15	235.3	110.5	55.4	.45	202.8	97.0	50.3	.75	170.3	83.5	45.2
.16	234.3	110.0	55.3	.46	201.7	96.5	50.1	.76	169.2	83.1	45.0
.17	233.2	109.6	55.1	.47	200.6	96.1	50.0	.77	168.1	82.6	44.9
.18	232.1	109.1	54.9	.48	199.6	95.6	49.8	.78	167.0	82.2	44.7
.19	231.0	108.7	54.8	.49	198.5	95.2	49.6	.79	165.9	81.7	44.5
.20	229.9	108.2	54.6	.50	197.4	94.7	49.5	.80	164.8	81.3	44.3
.21	228.9	107.8	54.4	.51	196.3	94.3	49.3	.81	163.7	80.8	44.2
.22	227.8	107.3	54.2	.52	195.2	93.9	49.1	.82	162.7	80.4	44.0
.23	226.7	106.9	54.1	.53	194.1	93.5	48.9	.83	161.6	79.9	43.8
.24	225.6	106.4	53.9	.54	193.0	93.0	48.8	.84	160.5	79.5	43.6
.25	224.5	106.0	53.7	.55	191.9	92.6	48.6	.85	159.4	79.1	43.5
.26	223.4	105.5	53.6	.56	190.9	92.1	48.4	.86	158.3	78.6	43.3
.27	222.3	105.1	53.4	.57	189.8	91.7	48.3	.87	157.3	78.2	43.1
.28	221.3	104.6	53.2	.58	188.7	91.2	48.1	.88	156.2	77.7	43.0
.29	220.2	104.2	53.1	.59	187.6	90.8	47.9	.89	155.1	77.3	42.8

Table of Powers continued.

Telescope 45.75 Inches.

Dist.	Lens No. 1.	Lens No. 2.	Lens No. 3.	Dist.	Lens No. 1.	Lens No. 2.	Lens No. 3.	Dist.	Lens No. 1.	Lens No. 2.	Lens No. 3.
.90	154.0	76.8	42.6	1.20	121.4	63.4	37.5	1.50	88.9	50.0	32.4
.91	152.9	76.4	42.4	1.21	120.4	63.0	37.3	1.51	87.8	49.6	32.2
.92	151.8	75.9	42.3	1.22	119.3	62.6	37.1	1.52	86.7	49.1	32.0
.93	150.7	75.5	42.1	1.23	118.2	62.1	37.0	1.53	85.6	48.7	31.9
.94	149.6	75.0	42.0	1.24	117.1	61.7	36.8	1.54	84.5	48.2	31.7
.95	148.5	74.6	41.8	1.25	116.0	61.2	36.6	1.55	83.5	47.8	31.5
.96	147.4	74.1	41.6	1.26	114.9	60.8	36.5	1.56	82.4	47.4	31.3
.97	146.3	73.7	41.4	1.27	113.9	60.3	36.3	1.57	81.3	46.9	31.2
.98	145.3	73.2	41.3	1.28	112.8	59.9	36.1	1.58	80.2	46.4	31.0
.99	144.2	72.8	41.1	1.29	111.7	59.4	36.0	1.59	79.1	46.0	30.9
1.00	143.1	72.3	40.9	1.30	110.6	58.9	35.8	1.60	78.0	45.5	30.7
1.01	142.0	71.9	40.7	1.31	109.5	58.5	35.6	1.61	76.9	45.1	30.5
1.02	140.9	71.5	40.6	1.32	108.4	58.0	35.4	1.62	75.8	44.7	30.4
1.03	139.8	71.1	40.4	1.33	107.4	57.5	35.3	1.63	74.7	44.3	30.2
1.04	138.7	70.6	40.2	1.34	106.3	57.1	35.1	1.64	73.6	43.8	30.0
1.05	137.6	70.2	40.1	1.35	105.2	56.6	34.9	1.65	72.6	43.4	29.9
1.06	136.6	69.7	39.9	1.36	104.1	56.2	34.8	1.66	71.5	42.9	29.7
1.07	135.5	69.3	39.7	1.37	103.0	55.7	34.6	1.67	70.4	42.4	29.5
1.08	134.4	68.8	39.5	1.38	101.9	55.3	34.4	1.68	69.3	42.0	29.4
1.09	133.3	68.4	39.4	1.39	100.8	54.8	34.2	1.69	68.2	41.5	29.2
1.10	132.3	67.9	39.2	1.40	99.7	54.4	34.0	1.70	67.2	41.1	29.0
1.11	131.2	67.5	39.0	1.41	98.6	53.9	33.9	1.71	66.1	40.6	28.9
1.12	130.1	67.0	38.8	1.42	97.5	53.5	33.7	1.72	65.0	40.1	28.7
1.13	129.0	66.6	38.7	1.43	96.5	53.0	33.6	1.73	63.9	39.7	28.5
1.14	127.9	66.1	38.5	1.44	95.4	52.6	33.4	1.74	62.8	39.2	28.4
1.15	126.8	65.7	38.3	1.45	94.3	52.2	33.2	1.75	61.7	38.8	28.2
1.16	125.7	65.2	38.2	1.46	93.2	51.8	33.1	1.76	60.7	38.3	28.0
1.17	124.7	64.8	38.0	1.47	92.1	51.3	32.9	1.77	59.6	37.9	27.9
1.18	123.6	64.3	37.8	1.48	91.0	50.9	32.7	1.78	58.5	37.5	27.7
1.19	122.5	63.9	37.7	1.49	89.9	50.5	32.5	1.79	57.4	37.0	27.5
								1.80	56.3	36.6	27.3

Table of Measures.

Powers.	1	2	3	4	5	6&7	8	9
10	17.80″	33.00″	58.60″	90.60″	131.00″	144.00″	234.90″	259.60″
11	16.18	30.00	53.27	82.36	119.09	130.91	213.55	236.00
12	14.83	27.50	48.83	75.50	109.17	120.00	195.75	216.33
13	13.70	25.38	45.08	69.69	100.77	110.77	180.69	199.70
14	12.71	23.57	41.86	64.71	93.57	102.86	167.78	185.43
15	11.87	22.00	39.07	60.40	87.33	96.00	156.60	170.07
16	11.12	20.63	36.62	56.62	81.88	90.00	146.81	162.25
17	10.47	19.41	34.47	53.30	77.06	84.70	138.18	152.70
18	9.90	18.33	32.55	50.33	72.77	80.00	130.50	144.22
19	9.37	17.37	30.84	47.68	68.95	75.80	123.63	136.63
20	8.90	16.50	29.30	45.30	65.50	72.00	117.45	129.80
21	8.47	15.71	27.90	43.14	62.28	68.57	111.86	123.62
22	8.09	15.00	26.64	41.18	59.54	65.46	106.77	118.00
23	7.74	14.35	25.48	39.39	56.96	62.61	102.13	112.86
24	7.42	13.75	24.42	37.75	54.59	60.00	97.87	108.17
25	7.12	13.20	23.44	36.24	52.40	57.60	93.96	103.84
26	6.85	12.69	22.54	34.85	50.39	55.39	90.34	99.85
27	6.60	12.22	21.70	33.55	48.52	53.33	87.00	96.15
28	6.36	11.78	20.93	32.36	46.78	51.43	83.89	92.72
29	6.14	11.38	20.21	31.24	45.17	49.65	81.00	89.51
30	5.93	11.00	19.53	30.20	43.66	48.00	78.30	86.53
31	5.74	10.65	18.90	29.23	42.26	46.45	75.77	83.74
32	5.56	10.31	18.31	28.31	40.94	45.00	73.45	81.18
33	5.39	10.00	17.76	27.45	39.70	43.64	71.18	78.67
34	5.24	9.70	17.24	26.65	38.53	42.35	69.09	76.35
35	5.09	9.43	16.74	25.90	37.43	41.14	67.11	74.17
36	4.94	9.17	16.28	25.16	36.38	40.00	65.25	72.11
37	4.81	8.92	15.84	24.48	35.40	38.92	63.48	70.14
38	4.68	8.69	15.42	23.84	34.48	37.90	61.82	68.32
39	4.57	8.47	15.02	23.23	33.57	36.91	60.23	66.56
40	4.45	8.25	14.65	22.65	32.75	36.00	58.73	64.90
41	4.34	8.05	14.29	22.10	31.95	35.12	57.29	63.32

Table of Measures continued.

Powers.	1	2	3	4	5	6 & 7	8	9
42	4″.23	7″.86	13″.95	21″.57	31″.14	34″.28	55″.93	61″.81
43	4.13	7.68	13.63	21.07	30.45	33.49	54.63	60.37
44	4.04	7.50	13.32	20.59	29.77	32.73	53.39	59.00
45	3.95	7.33	13.02	20.13	29.12	32.00	52.20	57.69
46	3.87	7.18	12.74	19.69	28.48	31.30	51.07	56.43
47	3.79	7.02	12.47	19.28	27.86	30.64	49.98	55.23
48	3.71	6.87	12.21	18.88	27.29	30.00	48.94	54.08
49	3.63	6.73	11.96	18.50	26.74	29.38	47.94	52.98
50	3.56	6.60	11.72	18.12	26.20	28.80	46.93	51.92
51	3.49	6.47	11.49	17.76	25.69	28.24	46.06	50.91
52	3.42	6.35	11.27	17.42	25.19	27.69	45.17	49.93
53	3.36	6.23	11.06	17.10	24.72	27.17	44.32	48.99
54	3.30	6.11	10.86	16.78	24.26	26.66	43.50	48.08
55	3.24	6.00	10.67	16.47	23.82	26.19	42.71	47.21
56	3.18	5.89	10.48	16.18	23.39	25.72	41.95	46.36
57	3.12	5.79	10.29	15.90	22.98	25.26	41.21	45.54
58	3.07	5.69	10.10	15.62	22.58	24.82	40.50	44.76
59	3.01	5.59	9.92	15.35	22.16	24.41	39.82	44.01
60	2.96	5.50	9.76	15.10	21.83	24.00	39.15	43.27
61	2.91	5.41	9.61	14.86	21.48	23.60	38.53	42.61
62	2.87	5.32	9.45	14.62	21.13	23.23	37.88	41.87
63	2.82	5.23	9.30	14.38	20.80	22.86	37.29	41.22
64	2.78	5.15	9.16	14.15	20.47	22.50	36.73	40.57
65	2.73	5.07	9.02	13.93	20.16	22.15	36.14	39.94
66	2.69	5.00	8.88	13.72	19.85	21.82	35.59	39.34
67	2.65	4.92	8.75	13.52	19.55	21.50	35.07	38.75
68	2.62	4.85	8.62	13.32	19.26	21.18	34.55	38.18
69	2.58	4.78	8.49	13.13	18.98	20.87	34.05	37.63
70	2.54	4.71	8.37	12.94	18.71	20.57	33.56	37.09
71	2.50	4.64	8.25	12.76	18.44	20.28	33.09	36.54
72	2.47	4.58	8.14	12.58	18.19	20.00	32.63	36.02
73	2.43	4.52	8.03	12.41	17.94	19.73	32.18	35.53

Table of Measures continued.

Powers.	1	2	3	4	5	6 & 7	8	9
74	2″.40	4″.46	7.92	12″.24	17″.70	19″.46	31″.74	35″.07
75	2.37	4.40	7.81	12.08	17.47	19.20	31.32	34.61
76	2.34	4.34	7.71	11.92	17.24	18.95	30.91	34.16
77	2.31	4.28	7.61	11.76	17.01	18.70	30.51	33.71
78	2.28	4.23	7.51	11.61	16.79	18.45	30.12	33.28
79	2.25	4.16	7.42	11.47	16.59	18.22	29.74	32.86
80	2.23	4.12	7.33	11.33	16.38	18.00	29.37	32.45
81	2.20	4.07	7.23	11.19	16.18	17.77	29.00	32.05
82	2.17	4.02	7.14	11.05	15.98	17.56	28.64	31.66
83	2.14	3.97	7.05	10.91	15.77	17.35	28.30	31.28
84	2.12	3.93	6.98	10.78	15.57	17.14	27.97	30.92
85	2.09	3.88	6.90	10.56	15.40	16.94	27.64	30.55
86	2.06	3.84	6.82	10.54	15.23	16.74	27.32	30.18
87	2.04	3.79	6.74	10.41	15.06	16.55	27.00	29.84
88	2.02	3.75	6.66	10.29	14.89	16.36	26.69	29.50
89	2.00	3.70	6.58	10.17	14.72	16.18	26.39	29.17
90	1.98	3.66	6.51	10.06	14.56	16.00	26.10	28.84
91	1.95	3.62	6.44	9.91	14.40	15.82	25.82	28.53
92	1.93	3.59	6.37	9.84	14.24	15.65	25.54	28.22
93	1.91	3.55	6.30	9.74	14.08	15.48	25.26	27.91
94	1.89	3.51	6.24	9.64	13.93	15.32	24.99	27.61
95	1.87	3.47	6.18	9.54	13.78	15.16	24.73	27.32
96	1.85	3.43	6.12	9 44	13.64	15.00	24.47	27.04
97	1.83	3.39	6.05	9.34	13.50	14.84	24.22	26.76
98	1.81	3.36	5.98	9.25	13.37	14.69	23.97	26.49
99	1.80	3.33	5.92	9.15	13.23	14.54	23.72	26.25
100	1.78	3.30	5.86	9.06	13.10	14.40	23.47	25.96
101	1.76	3.26	5.80	8.97	13.07	14.27	23.25	25.70
102	1.74	3.23	5.75	8.88	12.84	14.12	23.03	25.45
103	1.73	3.20	5.69	8.79	12.71	13.98	22.80	25.20
104	1.71	3.17	5.64	8.71	12.59	13.84	22.58	24.96
105	1.70	3.14	5.58	8.63	12.47	13.71	22.37	24.72

Table of Measures continued.

Powers.	1	2	3	4	5	6 & 7	8	9
	"	"	"	"	"	"	"	"
106	1.68	3.12	5.53	8.55	12.36	13.58	22.16	24.48
107	1.67	3.08	5.48	8.47	12.24	13.45	21.95	24.26
108	1.65	3.05	5.43	8.39	12.13	13.33	21.75	24.04
109	1.64	3.02	5.38	8.31	12.01	13.20	21.55	23.82
110	1.62	3.00	5.34	8.24	11.91	13.09	21.36	23.60
111	1.60	2.96	5.29	8.16	11.80	12.97	21.17	23.39
112	1.59	2.94	5.24	8.09	11.69	12.86	20.98	23.18
113	1.57	2.91	5.19	8.02	11.59	12.74	20.79	22.97
114	1.56	2.89	5.15	7.95	11.49	12.63	20.61	22.77
115	1.54	2.87	5.10	7.88	11.39	12.52	20.43	22.57
116	1.53	2.84	5.05	7.81	11.29	12.41	20.25	22.38
117	1.51	2.81	5.00	7.74	11.19	12.30	20.08	22.19
118	1.50	2.79	4.96	7.68	11.09	12.20	19.91	22.01
119	1.49	2.77	4.92	7.61	11.00	12.10	19.74	21.82
120	1.48	2.75	4.88	7.55	10.92	12.00	19.58	21.64
121	1.46	2.72	4.84	7.49	10.83	11.90	19.42	21.47
122	1.45	2.70	4.80	7.43	10.74	11.80	19.26	21.30
123	1.44	2.68	4.76	7.37	10.65	11.71	19.10	21.12
124	1.43	2.66	4.72	7.31	10.56	11.62	18.94	20.94
125	1.42	2.64	4.68	7.25	10.48	11.52	18.79	20.77
126	1.41	2.62	4.65	7.19	10.40	11.43	18.64	20.61
127	1.40	2.59	4.61	7.13	10.31	11.34	18.49	20.45
128	1.39	2.57	4.58	7.08	10.23	11.25	18.35	20.29
129	1.38	2.55	4.54	7.02	10.15	11.16	18.21	20.13
130	1.37	2.53	4.51	6.97	10.08	11.08	18.07	19.97
131	1.36	2.51	4.47	6.91	9.99	10.99	17.93	19.82
132	1.35	2.50	4.44	6.86	9.92	10.91	17.79	19.67
133	1.34	2.48	4.41	6.81	9.84	10.83	17.66	19.52
134	1.33	2.46	4.38	6.76	9.77	10.75	17.53	19.38
135	1.32	2.44	4.34	6.71	9.70	10.67	17.40	19.23
136	1.31	2.42	4.31	6.66	9.63	10.59	17.27	19.09
137	1.30	2.40	4.28	6.61	9.56	10.51	17.14	18.95

Table of Measures continued.

Powers.	1	2	3	4	5	6 & 7	8	9
138	1″.29	2″.39	4″.25	6″.56	9″.49	10″.43	17″.02	18″.82
139	1.28	2.37	4.22	6.51	9.42	10.36	17.90	18.68
140	1.27	2.35	4.19	6.47	9.35	10.29	16.78	18.55
141	1.26	2.33	4.16	6.42	9.28	10.21	16.66	18.41
142	1.25	2.32	4.13	6.38	9.22	10.14	16.54	18.27
143	1.24	2.30	4.10	6.33	9.15	10.07	16.42	18.14
144	1.23	2.29	4.07	6.29	9.09	10.00	16.31	18.01
145	1.22	2.27	4.04	6.24	9.03	9.93	16.20	17.88
146	1.21	2.26	4.01	6.20	8.97	9.87	16.09	17.76
147	1.21	2.24	3.98	6.16	8.91	9.80	15.98	17.64
148	1.20	2.23	3.96	6.12	8.85	9.73	15.87	17.53
149	1.19	2.21	3.93	6.08	8.79	9.66	15.76	17.42
150	1.18	2.20	3.90	6.04	8.74	9.60	15.66	17.31
151	1.17	2.19	3.88	6.00	8.68	9.54	15.55	17.19
152	1.17	2.17	3.86	5.96	8.62	9.48	15.45	17.07
153	1.16	2.16	3.83	5.92	8.56	9.41	15.35	16.96
154	1.15	2.14	3.80	5.88	8.50	9.35	15.25	16.85
155	1.15	2.13	3.78	5.84	8.44	9.29	15.15	16.74
156	1.14	2.11	3.76	5.80	8.38	9.23	15.06	16.64
157	1.13	2.10	3.74	5.77	8.33	9.17	14.97	16.53
158	1.12	2.08	3.72	5.73	8.29	9.11	14.87	16.43
159	1.12	2.07	3.69	5.69	8.24	9.05	14.78	16.32
160	1.11	2.06	3.66	5.66	8.19	9.00	14.68	16.22

VI. *On the construction of a new Position-Micrometer, depending on the doubly-refractive power of Rock Crystal. By the* Rev. W. Pearson, *LL.D. & F.R.S., Treasurer of this Society.*

Read June 8, 1821.

Since I had the honour of laying before the Astronomical Society of London two Memoirs, the one relating to the principle of double refraction, considered as the basis of an ocular micrometer, and the other to the construction of a celestial eye-piece of a telescope having variable powers, I have had occasion to modify the said eye-piece in such manner, as to render it subservient to a new purpose, not less interesting to the practical astronomer, than the measurement of small angles, which was the original object. But, before I proceed to describe the addition I have made to the former instrument, I shall just mention a method of varying the constant angle of a prismatic solid, by the juxta-position of a second solid of double refraction, which to me is new, but which probably may be known to those philosophers, who have studied more minutely the laws of the polarization of light. In trying what effect would be produced by placing over one another two doubly-refracting solids of crystal, face to face, when united in the way that has been before described; upon turning them round in contact, and looking across both in different positions, I perceived that there are only *four* positions, in which a distant object appears double and well defined: in all the other situations of the revolving prismatic solids, several faint images are seen confounded with one another. I had the points marked, to show the four positions where double images take place, and ascertained the constant angles due to these four positions, by means of a spider's-line micrometer. If we denominate the constant angular measure of one of the two prismatic solids a, and that of the other b, I found that, in the first position, considered as zero, when determined by the rotation of the prisms, the quantity measured will be $a-b$, and in the third position $a+b$; these po-

sitions being diametrically opposite each other: but at the points 90° and 270° in the circle of rotation, the two measures are found to be alike; though the quantity is not a mean between the two extremes. Figure 5 of Plate I. exhibits a section of two solids, put together in the usual way, and laid over one another, as seen in the double cap represented by figure 6; which may form a part of the eye-piece seen in figure 2, when screwed to it. Each part of the cap contains a separate solid, and the one in the exterior revolving portion, is so fitted by friction, as to be capable of making a rotation over the face of the other, while the outer portion of the double cap revolves. On the lid of the larger or stationary portion of this cap, are drawn four strokes, at 90° from each other, as seen in the figure; and a single line drawn on the circumference of the revolving portion, indicates the positions of the outer prism in every part of its rotation. The constant angle of the internal or fixed prismatic solid is $=58'=a$, and that of the external one $=54'=b$, when separately measured; and on examining the combinations at the different positions, I found $a-b=(58'-54')=4'$ at the first position, when nicely adjusted for giving double images; and also $a+b=(58'+54')=112'$: but at each of the two intermediate positions, the measure varied from 76′ to 78′ or 80′, according to slight variations in the position; which however cannot be much varied without producing a multiplicity and confusion of images, primary and secondary. In this combination there are *five* effective changes of measure, which may be considered as equal to the measures arising out of five separate prismatic solids: and if the refracting angles of the two solids were properly proportioned, they would thus be sufficient to constitute a complete set. For instance, if a and b were made respectively 30′ and 20′, there would be moreover $30'-20'=10'$, $30'+20'=50'$, and about 36 in each of the two quadrantal positions: which series would be suitable for an ordinary eye-piece with variable powers, to give all the intermediate measures. In such a combination there is neither so much loss of light, nor diminution of the field of view, as might have been expected.

I come now to describe what was intended to be the principal subject of this communication: viz. the arrangement that converts the eye-piece micrometer, with double images, into a *position-micrometer*. When a crystal of the micrometer, represented in figure 2, was applied before the eye-piece of a transit instrument, all the spider's lines, as was expected, were seen double; as was also a star, or other luminous point placed at a distance. But turning the prism round a little, soon brought all the images of the vertical lines into

contact with the lines themselves, and the coincidence was perfect as to breadth, but not as to length of the lines in question: the image of the star in the mean time revolved round the star itself without coming into contact. Likewise when two stars, in the same field of view, are examined through a doubly-refracting prism, a line connecting either star and its own image will be *truly vertical,* when the image of the vertical line is coincident with the line itself, which may always be made so by turning round the prism. While the image of the vertical line was separated from the line itself to its greatest distance, by turning the prism, the image of the star circulated round the star the space of an exact quadrant. In this situation the horizontal line and its image coincided as to breadth, but not as to length, just as the vertical line and its image had done before: and separating them to their greatest distance, brought the vertical line and its image again into a state of coincidence; while the image of the star moved through another quadrant. The same appearances took place in the quadrantal point of the other semi-circle. This experiment led to an immediate conclusion, that if a vernier connected with the revolving prismatic solid, were made to travel along a graduated circle, until a pair of stars and their images are all seen arranged in one straight line, it would indicate, in that position, the angle that this line makes with the vertical or horizontal line, accordingly as the graduations might be figured on the limb; provided that the zero of the circle has been previously adjusted to the vernier, while one of the vertical or horizontal lines had its respective image coincident therewith.

Accordingly I had a circle of three inches diameter graduated, and adapted both to the eye-end of my transit instrument, and to hold a prism attached to the vernier, by Mr. Troughton; which appendage has the peculiar advantage of measuring the required angle of position without the admission of adventitious light, which is a desideratum in the spider's-line micrometer, not to be attained. It is indeed necessary to admit light, to show the lines while the adjustment of the vernier to zero of the circle is making; but when once the parallelism of the line joining the star and its image, as it regards the vertical line, is ensured, the light may be altogether withdrawn. Indeed if one of any two stars under examination is *very small,* the light must necessarily be withdrawn, before the observation, depending on the *straight line,* connecting the two stars with their images, can be completed; for otherwise the small star would disappear amidst the adventitious light. If the two stars, to be observed, happen to lie in a vertical line, when the adjustment of the vertical spider's-

line to coincidence with its image is finished, a line connecting their images will also lie in a vertical position, and will pass consequently through the stars themselves also. In this case no angle will be formed with the vertical; and the two stars will have the same right ascension. Also when two stars have the same declination, the line that joins them, and also passes through their images, will be parallel to the horizon, and consequently will cut the vertical at right angles. But at all intermediate positions of any two contiguous stars, a line joining the said stars, and also passing through their images, will cut both the vertical and horizontal lines, whether these lines are visible or not, in the angle and complement of the angle indicated by the vernier. If the angle made with the vertical is the one indicated, then its complement will be that made by intersecting the horizontal line: and *vice versâ*. Hence it will be obvious, that after the proper adjustment for zero is once made, carrying the vernier gradually along the limb of the circle, until the two stars, to be observed, are seen forming *one straight line with their images*, is all that is required, in taking the measure, which such line makes with the vertical or horizontal line, as the case may be: but then this measure, to be accurate, must be taken when the stars are passing the meridian of the place of observation. If observations of this kind be taken with great care, and repeated at remote intervals, it will ultimately appear, whether or not the obliquity of the measured angle is constant; or, in other words, whether one of the two stars changes its relative position, by revolving slowly round the other: and it is probably from such observations as detect slight changes of angular distance, and of position, in pairs of stars, that the constitution of the starry firmament must be finally determined.

This new method of making delicate observations, on the position of stars, will be rendered more intelligible by a reference to the diagram contained in figure 9, in which *a b* is one of the vertical spider's-lines of a transit instrument, covered by its image; *d* and *e*, two stars in the field of view, when passing the meridian, with *f* and *g* their respective images: in which situation the short lines *d f*, and *e g*, connecting the separate stars with their own images, are both parallel to the vertical line *a b*; and the vernier is supposed to indicate *o* on the circle, that surrounds the eye-piece. Now when the vernier is made to pass along the arc *a c*, the two images *f* and *g* will also pass along their small arcs, until they arrive at the line *c b*, which connects the stars; and the angles *f d c*, and *g e c*, will each be equal to the angle *a b c*, which was to be measured; and which, in this position, will be

indicated by the vernier, as the angle of position. It is in this position also that the small angular distance between the stars themselves must be measured, by an eye-piece having variable powers, without reference to the circle. For, when the power of the telescope is increased or diminished, as the case may require, until the image of star *e* covers the star *d*, the distance between the two lenses, indicated on the graduated tube, will afford datum for finding first the *power*, and then the corresponding tabular *measure*, as before directed, with the utmost accuracy.

After I had been successful in the application of the position-micrometer, which is represented by the circle and vernier, at one end of the instrument seen in figure 7, and also in section in figure 8, I was desirous of uniting in one eye-piece the properties of both my micrometers; viz. that which I have before described, as measuring angular distances, and this which I have now described as measuring angles of position, when forming an appendage to the transit instrument. It was easy to perceive, that such union would be practicable, provided an adjustable vertical line could be introduced, that would not be displaced or broken by the sliding lens of the eye-piece already constructed. At first I introduced a spider's-line for this purpose, in a small piece of tube that held the sliding lens, and had the instrument completed by Fayrer, of White-Lion-Street, Pentonville, to my entire satisfaction, after I had supplied him with suitable prisms. A difficulty however soon presented itself, which I had to remedy: for the lens could not be cleaned on account of the attached spider's-line, near one of its faces. But on mentioning this unpleasant circumstance to Mr. Thomas Jones of Charing-cross, he drew me a fine line with a diamond diametrically across the plane face of the lens; which, being as fine as the spider's-line, completely answers the same purpose: and, as the piece of tube holding the lens will turn round by means of an adjusting fork, taking hold of two holes, the line thus made on the face of the lens can be readily made vertical, after the eye-piece is screwed home, in any telescope whatever. In this instrument the inner lens, or that which has the diametrical stroke of the diamond, is made to approach the eye-lens by a rack and pinion, in order that the vertical line, once adjusted, may not be turned aside from its proper position. The entire instrument is represented in figure 7; and figure 8 is a section of the same. The vernier piece is moved, along with the inner lens, by the rackwork, and shows on the external tube the distance between the two lenses in any given position; while the eye-lens remains stationary: which is just the reverse of the motion given in the eye-

piece denoted by figures 2 and 3. That portion of the instrument, which lies between the eye of the observer and the graduated circle, contains the prismatic solid and vernier connected together, which piece keeps its place by friction, but will revolve round the outer lens, on a fixed ring, that admits of the vernier's coming just in contact with the divided circle. In order that the scale of measurement of the powers may be extended, three eye-lenses will screw, in succession, into the same hole; which lenses have different focal lengths: and when any of them requires to be changed, the vernier piece must be removed from its place; but the prisms can be successively applied at the extreme end, close to the eye, without such removal.

When this micrometer is used as a micrometer for small angular distances, the thumb-screw of the rack, and the screw for distinct vision of the telescope, must both be used alternately, until the measure is correctly taken. But when it is used as a *position-micrometer*, the diamond line must be first distinctly seen, and placed vertical; and as there is but *one distance* between the lenses, that will give distinct vision of this line, it may be desirable, after having found this position, not to alter it, until the angle of position is taken: unless the power requires to be altered for a particular purpose. Because when the vertical line remains visible, by the introduction of lamp- or candle-light, at the side, or object-end of the telescope, after the angle of position has been taken, it may be presumed, that the images of the stars were properly adjusted when the measure was taken. In taking the angle of position, the vernier must be moved to the right or left of zero, accordingly as the upper star precedes or follows the lower: which a little practice will render familiar. And (what is an advantage in this eye-piece, as in other circular instruments,) the measure either of distance, or of position, may be repeated at the opposite end of the diameter, by carrying the vernier just a semicircle from its first measuring position. At some future time, probably, I may be enabled to lay before this society some actual observations, that may show what degree of confidence may be placed in measures taken by this position-micrometer.

VII. *Observations on the best mode of examining the double or compound Stars; together with a Catalogue of those whose places have been identified. By* James South, *Esq. F.R.S. F.L.S. Honorary Member of the Cambridge Philosophical Society, &c.*

Read May 12, 1820.

"Os homini sublime dedit, cœlumque tueri
Jussit, et erectos ad sidera tollere vultus."

AMONGST the various subjects, which engage the attention of practical astronomers, that of the double or compound stars, has perhaps scarcely had so much as might be wished for; and indeed to the venerable President of this Society it is, that astronomy is principally indebted for all she knows, relative to the interesting phænomena they present.

A period little short of forty years has now elapsed since Sir William Herschel communicated to the Royal Society his catalogues of compound stars; and to the present period, I am not aware of an attempt which has been made to repeat, upon any extensive scale, his observations. The Rev. Mr. Wollaston, it is true, in his valuable *Fasciculus*, has described some from his own observations; these, however, are but few in number, and when it is remembered that they were made with an instrument magnifying only 40 times, their importance seems inconsiderable.

Mindful therefore of this circumstance, and thinking as I do, that astronomers require little stimulus to induce them to enter upon this curious subject of inquiry, I beg to offer, through the medium of this society, the means by which, with little trouble, they may establish the truth of, or correct the errors (if any) of that indefatigable philosopher, who has made this province of astronomy peculiarly, if not exclusively, his own.

During the last ten years I have possessed telescopes of different sorts and magnitudes, not however very large, as the means of procuring such, not less

than the opportunity of using them, were denied me. The principal of these is a Gregorian reflector, whose speculum is six inches in diameter, and focal distance thirty inches, and an achromatic of five feet focal length, and 3¾ inches aperture. The former of these was made by WATSON, and is mounted plainly on a pillar and claw stand; the latter is the work of the late Mr. DOLLOND, and is attached to the declination circle of my equatorial.

With the first of these instruments, very many of the stars forming Sir WILLIAM'S catalogues have been observed; but the labour attending the observations soon becomes fatiguing. To practical astronomers it is needless for me to say, that the finding many of the stars with a plainly-mounted telescope is no easy task; and unless indeed the observer is very conversant with the names and relative situations of the stars, it becomes absolutely impossible. Hence it is, that young astronomers, uncertain whether to blame themselves, or their instruments, are often led to relinquish the pursuit with despair.

To avoid this constant labour, and not unfrequent source of mortification, I would propose to conduct the observations of these objects on the meridian; and for this purpose, telescopes such as I have alluded to, if well mounted, and furnished with the requisite appendages, may be advantageously employed. In proposing this mode, I know I am deviating from the plan recommended by Sir W. HERSCHEL, who, in the Philosophical Transactions, vol. 75, p. 42, after describing his own manner of finding the stars, says:—"Before I quit this subject, I must remark, that it will be found on trial that this method of pointing out a double star is not only equal but indeed superior to having its right ascension and declination given; for since it is to be viewed with very high powers, not such as fixed instruments are generally furnished with, the given right ascension and declination would be of no service. We might indeed find the star by a fixed or equatorial instrument, and, taking notice of its situation with regard to other neighbouring stars, find and view it afterwards by a more powerful telescope; but this will nearly amount to the very same way which here is pursued with more deliberate accuracy than we are apt to use while we are employed in seeking out an object to look at."

After such arguments, from such authority, it may be deemed presumptuous, or perhaps indecent in me, to withhold my approval of them: I trust, however, I shall be exonerated from either of these imputations, if I can show that the grounds of my dissent are well founded.

On referring to the principle upon which Sir WILLIAM'S preference to his own mode was founded, it will be seen that he was led to this conclusion by

the fact, that the fixed instruments in our observatories were inadequate to the purpose; and here is an indisputable truth, for at the time his paper was written, and indeed for many years afterwards, the Royal Observatory at Greenwich (which may be fairly taken as a specimen of the rest) could only produce three instruments at all suitable for the task; these were the mural quadrant, the transit instrument, and the equatorial.

The first of these had an achromatic object-glass, of 2.5 inches in diameter, and eight feet focal length; its deepest magnifying power was 60. The second was provided with an object-glass of 2.8 inches in diameter, and eight feet focal length, and its only magnifying power was 80. And the last, although of larger aperture, was only supplied with a power of 70. Hence, therefore, it is very clear that instruments such as these were perfectly inadequate to the purpose.

Now however things are different; telescopes of considerable magnitude are attached to our fixed instruments, capable of bearing magnifying powers of 5 or 600. Hence, therefore, the observations of Sir William Herschel, relative to the inadequacy of fixed instruments, ceasing to apply, become groundless; and it rests for me to proceed to the next part of my subject, which is, to point out their peculiar fitness.

Their claim to this attribute may be substantiated by their superior steadiness; by the unerring certainty with which they may be directed to the wished-for star; by the opportunity they afford us of examining any star, at its most advantageous situation; by the uniformity in the appearance of the compound stars, which they present to the eye and position of the observer; thereby materially assisting him in subsequent observations; and lastly, by the facility which they afford to the dispatch of business.

1st. As to their superior steadiness; of this much need not be said: no one will doubt the fact, who has ever seen a star pass through the field of a fixed instrument.

2dly. As to the certainty with which they may be directed to the required star; this is self-evident, and requires no comment.

3dly. As to the opportunity they afford us of examining any star, at its most advantageous situation; this also needs but little to be insisted upon: for it must be allowed, that objects are viewed with more or less accuracy, according as the medium through which they are observed is more or less dense; and *cæteris paribus*, when on the meridian there must be less obstructing medium than in their lower altitudes.

4thly. As to the uniformity in the appearance of the compound stars, which they present to the eye and position of the observer, whereby he is materially assisted in his future observations of them; this is a position which it is perhaps necessary to explain; its truth however will be best shown, when describing the mode of observing which I pursue with my own instrument.

Lastly. The facility which these instruments afford to the dispatch of observations. Now this will admit of no doubt, if the observer be supplied with a catalogue of these stars, arranged according to their order of right ascension; and of such a catalogue I beg to crave this society's acceptance. For the removal of doubt, however, if any should exist, as to the accuracy of this assertion, I must, as in the last instance, refer the members to a future part of the paper.

The instrument I make use of is an equatorial; by contrivances, however, which it is needless now to enumerate, it becomes a very good transit, and as such only is frequently employed: to enter into a description of it would be foreign to the purpose of the present paper; suffice it therefore to say, that its object-glass, under favourable circumstances, will bear a magnifying power as high as 5 or 600.

Placed in the plane of the meridian, it is supplied with such a power, as best suits the state of the atmosphere. When the star has advanced into the field sufficiently to allow the moveable eye-piece to be brought directly over it, the light illuminating the wires is gradually removed, and the faint star or stars, if any be present, are thus rendered visible: satisfied upon this point, I then cautiously re-admit the light, so as to make the position of the wires barely perceptible: by a slight motion of the declination circle, the larger star is made to run just above or just below the horizontal wire, as near as may be, so as not to touch it. During this time, the eye is capable, in most instances, of following the small star or stars, although perhaps there may be too much light to have enabled the observer in the first instance to have discovered them. Allowing, therefore, the small star to reach the meridian wire, the situation of it, with regard to the larger or bright star, is immediately seen, and may be transferred to paper without difficulty.

To the relative situation and magnitude of the stars should be added their difference in colour (where any exists); and the magnifying power employed, together with the state of the weather, should on no account be omitted.

After this manner, in a night of six hours, I have examined upwards of

100 stars; out of this number 70 were separated, and diagrams made of them at the time.

That the diagrams thus made possess a tolerable degree of accuracy, may be inferred from the circumstance that a friend of mine, a member of this Society, much accustomed to observations of this sort, whilst passing the evening with me, examined and made diagrams of fifty of the same stars, which, on comparison with mine, were found to differ only in one solitary instance.

These therefore, when carefully taken, will materially expedite our application of micrometrical instruments, for the purpose of ascertaining their distances and relative situations; for as yet, these are matters involved in some uncertainty. It is to be hoped, however, that ere long observations will not be wanting, sufficient to remove from minds the most sceptical, all doubts as to these curious and interesting points.

Although it is not my intention to say, that a telescope such as I have described, is adequate distinctly to separate all of Sir W. HERSCHEL'S compound stars, still I am persuaded, that most of them are within its reach; it probably may discover some not yet described by him, and perhaps in a few instances may correct the descriptions he has given: having however examined most of them, I feel warranted in saying these will be but few.

As to the prevalent opinion that the compound stars cannot be well seen, except when the weather is particularly favourable, and when twilight and moon-light are absent, it may be right to observe, that this opinion should not be carried too far; for although it is true that some of them cannot be separated at all, unless under the most advantageous circumstances possible, yet it is as true, that many may be seen during twilight or moon-light, and that several may be more neatly defined, whilst the sun is shining with all its splendour on the meridian, than they can be in the darkest period of the night.

The catalogue accompanying this paper is arranged from that of BODE, published at Berlin, in the year 1801. The reductions are made to the 1st of January 1821. The first column contains the number in BODE'S catalogue; the second, that of FLAMSTEED'S; the third, the name or character of the star; the fourth, its magnitude; the fifth, its right ascension converted into time; the sixth, its annual variation in right ascension; the seventh, its declination; the eighth, its annual variation in declination; the ninth, its class and number in Sir WILLIAM'S catalogue; and the tenth contains the distances between the stars.

It is right for me to observe, that Sir WILLIAM HERSCHEL has described many compound stars, which are not to be found in this catalogue. Their right ascension and declination I am at present unacquainted with.

Having now come to the conclusion of my subject, it remains for me only to apologize to the society for the large demand I have made upon their patience; and to express a hope, and a sincere one it is, that nothing may have escaped my pen, calculated to occasion the least offence towards the illustrious individual, with whom, as to the best mode of observing these objects, I have presumed to differ. Censure I have none to offer; commendation such as mine is unworthy his acceptance.

My sole object has been to facilitate the investigation of this branch of practical astronomy, at the same time showing, that to substantiate the general accuracy of this portion of Herschelian discovery, an unwieldy apparatus is not necessary; but that an observer with moderate instruments, if diligent in the pursuit of, and faithful in narrating observations, may perhaps add something to our present stock of knowledge, alike gratifying to himself, and beneficial to astronomy.

JAMES SOUTH.

Blackman Street,
May 10, 1820.

CATALOGUE.

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.&N.	Dist.
				h ′ ″	″	° ′ ″	″		″
27		Ceti	7	0 2 23	+ 3.0	4 3 27 S	−20.0	—	—
91	35	Piscium	6	0 5 45	+ 3.0	7 49 37 N	+20.0	3.62	12½
96	38	Piscium	7	0 8 9	+ 3.0	7 52 9 N	+20.0	2.50	—
122	51	Piscium	6	0 23 9	+ 3.0	5 57 58 N	+20.0	4.70	22
43		β Toucan	4	0 23 18	+ 2.8	63 56 43 S	−20.0	—	—
44		β Toucan	4	0 23 18	+ 2.8	63 57 9 S	−20.0	—	—
116	29	π Andromedæ	4	0 27 8	+ 3.1	32 43 32 N	+19.9	5.17	34
66	18	α Cassiopeiæ	3	0 30 23	+ 3.3	55 33 22 N	+19.9	5.18	53
142		Andromedæ	6	0 36 49	+ 3.1	29 59 15 N	+19.8	5.123	—
79	24	η Cassiopeiæ	4	0 38 11	+ 3.3	56 52 1 N	+19.8	3.3	11
152	65	Piscium	6	0 40 16	+ 3.1	26 44 9 N	+19.8	2.84	—
164		Andromedæ	6	0 49 56	+ 3.3	43 45 1 N	+19.6	—	—
155	26	Ceti	6	0 54 36	+ 3.0	0 24 28 N	+19.5	4.83	17
188	74	ψ Piscium	5	0 56 4	+ 3.1	20 30 24 N	+19.5	4.9	27
189	76	σ Piscium	6	0 56 21	+ 3.2	31 13 22 N	+19.5	5.16	48
190	77	Piscium	6	0 56 33	+ 3.0	3 57 23 N	+19.5	4.68	29½
2	1	α Ursæ Minor.	2	0 56 49	+13.0	88 21 15 N	+19.6	4.1	17
110	31	Cassiopeiæ	6	0 58 39	+ 3.8	67 49 9 N	+19.4	4.16	20
216	86	ζ Piscium	4	1 4 23	+ 3.1	6 37 42 N	+19.3	4.8	22
118	34	φ Cassiopeiæ	6	1 8 52	+ 3.6	57 17 13 N	+19.2	3.23	12
121	35	Cassiopeiæ	7	1 9 53	+ 3.8	64 3 17 N	+19.2	5.81	43
122	36	ψ Cassiopeiæ	5	1 13 23	+ 4.0	67 11 25 N	+19.1	5.83	33
220		Ceti	7	1 21 57	+ 2.9	19 56 14 S	−18.8	—	—
272	100	Piscium	6	1 25 21	+ 3.1	11 38 24 N	+18.7	4.131	16
304		Piscium	6	1 40 15	+ 3.2	21 23 10 N	+18.3	1.73	—
8		Arietis et Muscæ	6	1 40 15	+ 3.2	21 23 10 N	+18.3	1.73	—
16	5	γ Arietis et Muscæ	4	1 43 42	+ 3.2	18 25 2 N	+18.1	3.9	10
14	47	Rangifer.	6	1 47 30	+ 5.4	76 24 50 N	+18.0	—	—
147	47	Cassiopeiæ	6	1 47 30	+ 5.4	76 24 50 N	+18.0	5.84	51
22	9	λ Arietis et Muscæ	5	1 47 58	+ 3.3	22 43 20 N	+17.9	5.12	36
292		Ceti	7	1 50 39	+ 2.8	23 47 17 S	−17.8	2.58	—
313	113	α Piscium	3	1 52 47	+ 3.0	1 53 48 N	+17.7	2.12	5

B.	Fl.	Stars' Names.	M.	Rt. A.			A. V. Æ.	Declination.				A. V. D.	C.& N.	Dist.
				h	′	″	″	°	′	″		″		″
250	57	γ Andromedæ	2	1	52	56	+3.6	41	28	2	N	+17.7	3.5	9
304	61	Ceti	7	1	54	38	+3.0	1	12	46	S	−17.7	5.102	38
49	14	Arietis et Muscæ	6	1	59	10	+3.3	25	5	6	N	+17.5	6.69	89
255	59	Andromedæ	6	2	0	4	+3.5	38	11	6	N	+17.4	4.129	15
155	55	Cassiopeiæ	6	2	0	33	+4.5	65	40	44	N	+17.4	1.34 or	3.4
27	6	ι Trianguli	6	2	1	54	+3.4	29	28	44	N	+17.3	2.34	—
334	66	Ceti	6	2	3	8	+3.0	3	16	53	S	−17.3	4.25	17
33	10	Trianguli	6	2	8	23	+3.4	27	48	47	N	+17.0	4.40	17
353	68	ο Ceti	2	2	10	18	+3.0	3	47	23	S	−17.0	—	—
378		Ceti	6	2	17	28	+2.8	16	7	45	S	−16.6	—	—
95	30	Arietis et Muscæ	7	2	26	41	+3.4	23	51	56	N	+16.2	5.49	31
101	33	Arietis et Muscæ	5	2	30	9	+3.4	26	17	9	N	+16.0	4.9	25
68	13	θ Persei	4	2	31	57	+3.9	48	28	5	N	+15.9	3.58	13
80		η Persei	4	2	37	41	+4.2	55	8	40	N	+15.5	4.4	26
118	42	π Arietis et Muscæ	6	2	39	18	+3.3	16	42	55	N	+15.5	1.64	—
119	41	Arietis et Muscæ	3	2	39	27	+3.4	26	31	15	N	+15.5	6.5	125
89	20	Persei	6	2	42	21	+3.7	37	36	15	N	+15.3	3.60	14
41		Appar. Chemic.	6	2	49	20	+2.6	25	41	57	S	−14.9	—	—
77		θ Eridani	3	2	51	28	+2.2	41	1	22	S	−14.8	—	—
499		Ceti	7	2	59	40	+3.1	6	46	17	N	+14.3	—	—
143	33	α Persei	2	3	11	34	+4.1	49	13	3	N	+13.6	—	—
32		ξ { Horol. Pend.	6	3	13	39	+1.0	63	16	24	S	−13.3	—	—
33		ξ { Horol. Pend.	6	3	14	5	+1.0	63	12	20	S	−13.3	—	—
159		Persei	6	3	21	16	+3.7	34	50	47	N	+13.0	—	—
25	7	Tauri	6	3	23	50	+3.5	23	51	24	N	+12.7	4.88	20
168	40	ο Persei	4	3	32	55	+3.7	31	31	19	N	+12.1	3.39	15
114	30	Tauri	5	3	38	27	+3.2	19	35	13	N	+11.7	3.6	11
184	44	ζ Persei	3	3	42	52	+3.7	31	20	50	N	+11.4	6.96	71.90
186	43	Persei	5	3	43	19	+4.3	50	10	9	N	+11.4	5.41	50
37	32	Harp Georgii	4	3	45	18	+3.0	3	29	32	S	−11.2	2.36	4
216	32	Eridani	4	3	45	18	+3.0	3	29	32	S	−11.2	2.36	4
190	45	ε Persei	3	3	45	52	+3.9	39	29	25	N	+11.2	2.22	—
203	51	μ Persei	4	4	1	46	+4.3	47	56	56	N	+10.5	6.20	90
209	52	φ Tauri	5	4	9	21	+3.6	26	54	58	N	+ 9.4	6.5.13	55

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.&N.	Dist.
				h ′ ″	″	° ′ ″	″		″
223	59	χ Tauri	5	4 11 41	+3.6	25 11 58 N	+9.2	7.4.10	18
229	62	Tauri	7	4 13 11	+3.5	23 52 31 N	+9.1	4.109	28
30	1	Camelopard.	6	4 17 52	+4.6	53 30 42 N	+8.8	—	—
225	57	Persei	6	4 20 55	+4.1	42 39 41 N	+8.6	6.99	96
229	58	Persei	5	4 24 21	+4.1	40 53 34 N	+8.3	3.65	11
283	87	α Tauri	1	4 25 38	+3.4	16 8 42 N	+8.2	6.66	88
285	88	Tauri	5	4 25 48	+3.2	9 47 20 N	+8.1	6.31	70
19		Scept. Brand.	7	4 26 57	+2.8	9 27 20 S	−8.0	—	—
295	94	σ Tauri	5	4 31 29	+3.5	22 36 23 N	+7.7	6.7	71
357	55	Eridani	6	4 35 0	+2.8	9 8 17 S	−7.4	3.99	9
25	4	ω Aurigæ	5	4 47 6	+4.0	37 36 52 N	+6.4	2.14	—
58	10	Camelopard.	4	4 47 31	+5.2	60 10 2 N	+6.5	6.36	90
385	62	Eridani	6	4 47 33	+2.9	5 27 25 S	−6.4	6.106	60
26		Orionis	6	4 48 49	+3.3	14 15 39 N	+6.2	—	—
36	13	Aurigæ	4	4 52 43	+4.7	52 13 55 N	+6.0	—	—
63	13	Camelopard.	4	4 53 7	+5.4	52 13 55 N	+6.0	6.35	120
334	103	Tauri	6	4 56 23	+3.6	24 1 52 N	+5.6	5.114	30
66	14	Camelopard.	5	4 56 53	+5.5	62 27 16 N	+5.7	—	—
336	105	Tauri	6	4 57 13	+3.5	21 27 18 N	+5.5	6.105	101
400	69	λ Eridani	4	5 0 34	+2.8	8 59 16 S	−5.3	4.43	—
56	14	Aurigæ	5	5 3 46	+3.8	32 27 58 N	+5.0	4.19	16
13	3	ι Leporis	5	5 4 0	+2.7	12 5 53 S	−5.0	3.67	12
68	19	β Orionis	1	5 5 56	+2.8	8 24 52 S	−4.8	1.33	6
74	20	τ Orionis	4	5 8 54	+2.9	7 2 35 S	−4.5	5.25	30
82		Orionis	6	5 11 46	+2.8	8 12 40 S	−4.3	4.87	—
85	23	Orionis	6	5 13 22	+3.1	3 21 27 N	+4.2	4.84	26
350	111	Tauri	6	5 13 53	+3.4	17 12 58 N	+4.1	5.110	47
95	28	η Orionis	3	5 15 28	+3.0	2 34 3 S	−3.9	6.67	—
361	114	Tauri	5	5 16 52	+3.5	21 46 27 N	+3.8	5.115	51½
363	117	Tauri	7	5 17 36	+3.4	17 4 47 N	+3.8	3.93	12
364	118	Tauri	6	5 18 15	+3.6	24 59 55 N	+3.8	2.75	5
117	32	Orionis	5	5 21 9	+3.2	5 48 10 N	+3.5	1.25	—
120	33	Orionis	6	5 21 47	+3.1	3 8 44 N	+3.5	1.22	—
123	34	δ Orionis	2	5 22 52	+3.0	0 26 19 S	−3.4	5.10	53
129	39	λ Orionis	4	5 25 15	+3.2	9 48 28 N	+3.1	2.49	—

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.&N.	Dist.
137	41	θ Orionis	6	h ′ ″ 5 26 30	″ +2.9	° ′ ″ 5 31 28 S	″ −3.0	3.1	″ —
139	43	θ Orionis	4	5 26 35	+2.9	5 33 11 S	−3.0	—	—
142	44	ι Orionis	4	5 26 40	+2.9	6 2 4 S	−3.0	3.12.13 & 14	
115	26	Aurigæ	6	5 27 9	+3.8	30 22 20 N	+3.0	3.64	13½
164	48	σ Orionis	4	5 29 45	+3.0	2 42 35 S	−2.7	2.10 & 11	
170	50	ζ Orionis	2	5 31 43	+3.0	2 2 39 S	−2.6	4.21	25
90	29	Camelopard.	6	5 35 18	+5.1	56 50 46 N	+2.4	4.125	22
136	29	τ Aurigæ	5	5 36 47	+4.1	39 7 5 N	+2.2	5.21	30
53	13	γ Leporis	4	5 37 1	+2.5	22 30 37 S	−2.1	5.50	40
196		Orionis	6	5 38 23	+3.2	6 23 16 N	+2.0	1.20	—
140	32	ν Aurigæ	5	5 39 3	+4.1	39 5 35 N	+2.0	5.90	53
222	58	α Orionis	1	5 45 28	+3.2	7 21 59 N	+1.4	—	—
158	34	β Aurigæ	2	5 46 24	+4.4	44 55 8 N	+1.4	6.88	169
161		Aurigæ	6	5 47 14	+4.3	44 34 34 N	+1.2	5.91	—
163	37	θ Aurigæ	4	5 47 31	+4.1	37 11 45 N	+2.0	6.34	2′½
233	59	Orionis	6	5 49 6	+3.1	1 49 2 N	+1.1	5.100	37
191	41	Aurigæ	6	5 57 52	+4.5	48 44 18 N	+0.4	3.82	8
9	5	Lyncis	6	6 11 9	+5.2	58 30 6 N	−0.7	6.102	88
35	8	Monocerot.	4	6 14 16	+3.1	4 40 51 N	−1.1	3.29	12
38	15	Geminor.	7	6 17 13	+3.9	20 53 33 N	−1.4	5.56	33
11		Argo Navis	7	6 17 55	+2.0	36 36 45 S	+1.4	—	—
229		Aurigæ	7	6 19 36	+4.8	52 35 1 N	−1.5	—	—
48	11	Monocerot.	5	6 20 4	+2.9	6 55 20 S	+1.6	—	—
45	20	Geminor.	6	6 21 51	+3.5	17 54 7 N	−1.7	4.46	—
15		Telescop. Hersch.	7	6 26 28	+4.2	41 40 16 N	−2.2	—	—
60	6	ν Canis Major.	6	6 28 35	+2.6	18 30 56 S	+2.4	6.81	18
23	12	Lyncis	6	6 30 22	+5.3	59 36 30 N	−2.4	3.22 or 1.6	
37		Argo Navis	6	6 30 33	+1.3	52 41 3 S	+2.6	—	—
64	27	ε Geminor.	3	6 32 53	+3.6	25 17 58 N	−2.7	6.73	110
252	56	Aurigæ	5	6 33 55	+4.3	43 45 14 N	−2.7	5.107	53
20	56	Telescop. Hersch.	5	6 33 55	+4.3	43 45 14 N	−2.7	—	—
254	59	Aurigæ	6	6 40 39	+4.1	39 4 30 N	−3.3	4.102	23
23	59	Telescop. Hersch.	6	6 40 39	+4.1	39 4 30 N	−3.3	4.102	—
95		Canis Major.	6	6 43 39	+2.2	31 30 31 S	+3.7	5.108	—
96	38	Geminor.	6	6 44 31	+3.3	13 23 57 N	−3.7	3.47	8

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C. & N.	Dist.
105	17	π Canis Major.	7	6h 47′ 16″	+2.″5	20° 11′ 54″ S	+ 4.″0	5.65	—″
112	43	ζ Geminor.	3	6 53 28	+3.5	20 49 30 N	− 4.5	6.9	92
83		Argo Navis	6	6 58 27	+1.8	43 22 4 S	+ 5.0	—	—
127	51	Geminor.	5	7 3 4	+3.4	16 27 21 N	− 5.3	6.74	—
100		Argo Navis	7	7 5 45	+1.1	56 4 13 S	+ 5.6	—	—
48	19	Lyncis	5	7 8 15	+4.9	55 36 38 N	− 5.5	3.84	7
139	55	δ Geminor.	3	7 9 24	+3.5	22 18 15 N	− 5.8	2.21	—
151	63	Geminor.	6	7 17 5	+3.5	21 48 19 N	− 6.5	5.43	44
144		Argo Navis	7	7 21 0	+0.7	61 55 24 S	+ 6.9	—	—
165	66	α Geminor.	2	7 23 9	+3.8	32 16 26 N	− 6.9	2.1	5
34		Officin. Typog.	6	7 28 18	+2.7	14 5 55 S	+ 7.4	—	—
31		Canis Minor.	7	7 30 36	+3.1	5 38 9 N	− 7.6	1.23	—
196	78	β Geminor.	2	7 34 23	+3.7	28 27 7 N	− 7.9	6.42	—
198	80	π Geminor.	5	7 35 56	+3.8	33 50 49 N	− 7.9	4.53	21½
194	2	Argo Navis	6	7 37 13	+2.7	14 15 30 S	+ 8.1	4.91	17
51	2	Officin. Typog.	6	7 37 13	+2.7	14 15 30 S	+ 8.1	—	—
201		Geminor.	7	7 37 58	+3.4	18 46 30 N	− 8.2	2.64	—
238		Argo Navis	6	7 44 29	+1.2	56 1 37 S	+ 8.8	—	—
1		Ursæ Major.	7	7 47 58	+5.5	64 9 4 N	− 8.8	—	—
53	14	Canis Minor.	6	7 49 4	+3.1	2 41 29 N	− 9.1	6.84	—
8	4	ω Cancri	6	7 50 54	+3.6	25 34 34 N	− 9.2	6.75	75
186		Monocerot.	6	7 53 35	+2.9	5 49 30 S	+ 9.5	—	—
24	11	Cancri	6	7 57 50	+3.6	27 59 48 N	− 9.7	1.11	—
191	29	Monocerot.	6	7 59 35	+3.0	2 28 28 S	+ 9.9	4.97	30
32	16	ζ Cancri	5	8 1 54	+3.4	18 10 57 N	−10.0	1.24 or 3.19.8″	
300	19	Argo Navis	4	8 2 52	+2.8	12 23 57 S	+10.1	4.26	25
37		Cancri	7	8 3 55	+3.4	18 9 0 N	−10.2	6.78	—
320		Argo Navis	7	8 7 15	+2.2	35 47 20 S	+10.5	—	—
58	22	φ Cancri	7	8 15 32	+3.6	28 25 50 N	−11.0	6.109	—
60	23	φ Cancri	6	8 15 56	+3.6	27 30 48 N	−11.0	2.40	—
62	24	υ Cancri	6	8 16 1	+3.5	25 7 2 N	−11.1	2.41	—
64		Cancri	6	8 16 17	+3.2	8 8 33 N	−11.1	5.109	—
207	30	Monocerot.	4	8 16 42	+3.0	3 19 34 S	+11.1	6.118	211
76	31	θ Cancri	5	8 21 21	+3.4	18 41 41 N	−11.5	5.59	45

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.& N.	Dist.
18		Hydræ	7	h 8 ′ 26 ″ 19	″ +3.2	° 7 ′ 14 ″ 27 N	″ −11.9	3.49	″ 12
377		Argo Navis	6	8 31 5	+1.4	57 23 43 S	+12.2	—	—
378		Argo Navis	6	8 31 7	+1.4	57 36 48 S	+12.2	—	—
231		Monocerot.	7	8 33 46	+2.9	7 51 32 S	+12.4	6.107	—
383		Argo Navis	7	8 33 48	+2.1	41 50 55 S	+12.4	—	—
232	31	Monocerot.	4	8 34 52	+2.9	6 35 29 S	+12.4	6.82	70
124	48	ε Cancri	5	8 35 50	+3.6	29 24 33 N	−12.5	4.52	30
140	54	Cancri	7	8 41 2	+3.3	16 0 30 N	−12.8	4.111	17
144	51	σ Cancri	6	8 41 28	+3.7	33 8 17 N	− 2.9	6.86	—
242	15	Monocerot.	6	8 42 45	+2.9	6 30 56 S	+13.0	—	—
47	15	Hydræ	6	8 42 45	+2.9	6 30 56 S	+13.0	5.120	43
152	57	ι Cancri	5	8 43 17	+3.6	31 15 7 N	−12.9	1.30	—
246	17	Monocerot.	6	8 46 40	+2.9	7 17 46 S	+13.2	—	—
55	17	Hydræ	6	8 46 40	+2.9	7 17 46 S	+13.2	2.77	—
174	64	σ Cancri	6	8 48 31	+3.7	33 6 52 N	−13.3	6.87	86
180	67	ρ Cancri	6	8 51 7	+3.6	28 36 0 N	−13.4	6.41	96
49	13	σ Ursæ Major.	5	8 54 29	+5.4	67 51 3 N	−13.6	3.54	8
50	14	τ Ursæ Major.	5	8 56 1	+5.0	64 14 3 N	−13.7	5.73	55
194		Cancri	6	8 57 6	+3.4	23 41 47 N	−13.9	—	—
53		Ursæ Major.	8	8 58 31	+4.9	62 25 50 N	−13.9	—	—
54	16	Ursæ Major.	5	9 0 4	+4.8	62 9 2 N	−14.0	5.15	49
91	22	θ Hydræ	4	9 4 58	+2.9	3 4 3 N	−14.5	5.54	60
154	38	Lyncis	4	9 7 39	+3.7	37 33 23 N	−14.4	1.9	—
159	40	Lyncis	4	9 10 7	+3.7	35 8 40 N	−14.7	3.84	7
107	27	Hydræ	6	9 11 42	+2.9	8 48 30 S	+14.8	6.85	—
73	21	Ursæ Major.	6	9 12 52	+4.3	54 46 44 N	−14.8	2.73	—
76	23	Ursæ Major.	4	9 17 13	+4.8	63 50 13 N	−15.1	4.29	19
476		Argo Navis	6	9 17 34	+0.0	73 57 39 S	+15.2	—	—
477		Argo Navis	6	9 17 39	+0.0	74 7 20 S	+15.2	—	—
14	2	ω Leonis	5	9 18 44	+3.2	9 49 55 N	−15.2	1.26	—
16	3	Leonis	6	9 18 54	+3.2	8 57 36 N	−15.3	4.47	24
127	31	τ Hydræ	5	9 20 1	+3.0	2 0 0 S	+15.3	6.71	62
27	6	Leonis	6	9 22 20	+3.2	10 30 3 N	−15.4	4.26	16
4		ζ Antliæ Pneum.	6	9 23 7	+2.5	31 6 8 S	+15.5	—	—
21		Leonis Minor.	6	9 23 49	+3.7	40 24 45 N	−15.5	—	—

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.&N.	Dist.
				h ′ ″	″	° ′ ″	″		″
5		ζ Antliæ Pneum.	6	9 23 51	+2.5	31 6 28 S	+15.5	—	—
37	7	Leonis	6	9 26 1	+3.6	15 9 48 N	−15.6	5.58	42
494		Argo Navis	7	9 27 23	+2.1	48 56 55 S	+15.7	—	—
50	14	Leonis	4	9 31 34	+3.2	10 42 18 N	−15.9	6.76	63
86	25	Leonis	6	9 48 18	+3.3	22 10 16 N	−16.8	5.63	53
113	32	α Leonis	1	9 58 50	+3.2	12 50 21 N	−17.2	6.11	168
143	41	γ Leonis	2	10 10 4	+3.3	20 44 47 N	−17.7	1.28	—
145		Leonis	8	10 11 11	+3.1	7 20 58 N	−17.8	2.43	—
155		Leonis	7	10 14 0	+3.1	6 37 33 N	−17.9	5.64	—
557		Argo Navis	7	10 23 23	+2.5	41 18 51 S	+18.3	—	—
43		θ Rob. Caroli	3	10 36 35	+2.1	63 27 29 S	+18.7	—	—
245	54	Leonis	5	10 45 53	+3.2	25 42 13 N	−18.9	—	—
253	57	Leonis	6	10 46 56	+3.0	1 22 25 N	−19.0	5.62	33
326	74	φ Leonis	4	11 7 34	+3.0	2 40 21 S	+19.5	6.79	99
244	53	ξ Ursæ Major.	4	11 8 45	+3.2	34 4 16 N	−19.5	1.2	—
256		Ursæ Major.	7	11 15 58	+3.2	30 58 52 N	−19.7	—	—
372	81	Leonis	6	11 16 15	+3.1	17 26 28 N	−19.6	5.61	57
382	83	Leonis	8	11 17 44	+3.0	3 59 10 N	−19.7	4.13	29
386	84	τ Leonis	4	11 18 42	+3.0	3 50 29 N	−19.7	6.12	83
262	57	Ursæ Major.	6	11 19 26	+3.2	40 19 49 N	−19.7	3.86	—
274		α Hydræ	7	11 21 1	+2.9	37 28 26 S	+19.7	3.96	10
407	88	Leonis	6	11 22 33	+3.1	15 21 39 N	−19.7	3.51	15
415	90	Leonis	6	11 25 19	+3.1	17 47 7 N	−19.8	1.27	—
32		Centauri	7	11 27 26	+2.8	49 47 29 S	+19.8	—	—
35		Centauri	7	11 28 5	+2.9	38 0 30 S	+19.9	—	—
7	4	ξ Virginis	6	11 38 42	+3.0	9 14 24 N	−19.9	6.113	146
471	93	Leonis	4	11 38 44	+3.1	21 12 48 N	−19.9	6.80	70
338	65	Ursæ Major.	7	11 45 44	+3.1	47 28 27 N	−20.0	1.72	60
490	95	ο Leonis	6	11 46 26	+3.0	16 38 34 N	−20.0	6.13	90
4	2	Comæ Berenic.	6	11 55 7	+3.0	22 27 37 N	−20.0	2.47	—
74	12	Virginis	6	12 4 18	+3.0	11 15 35 N	−20.0	4.114	23
1		Apis	7	12 4 32	+3.0	65 31 21 S	+20.0	—	—
15	2	Canum Venat.	5	12 7 9	+3.0	41 39 26 N	−20.0	3.85	12

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C. & N.	Dist.
				h ′ ″	″	° ′ ″	″		″
55		Comæ Berenic.	7	12 11 43	+3.0	28 4 28 N	−20.0	—	—
124	17	Virginis	6	12 13 26	+3.0	6 18 12 N	−20.0	5.50	20
61	12	Comæ Berenic.	5	12 13 29	+3.0	26 50 37 N	−20.0	5.121	59
39	7	δ Corvi	3	12 20 37	+3.1	15 30 48 S	+20.2	5.4.105	23
95	24	Comæ Berenic.	5	12 26 9	+3.0	19 21 41 N	−19.9	4.27	18
278	27	Virginis	6	12 32 36	+3.0	11 24 1 N	−19.9	6.81	89
279	29	γ Virginis	3	12 32 37	+3.0	0 27 57 S	+19.9	3.18	7
138	35	Comæ Berenic.	4	12 44 28	+2.9	22 13 24 N	−19.6	5.130	31
79	12	Canum Venat.	3	12 47 38	+2.8	39 17 17 N	−19.6	4.17	20
212		Camelopard.	5	12 47 57	+0.2	84 23 3 N	−19.6	4.15	20
403	44	Virginis	6	12 50 26	+3.0	2 50 37 S	+19.6	4.51	22
451	51	θ Virginis	4	13 0 40	+3.0	4 34 47 S	+19.4	6.43	64
465	54	Virginis	6	13 3 49	+3.1	17 52 6 S	+19.3	2.45	—
488	61	Virginis	5	13 9 7	+3.1	17 17 37 S	+19.2	6.90	73
423	79	ζ Ursæ Major.	2	13 16 42	+2.4	55 51 47 N	−19.0	3.2	14
426		Ursæ Major.	5	13 21 50	+2.2	60 52 16 N	−18.9	6.22	3½
557	81	Virginis	6	13 28 17	+3.1	6 57 22 S	+18.6	1.80	—
565	84	Virginis	6	13 34 3	+3.0	4 27 1 N	−18.4	2.44	—
218	3	Centauri	4	13 41 31	+3.4	32 6 4 S	+18.2	3.101	12
222		Centauri	7	13 43 0	+3.5	37 22 15 S	+18.1	—	—
42	8	η Bootis	3	13 46 9	+2.8	19 18 15 N	−18.0	6.95	1½
617	93	τ Virginis	5	13 52 31	+3.0	2 25 33 N	−17.8	6.77	68
86	13	Bootis	6	14 1 35	+2.2	50 18 34 N	−17.3	6.112	78
113	17	κ Bootis	6	14 7 2	+2.1	52 38 0 N	−17.1	3.11	12½
136	21	ι Bootis	4	14 9 48	+2.1	52 11 47 N	−17.0	5.9	37
9		χ Turdi Solitar.	7	14 15 27	+3.3	19 8 36 S	+16.7	—	—
10		χ Turdi Solitar.	7	14 15 29	+3.3	19 8 53 S	+16.7	—	—
309		α { Centauri	4	14 28 2	+4.4	60 6 31 S	+16.1	—	—
310		α { Centauri	1	14 28 3	+4.4	60 6 15 S	+16.1	—	—
249	29	π Bootis	4	14 32 18	+2.8	17 11 29 N	−16.0	3.8	6
252	30	ζ Bootis	3	14 32 35	+2.8	14 30 11 N	−15.9	6.104	90
30	73	Turdi Solitar.	5	14 35 37	+3.4	24 40 4 S	+15.7	3.97	11
397	73	Hydræ	5	14 35 37	+3.4	24 40 4 S	+15.7	3.97	11
276	36	ε Bootis	3	14 37 9	+2.6	27 50 8 N	−15.6	1.1	—

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.& N.	Dist.
				h ' '	"	° ' "	"		"
34		Libræ	6	14 41 50	+3.0	0 29 21 N	−15.3	1.81	—
300	37	ξ Bootis	4	14 43 7	+2.7	19 51 2 N	−15.2	2.18	3
303	39	Bootis	6	14 43 34	+2.0	49 27 42 N	−15.2	2.79	—
44	15	ξ Libræ	6	14 47 3	+3.2	10 40 50 S	+15.0	2.20 and 21	
52	18	Libræ	6	14 49 13	+3.2	10 24 55 S	+14.9	4.56	—
33		Ursæ Minor.	6	14 56 21	−0.5	75 36 2 N	−14.4	—	—
358	44	Bootis	6	14 57 54	+2.0	48 21 22 N	−14.4	1.15	—
89	24	ι Libræ	4	15 2 1	+3.3	19 6 17 S	+14.2	6.44	59
97		Libræ	6	15 4 20	+3.3	17 44 55 S	+14.0	—	—
396	49	δ Bootis	3	15 8 16	+2.4	33 59 26 N	−13.7	6.16	2¼
6	5	Serpent. Oph.	6	15 10 7	+3.0	2 27 24 N	−13.6	3.106	—
117	5	Libræ	6	15 10 7	+3.0	2 27 24 N	−13.6	—	—
10	2	η Coronæ Boreal.	5	15 15 46	+2 4	30 56 33 N	−13.3	1.16	—
46	12	Ursæ Minor.	7	15 17 10	+0.0	71 51 50 N	−13.1	5.86	—
409	51	μ Bootis	4	15 17 44	+2.2	38 0 38 N	−13.1	2.17	128
33	13	Serpent. Oph.	3	15 26 15	+2.8	11 8 44 N	−12.6	1.42	—
178		Libræ	6	15 29 0	+3.2	8 11 53 S	+12.4	—	—
35	7	ζ Coronæ Boreal.	4	15 32 39	+2.2	37 13 27 N	−12.1	2.8	5
69	28	β Serpent. Oph.	3	15 37 55	+2.7	15 59 28 N	−11.8	4.36	24
61	12	λ Coronæ Boreal.	5	15 49 14	+2.1	38 28 12 N	−10.9	6.94	95
112		Serpent. Oph.	7	15 52 12	+2.6	22 8 7 N	−10.7	—	—
71	15	ρ Coronæ Boreal.	5	15 54 9	+2.3	33 51 6 N	−10.6	6.93	88
211	51	ξ Libræ	4	15 54 32	+3.2	10 52 11 S	+10.6	1.33	—
53	51	ξ Scorpii	4	15 54 32	+3.2	10 52 11 S	+10.6	1.33	—
55	8	β Scorpii	2	15 55 2	+3.4	19 18 22 S	+10.5	3.7	—
20	7	κ Herculis	7	15 59 55	+2.7	17 31 46 N	−10.1	5.8	40
78	14	ν Scorpii	8	16 1 35	+3.4	18 58 26 S	+10.0	5.6	38
133	49	Serpent. Oph.	6	16 3 58	+2.7	14 1 6 N	− 9.8	1.82	—
87	17	σ Coronæ Boreal.	5	16 8 3	+2.2	34 19 11 N	− 9.5	1.3	—
89	18	υ Coronæ Boreal.	6	16 9 32	+2.3	29 35 59 N	− 9.4	5.37	50
104	20	σ Scorpii	5	16 10 18	+3.6	25 9 16 S	+ 9.4	4.121	22
50		γ Herculis	3	16 14 0	+2.6	19 34 49 N	− 9.1	5.19	42
12	5	Ophiuchi	5	16 14 51	+3.5	23 1 27 S	+ 9.0	2.19	—
55	23	Herculis	6	16 16 3	+2.3	32 45 8 N	− 8.9	5.38	36

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.&N.	Dist.
				h ′ ″	″	° ′ ″	″		″
28	10	λ Ophiuchi	4	16 21 52	+3.0	2 23 9 N	−8.4	1.83	—
116	36	Herculis	6	16 31 36	+2.9	4 32 51 N	−7.7	5.72	60
117	37	Herculis	6	16 31 39	+2.9	4 34 27 N	−7.7	5.72	60
107	16	Draconis	5	16 31 57	+1.4	53 15 53 N	−7.5	1.4	—
108	17	Draconis	5	16 31 59	+1.4	53 17 20 N	−7.5	1.4	—
126	42	Herculis	5	16 33 53	+1.6	49 17 0 N	−7.4	4.63	21
128	40	ζ Herculis	3	16 34 33	+2.3	31 55 54 N	−7.2	1.36	—
137	43	Herculis	5	16 37 9	+2.8	8 54 49 N	−7.2	3.41	12
140	46	Herculis	7	16 37 49	+2.3	28 41 29 N	−7.1	1.79	—
58	19	Ophiuchi	6	16 38 7	+3.0	2 23 27 N	−7.0	4.123	20
192		Herculis	6	16 53 34	+2.7	15 12 2 N	−5.9	—	—
122	20	Draconis	6	16 55 32	+0.2	65 18 42 N	−5.6	1.19	—
208	60	Herculis	6	16 57 6	+2.7	12 59 28 N	−5.6	5.133	49
128	121	μ Draconis	5	17 1 37	+1.2	54 42 37 N	−5.1	2.13	4
148	36	Ophiuchi	5	17 4 23	+3.7	26 18 24 S	+4.8	—	—
234	64	α Herculis	3	17 6 29	+2.7	14 36 3 N	−4.6	—	—
155	38	Ophiuchi	6	17 6 34	+3.7	26 25 27 S	+4.7	1.35	—
184	31	Scorpii	6	17 6 34	+3.7	26 25 27 S	+4.7	1.35	—
156	39	ο Ophiuchi	6	17 7 5	+3.6	24 4 48 S	+4.7	3.25	10
238	65	δ Herculis	4	17 7 40	+2.4	25 3 37 N	−4.6	5.1	34
157	53	ν Serpent. Oph.	4	17 10 44	+3.3	12 39 10 S	+4.4	5.29	32
273	75	ρ Herculis	4	17 17 30	+2.0	37 19 6 N	−3.8	2.4	3
242	54	Ophiuchi	6	17 26 5	+2.7	13 17 27 N	−3.0	3.35	8
241	53	Ophiuchi	6	17 26 6	+2.8	9 43 6 N	−3.0	5.30	32
138	24	ν Draconis	4	17 28 38	+1.1	55 18 34 N	−2.8	5.11	55
139	25	ν Draconis	4	17 28 43	+1.1	55 17 51 N	−2.8	5.11	55
272	61	Ophiuchi	6	17 35 33	+3.0	2 41 3 N	−2.2	4.32	19
334	86	μ Herculis	4	17 39 27	+2.3	27 50 19 N	−1.9	4.41	18
295		Ophiuchi	7	17 42 59	+3.0	1 9 59 N	−1.6	—	—
147	31	ψ Draconis	4	17 45 7	−1.1	72 14 6 N	−1.2	4.7	28
15	4	Sagittarii	6	17 48 51	+3.6	23 47 14 S	+1.1	—	—
312	67	Ophiuchi	4	17 51 40	+3.0	2 56 59 N	−0.9	6.2	14
13	67	Tauri Poniat.	4	17 51 40	+3.0	2 56 59 N	−0.9	6.2	—
315	69	τ Ophiuchi	5	17 53 17	+3.2	8 10 29 S	+0.7	1.88	—
385	95	Herculis	4	17 53 53	+2.5	21 36 15 N	−0.7	3.26	6

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.&N.	Dist.
317	70	Ophiuchi	4	17h 56′ 23″	+3.0″	2° 33′ 29″ N	−0.5″	2.4	—″
18	70	Tauri Poniat.	4	17 56 23	+3.0	2 33 29 N	−0.5	—	—
320	73	Ophiuchi	6	18 0 39	+2.9	3 57 13 N	−0.0	1.87	—
24	73	Tauri Poniat.	6	18 0 39	+2.9	3 57 13 N	−0.0	1.87	—
55	13	μ Sagittarii	4	18 3 2	+3.5	21 5 47 S	−0.1	5.7	—
185	58	η Serpent. Oph.	3	18 12 1	+3.0	2 55 49 S	−0.9	5.14	81
5	40	Cephei	5	18 13 25	−4.4	79 57 50 N	+1.4	4.67	21
164	40	Draconis	5	18 13 25	−4.4	79 57 50 N	+1.4	4.67	21
6	41	Cephei	5	18 13 32	−4.4	79 58 6 N	+1.4	4.67	21
165	41	Draconis	5	18 13 32	−4.4	79 58 6 N	+1.4	4.67	21
192	59	Serpent. Oph.	6	18 18 2	+3.0	0 5 31 N	+1.5	1.12	—
175	39	Draconis	5	18 21 16	+0.8	58 41 56 N	+1.8	1.7	—
7		κ Coronæ Austral.	6	18 21 53	+4.1	38 51 9 S	−1.6	—	9
22		Pavonis	6	18 24 53	+5.8	64 46 58 S	−1.9	—	—
23		Pavonis	6	18 26 8	+5.8	64 41 55 S	−2.0	—	—
40	3	α Vulturis et Lyræ	1	18 30 51	+2.0	38 36 59 N	+2.6	5.39	37
2	2	Aquilæ et Antinoi	5	18 32 28	+3.2	9 12 34 S	−2.7	5.36	43
49	2	Scuti Sobieskii	5	18 32 28	+3.2	9 12 34 S	−2.7	—	—
57		Vulturis et Lyræ	7	18 37 57	+2.1	33 51 39 N	+3.2	—	—
59	4	ε Vulturis et Lyræ	5	18 38 18	+1.9	39 29 15 N	+3.2	2.5	—
60	5	Vulturis et Lyræ	6	18 38 20	+1.9	39 25 41 N	+3.2	2.6	—
61	6	ζ Vulturis et Lyræ	5	18 38 32	+2.0	37 25 30 N	+3.2	5.2	42
200	46	Draconis	5	18 39 10	+1.1	55 21 35 N	+3.4	6.37	3½
72	8	ν Vulturis et Lyræ	6	18 42 6	+2.2	32 36 59 N	+3.6	5.40	—
76	10	β Vulturis et Lyræ	3	18 43 28	+2.2	33 9 51 N	+3.6	5.3	—
208	63	θ Serpent. Oph.	3	18 47 19	+2.9	3 58 32 N	+4.0	4.6	19
88	11	δ Vulturis et Lyræ	5	18 47 26	+2.0	36 45 10 N	+4.0	6.3	—
212	47	ο Draconis	4	18 48 32	+0.8	59 10 19 N	+4.2	4.20	27
20	11	Aquilæ et Antin.	6	18 50 52	+2.7	13 23 27 N	+4.3	3.32	7
169	38	ζ Sagittarii	4	18 51 12	+3.8	30 7 31 S	−4.3	5.78	—
32	15	Aquilæ et Antin.	6	18 55 30	+3.1	4 17 27 S	−4.7	5.33	34
233		Draconis	7	18 59 22	−1.9	75 33 15 N	+5.2	—	—
8	56	Cephei	6	19 5 22	−2.3	76 47 19 N	+5.7	—	—
237	56	Draconis	6	19 5 22	−2.3	76 47 19 N	+5.7	2.31	5

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.& N.	Dist.
6		Cygni	6	19h 7′ 26″	+1″.5	49° 31′ 10″ N	+ 5″.7	—	—″
144	20	η Vulturis	6	19 7 42	+2.0	38 50 43 N	+ 5.8	4.2	26
215		Sagittarii	7	19 8 15	+3.5	19 2 47 S	− 5.8	5.77	36
77	24	Aquilæ et Antin.	7	19 9 39	+3.0	0 1 7 S	− 5.9	1.14	—
151	21	θ Vulturis	6	19 10 8	+2.0	37 49 18 N	+ 6.0	6.56	90
226		Sagittarii	7	19 11 6	+3.5	19 33 25 S	− 6.0	6.120	74
85	28	Aquilæ et Antin.	6	19 11 17	+2.8	12 1 15 N	+ 6.1	5.34	35
27		Cygni	6	19 21 0	+1.8	44 39 21 N	+ 6.9	—	—
30	6	β Cygni	4	19 23 29	+2.4	27 35 27 N	+ 7.1	5.5	39
11	4	ε Sagittæ	5	19 29 7	+2.7	16 3 47 N	+ 7.5	6.26	92
151		Aquilæ et Antin.	7	19 33 22	+3.2	8 43 11 S	− 7.9	—	—
72	16	Cygni	5	19 37 6	+2.2	33 44 28 N	+ 8.2	5.46	30
80	18	δ Cygni	3	19 39 22	+1.8	44 41 46 N	+ 8.3	1.94	—
81	17	χ Cygni	5	19 39 31	+2.2	33 20 0 N	+ 8.3	—	25 Variable.
171	52	π Aquilæ et Antin.	6	19 40 23	+3.3	11 22 37 N	+ 8.4	1.92	—
22	8	ζ Sagittæ	6	19 41 0	+2.6	18 41 56 N	+ 8.4	2.30	5½
177	53	α Aquilæ et Antin.	1	19 42 1	+2.9	8 23 59 N	+ 9.2	6.16	143
187	57	Aquilæ et Antin.	6	19 44 54	+3.2	8 41 19 S	− 8.8	4.14	29½
260	63	ε Draconis	5	19 48 46	−0.1	69 48 52 N	+ 9.2	1.8	—
114	24	ψ Cygni	5	19 50 59	+1.5	51 57 51 N	+ 9.3	2.15	—
30	13	χ Sagittæ	6	19 51 54	+2.7	17 1 44 N	+ 9.3	4.64	23
342	64	Sagittarii	6	19 55 11	+3.3	12 5 50 S	− 9.5	4.3	25
51		Octant. Naut.	6	19 55 49	+9.4	79 29 1 S	− 9.3	—	—
131	26	Cygni	6	19 56 16	+1.6	49 36 48 N	+ 9.7	5.47	39
52		Octant. Naut.	6	19 56 59	+9.5	79 35 44 S	− 9.4	—	—
263	64	Draconis	5	19 59 32	+0.6	64 19 20 N	+10.0	6.38	120
264	65	Draconis	6	20 0 18	+0.6	64 7 50 N	+10.0	6.38	120
38	17	θ Sagittæ	6	20 2 0	+2.6	20 22 43 N	+10.1	3.24	—
13	5	α Capricorni	4	20 7 42	+3.3	13 3 18 S	−10.4	6.4	75
156	31	ο Cygni	5	20 7 59	+1.8	46 11 58 N	+10.5	6.10	100
15	7	σ Capricorni	7	20 9 2	+3.4	19 40 11 S	−10.6	5.87	50
164	32	Cygni	5	20 9 56	+1.8	47 10 8 N	+10.7	6.33	120
18	9	β Capricorni	3	20 10 55	+3.3	15 20 23 S	−10.7	6.28	180
32	11	ρ Capricorni	6	20 18 37	+3.4	18 23 52 S	−11.3	6.29 or	2.51
37	12	Capricorni	7	20 19 37	+3.4	19 9 59 S	−11.3	4.71	23

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.&N.	Dist.
				h ′ ″	″	° ′ ″	″		″
35	1	κ Cephei	5	20 21 36	+1.6	77 11 28 N	+11.6	3.70	6
15		Delphini	6	20 22 29	+2.8	10 45 52 N	+11.6	3.16	12
209	45	ω Cygni	5	20 24 30	+1.8	48 21 10 N	+11.7	4.23	30
211	46	ω Cygni	5	20 25 45	+1.8	48 37 11 N	+11.8	4.24	—
26	6	β Delphini	3	20 29 8	+2.8	13 58 49 N	+12.1	5.35	26
228	49	Cygni	6	20 33 40	+2.4	31 40 22 N	+12.4	2.98	—
237	52	Cygni	6	20 38 9	+2.4	30 4 7 N	+12.7	2.25	—
44	12	γ Delphini	3	20 38 20	+2.7	15 29 17 N	+12.7	3.9	10
240	54	λ Cygni	4	20 40 25	+2.3	35 50 14 N	+12.8	6.32	—
13	4	Aquarii	6	20 41 54	+3.1	6 17 29 S	−12.9	1.44	—
91	19	Capricorni	7	20 44 40	+3.4	18 35 41 S	−13.1	—	—
1	1	ε Equulei	5	20 49 56	+3.0	3 36 33 N	+13.5	3.21	9
296		Cygni	6	21 0 5	+2.0	46 56 14 N	+14.2	4.22	18
19		Equulei	7	21 4 37	+2.9	6 28 16 N	+14.4	—	—
21	7	δ Equulei	4	21 5 45	+2.9	9 17 33 N	+14.5	4.37	19½
339	69	Cygni	6	21 18 27	+2.4	35 54 1 N	+15.2	5.44	—
66		γ { Octant. Naut.	6	21 21 10	+7.2	78 9 47 S	−15.2	—	—
67		Octant. Naut.	6	21 21 10	+7.2	78 10 2 S	−15.2	—	—
74	22	β Aquarii	3	21 22 7	+3.1	6 21 14 S	−15.4	5.76	33
117	8	β Cephei	3	21 26 19	+0.8	69 46 34 N	+15.7	3.6	13
135	39	ε Capricorni	4	21 27 2	+3.3	20 15 48 S	−15.5	6.6	75
36	3	Pegasi	6	21 28 42	+2.9	5 49 9 N	+15.8	5.98	35
84	24	Aquarii	6	21 30 22	+3.0	0 50 41 S	−15.8	4.38	25
54		Indi	6	21 31 12	+4.3	58 7 53 S	−15.8	—	—
55		Indi	6	21 31 14	+4.3	58 5 13 S	−15.8	—	—
374	76	Cygni	6	21 34 23	+2.4	40 0 6 N	+16.1	5.43	48
50	8	ε Pegasi	2	21 35 22	+2.9	9 3 35 N	+16.1	6.103	91
382	79	Cygni	6	21 35 58	+2.4	37 29 39 N	+16.3	6.57	100
383	78	μ Cygni	3	21 36 7	+2.6	27 56 28 N	+16.2	3.15	7
154	13	μ Cephei	6	21 48 52	+2.0	55 44 40 N	+16.8	4.79	21
173	17	ξ Cephei	5	21 58 33	+1.6	63 45 42 N	+17.2	2.16	5
1		α Toucan.	3	22 6 9	+4.2	61 8 51 S	−17.5	—	—
169	51	Aquarii	6	22 14 42	+3.1	5 44 14 S	−17.9	5.95	—
171	33	Pegasi	6	22 15 0	+2.8	19 57 0 N	+18.0	5.99	45

B.	Fl.	Stars' Names.	M.	Rt. A.	A. V. Æ.	Declination.	A. V. D.	C.&N.	Dist.
				h ′ ″	″	° ′ ″	″		″
202		Cephei	6	22 16 14	+1.7	65 47 45 N	+17.9	—	—
181	55	ζ Aquarii	4	22 19 35	+3.0	0 56 1 S	−18.1	2.7	—
211	27	δ Cephei	4	22 22 30	+2.1	57 29 58 N	+18.2	5.4	38
12	10	Honor. Fred.	6	22 31 10	+2.6	38 7 33 N	+18.5	—	—
50	10	Lacertæ	6	22 31 10	+2.6	38 7 33 N	+18.5	5.97	52
15	12	Honor. Fred.	6	22 33 25	+2.6	39 17 42 N	+18.6	—	—
53	12	Lacertæ	6	22 33 25	+2.6	39 17 42 N	+18.6	6.121	1
213		Aquarii	7	22 33 39	+3.1	9 14 36 S	−18.6	1.50	—
233	44	η Pegasi	3	22 34 36	+2.7	29 17 22 N	+18.7	6.21	135
222		Aquarii	8	22 35 55	+3.1	10 35 0 S	−18.7	3.69	—
230	69	τ Aquarii	6	22 38 10	+3.1	14 59 52 S	−18.7	5.80	37
231		Aquarii	7	22 38 35	+3.1	5 9 4 S	−18.8	2.57	—
234	71	τ Aquarii	5	22 40 5	+3.1	14 32 3 S	−18.8	6.97	124
65	16	Lacertæ	6	22 48 11	+2.7	40 39 26 N	+19.0	4.85	—
41	16	Honor. Fred.	6	22 48 13	+2.7	40 39 9 N	+19.1	—	—
261	2	Cephei	7	23 2 5	+2.5	58 21 47 N	+19.4	—	—
2	2	Cassiopeiæ	7	23 2 5	+2.5	58 21 47 N	+19.4	6.55	2½
294	91	ψ Aquarii	5	23 6 28	+3.1	10 3 40 S	−19.5	4.12	—
311	94	Aquarii	6	23 9 39	+3.1	14 25 34 S	−19.5	3.34	14
322		Aquarii	7	23 14 28	+3.1	9 26 19 S	−19.7	—	—
277	3	Cephei	6	23 16 14	+2.6	57 41 21 N	+19.7	—	—
3	3	Cassiopeiæ	6	23 16 14	+2.6	57 41 21 N	+19.7	6.25	2¼
280	4	Cephei	5	23 16 55	+2.6	61 18 4 N	+19.7	—	—
4	4	Cassiopeiæ	5	23 16 55	+2.6	61 18 4 N	+19.7	6.24	—
24	8	κ Piscium	5	23 17 45	+3.0	0 16 46 N	+19.7	6.62	120
372	107	Aquarii	6	23 36 43	+3.1	19 40 24 S	−19.9	2.24	—
33		Andromedæ	7	23 48 57	+3.0	30 45 14 N	+19.9	—	—
20	8	σ Cassiopeiæ	6	23 49 57	+2.9	54 45 32 N	+20.0	1.5	
37		Andromedæ	6	23 50 57	+3.0	32 44 9 N	+20.0	—	—
310		Cephei	7	23 54 35	+2.9	65 42 56 N	+20.0	—	—
311	9	Cephei	6	23 55 3	+3.0	61 17 30 N	+20.0	—	—
24	9	Cassiopeiæ	6	23 55 3	+3.0	61 17 30 N	+20.0	5.79	53
49	21	α Andromedæ	2	23 59 8	+3.0	28 6 19 N	+20.0	5.32	55

JAMES SOUTH.

Blackman Street,
May 12, 1820.

VIII. *On the new Meridian Circle at* GOTTINGEN. *Communicated by Professor* GAUSS, *in a Letter to the Foreign Secretary, of which the following is an abstract.*

Read November 10, 1820.

SIR, Observatory, Gottingen, June 3, 1820.

I RECEIVED on Tuesday last your favour of May 9th, with a copy of the Regulations of the Astronomical Society lately formed at London. * * * * As the society has declared it to be one of their purposes to gather and propagate notices of Astronomical instruments of superior merit, I take the liberty to annex to this letter a description (just leaving the press) of an astronomical circle, lately erected at the observatory under my direction.

I am, Sir, with high respect,

Your most obedient Servant,

CHARLES FREDERICK GAUSS.

To Mr. HERSCHEL, &c. &c.
Foreign Secretary of the Astronomical Society,
London.

The new meridian circle by REICHENBACH, received at our observatory during the course of last year, has already been some time fixed and in use; we have however judged it necessary, previous to any public notice of its merits, to wait the result of a considerable series of trials, so as to be enabled to pronounce with greater certainty on its performance, being on a construction now employed for the first time.

The instrument is adapted at once for a transit, and for the measurement of altitudes: and possesses (in common with the most perfect meridian telescopes) all the adjustments requisite for their purpose, which require no further description. The telescope is five Paris feet in focal length, and four

Paris inches in aperture. The four eye-pieces magnify respectively 68, 86, 120, and 170 times. (Mr. GAUSS uses the most powerful almost exclusively.) The cross-wires were obliged to be renewed here, some of them having become loosened. They now consist of seven vertical and two horizontal spider-threads. The intervals between the former are each traversed by an equatorial star in 14″. The horizontal ones are only 7″,6 asunder.

The axis (33 Paris inches in length) carries on one side two concentric circles, whose outer surfaces (or those furthest from the telescope) lie nearly in one plane. The exterior circle (which being fastened on the axis, revolves with the telescope,) bears the divisions, which are to every three minutes. The inner, or *Alidade* circle, would turn freely about the axis, were it not for a clamp fastened on the pillar. This allows it only a small delicate motion for the purpose of adjusting the level fastened on it. On this alidade circle are the four indices, each 45° from the vertical line, with their verniers; which subdivide the principal division into 90 parts each; and consequently from 2″ to 2″, and yet smaller parts admit of estimation. The diameter of the circle where the reading-off takes place is 35 Paris inches. That both circles, without being in actual contact, are yet separated by an interval scarce perceptible, and that in consequence the microscopes for reading-off are purposely set somewhat obliquely, the surface of the divided circle standing out a little, though but extremely little beyond that of the alidade circle, are adjustments which this instrument possesses in common with others by the same artist. *Three* sets of counter-weights (1°. for the whole instrument, 2°, for the alidade circle, and 3°. for the telescope,) are intended respectively to take off the pressure of the pivots on their beds, and that of the alidade circle on the axis, and to counteract the tendency of the telescope itself to bend by its own weight. In the suspended level, employed for levelling the axis, a motion of one Paris inch in the bubble corresponds to 22″; in the principal level to 17″,6. The latter serves to retain the alidade circle always in the same position, or to measure and take into account very small disturbances. (Mr. GAUSS usually leaves the adjusting-screw untouched, unless the displacement amount to 2″ or upwards, which is seldom the case.)

The inversion of the instrument, which is easily and securely performed by means of a very convenient apparatus, serves, where altitudes are in question, to eliminate the error of collimation, and convert measured into absolute zenith distances. But as it may appear hazardous to regard this error as quite unchangeable for a long lapse of time, and after considerable changes of tem-

perature, it is preferable (unless the instrument be very frequently reversed) to refer the observations to the pole rather than the zenith. It is so much the easier to determine the place of the pole on the limb by circumpolar stars, as the optical power of the instrument is surprisingly great. In this respect it is scarce inferior to Reichenbach's transit, described in the *Gött. Gelehrten Anzeigen*, 1819, p. 167; though the latter, with a greater focal length, has also a somewhat greater aperture, and the trifling difference which does exist is counterbalanced by the circumstance, that the meridian circle being susceptible of adjustment beforehand with the utmost exactness on the object, the attention of the observer need only be directed to a very small part of the field. Among the stars observed in the day-time are a multitude of the fourth magnitude. ι Draconis (Hevelii) and ω Cephei (Hevelii), which were observed nearly in their conjunction with the Sun at noon, are but of the fifth. So delicate is the construction of the reticule, that the day observations of these minute points, even when just visible, admit of the same accurate attention as those of the largest stars.

We subjoin here, as a proof of the admirable agreement of the results obtainable by this instrument, those of all the determinations of the place of the pole on the limb, from the opening of the day-book to the first reversion of the instrument, deduced from opposite culminations of circumpolar stars observed immediately after one another. The harmony of the results afforded by one and the same star, shows at once the nicety with which the observations may be made, and the constancy of the error of collimation throughout this interval. The no less remarkable coincidence of the results deduced from different stars proves the excellence of Reichenbach's division, which is the more to be admired considering the moderate size of the circle.

1820.	Names of Stars.	Mag.	Pl. of the Pole.	1820.	Names of Stars.	Mag.	Pl. of the Pole.
Feb. 26	α Cephei	3	321° 29′ 32″,14	Feb. 28	δ Cephei	4	321° 29′ 31″,12
	β Cephei	3	31,89	Mar. 9	δ Draconis	3	31,18
	δ Draconis	3	32,29		β Cephei	3	32,18
	η Cephei	3.4	32,24		τ Cephei	4	31,42
	α Cephei	3	31,16		ι Cephei	4	31,52
	β Cephei	3	31,65	10	β Cephei	3	32,05
27	δ Draconis	3	31,39	11	1 (H.) Draconis	5	31,97
	η Cephei	3.4	31,90	18	1 —— Draconis	5	32,04
	δ Draconis	3	31,54		3 Lacertæ	4	32,32
	η Cephei	3.4	31,63		α Ursæ Maj.	2	21,62
28	γ Cephei	3	31,54	19	α Ursæ Min.	2	32,81
	τ Cephei	4	32,74				

The mean is 321° 29′ 31″,84, from which no one of these 23 determinations differs by a whole second.

So far as the observations with this instrument are referred to the pole and not to the zenith, the perfectly accurate determination of the altitude of the pole is a subordinate object; and since, when seconds and fractions of seconds are in question, the greatest circumspection is required, and moreover so perfect a constancy in the error of collimation as the above statement exhibits cannot always be expected; it is advisable, when we would ascertain this element with the last exactness, to combine only such observations as immediately precede and follow an inversion of the axis. M. GAUSS does not venture, however, to employ his observations hitherto made for *this* purpose, as a considerable interval of unfavourable weather intervened after the first inversion; and at the second, on the 13th of April, a trifling accidental derangement of the cross threads rendered the *perfect* equality of the errors of collimation somewhat doubtful, though the results of his observations agree very nearly both with each other and with those obtained by another method, presently to be explained. The combination of the place of the pole in the first interval, with that in the second (38° 25′ 53″,54), gives the latitude 51° 31′ 49″,15; and the combination of thirteen observations of circumpolar stars, immediately before the second inversion, with the opposite culminations of the same stars taken immediately after it, gives 51° 31′ 47″,86.

A point which has occupied the attention of astronomers for some years, though it involves only a few seconds, is yet of the highest importance, both in reference to the art of astronomical observation, and on account of the numerous astronomical elements, whose exact determination depends on it; I mean the minute differences in the declinations of stars, the obliquity of the ecliptic, and the altitude of the pole, which appear in their determination by different though very excellent instruments. There is no doubt these differences arise from the action of gravity on the different parts of each instrument, though hitherto the mode of this action has not been clearly pointed out, nor is it possible to pronounce decidedly which instrument has afforded the right and which the wrong result. We know, in fact, very little of the extent to which the yielding of the metals may go; and it seems too hazardous to deny the possibility of this cause exercising a notable influence on the divisions, and in consequence on the observations in any instrument, whatever be its construction, without grounding such denial on sufficient proof. In our meridian circle, the great artist has done every thing to obviate the flexure of

the telescope by a well-adapted system of counterpoises: still a doubt may remain, whether all the flexure be done away with by that means, or rendered quite insensible; and the only direct means of ascertaining the point seems to be, the combination of immediate observations of a heavenly body, with those of its image reflected in an artificial horizon. Such observations must of course be frequently repeated to clear up a point of so much delicacy. M. GAUSS has already entered on this inquiry by observing the pole-star in a reflecting surface of water. It is perhaps the most striking proof of the astonishing optical power of the telescope, that the superior culmination may be very well observed in this manner even in the day-time. The result of the first *complete* observation of this kind was as follows:

May 13, 1820.—Zenith distance of the north star, free from refraction, but including the error of collimation.

Inferior culmination . .	Direct	319°	50′	20″,73.
	Reflected	220	5	3 ,94.
Superior do.	Direct	323	8	41 ,51.
	Reflected	216	46	44 ,31.

Hence we deduce the true zenith distance.

Inferior culmination . . .	40°	7′	21″,60.
Superior do. . . .	36	49	1 ,40.

And hence (the change of declination in 12 hours being $-0'',1$) the latitude of the place of the water-vessel is 51° 31′ 48″,45, and that of the centre of the circle 51° 31′ 48″,40.

This being nearly a mean between the two above given, it is rendered very probable that the effect of its weight on the observations with this instrument are either quite insensible, or at least extremely small. The declination of the North Star from the above observations comes out 0″,42 less than in BESSEL's tables. (The whole series of M. GAUSS's observations to the present time, give the value of this correction $-0'',67$.)

To give one more proof of what this instrument is capable of performing as a transit, we subjoin here some observations of Mars by M. GAUSS, about the time of the quadrature of that planet.

1820.	Mean time.	Æ of precs Limb.	Dec. of the Centre.
	h ′ ″	° ′ ″	° ′ ″
March 29	7 10 16,3	114 37 49,3	24 16 47,3 N
30	7 7 40,7	114 57 57,7	24 12 7,1
31	7 5 6,8	115 18 30,1	24 7 18,1
April 5	6 52 37,3	117 6 19,9	23 42 5,4
10	6 40 40,4	119 2 17,5	23 14 30,6
11	6 38 20,3	119 26 18,3	23 8 41,0
12	6 36 1,6	119 50 41,5	23 2 48,0
13	6 33 43,7	120 15 15,0	22 56 44,8
26	6 5 16,8	125 56 10,0	21 29 12,8

The comparison of these observations with M. Von Lindenau's tables of Mars was performed by M. Von Staudt, who devotes himself among us with distinguished zeal and success to the study of mathematical and astronomical science. They gave the following differences.

1820.	Differences.	
	Æ	Dec.
	″	″
March 29	−0,7	+5,1
30	−0,5	+3,9
31	−2,1	+6,1
April 5	−1,3	+3,9
10	−4,9	+2,9
11	−1,9	+3,6
12	−4,8	+1,9
13	−1,8	+4,2
26	−1,5	+1,5

IX. *On the Solar Eclipse which took place on September 7,* 1820. *By F.* Baily, *Esq. F.R.S. and L.S.*

Read December 8, 1820.

THE solar eclipse of the 7th of September last, having excited general attention throughout Europe, on account of its magnitude, I shall venture to lay before the society such observations as I myself made relative thereto, and also the result of such observations as have been communicated to me by others, whose accuracy I have no reason to doubt. These latter however are at present neither so numerous nor important as I had reason to expect, considering the number of good observers, who must have witnessed this phænomenon: nevertheless I flatter myself that the observations of such persons will eventually be communicated to the public in some other manner.

My own observations were made at Kentish Town, near the bottom of Highgate Hill, in N. Lat. 51° 33′ 34″, and W. Long. 35″,2 in time, from Greenwich. The state of the clock was determined by several altitudes of the sun, taken on the morning and evening of the 6th, 7th, and 8th, with a Troughton's reflecting circle; the results of which agreed with each other to great exactness. The following are the times of the beginning and end of the eclipse.

$$\left.\begin{array}{lll} \text{Beginning} & = & 0^{h}\ 21'\ 42'',4 \\ \text{End} & = & 3\ \ 13\ \ 41,1 \\ \hline \text{Duration} & = & 2\ \ 51\ \ 58,7 \end{array}\right\}\text{mean time at the place.}$$

In noting the time of the beginning of the eclipse, I have not made any allowance for the first second or two of time, which must (I think) in all cases elapse before the commencement of the eclipse can become visible to a spectator, even with the best telescopes. With respect to the termination of the

eclipse, I do not consider any such allowance to be necessary, as the eye can follow the moon till it is completely off the sun's disc. The telescope, made use of, was a $3\frac{1}{2}$ feet refracting telescope by Tulley, with an object-glass of $3\frac{3}{4}$ inches diameter, and magnifying 38 times: but the object end was covered with a brass cap, which reduced the aperture to two inches. The eye was protected from the rays of the sun by a dark glass of a *red* colour; a circumstance which I have thought proper to mention in this place, as it appears, from the remarks of M. Messier, that the *colour* of the glass is not immaterial in observations of this kind.

The sun was perfectly free from spots during the whole of the day; and had been so for the day or two previous thereto. Soon after the commencement of the eclipse, a succession of flying clouds prevented any correct measures being taken of the enlightened part of the sun's disc. Towards the middle of the eclipse however the clouds dispersed; and I had an excellent opportunity of measuring the diameter of the moon on the sun's disc, with one of Troughton's spider-line micrometers, attached to the telescope. By placing the two lines of the micrometer as tangents to the moon's disc, I found that the distance between them was 41.20 revolutions. But this was evidently too great by the thickness of one of the lines, which I found to be equal to five divisions: therefore the diameter of the moon was only 41.15 revolutions. The value of each revolution (by taking, as a standard, the diameter of the sun on that day, as given by Delambre's tables,) was 42″,999: therefore the apparent diameter of the moon, in the direction in which it was measured, was 29′ 29″,4. But this direction was inclined to the horizon about 75 degrees: which (on account of the refraction) diminished the true diameter in that direction exactly 1″,0: so that its apparent diameter, measured horizontally, would be 29′ 30″,4; and consequently its semidiameter equal to 14′ 45″,2.

Now the horizontal semidiameter of the moon, at noon on that day was, according to Burckhardt's tables, 14′ 41″,02; to which must be added 8″,71 for the augmentation at 2 o'clock (the hour of observation): thus making the apparent semidiameter at that time equal to 14′ 49″,73, or 4″,53 more than the above observation. I would here remark that, according to Burgh's tables, the semidiameter of the moon was 14′ 43″,13: which, allowing for the augmentation, would make the apparent semidiameter 6″,64 more than the above observation.

After the middle of the eclipse (the atmosphere remaining beautifully clear) I proceeded to measure the enlightened part of the sun's disc, or the distance

of the borders of the sun and moon, with a telescope fitted up for the occasion by the Rev. Dr. PEARSON, with a ROCHON'S prismatic micrometer. It was only 19 inches long, 1 inch diameter, and magnified about 30 times: but it was admirably adapted for the purpose intended. The prism moved through the whole length of the tube by means of a rack and pinion; and took in a scale of 36′, divided into seconds by means of a vernier. The advantages attending an instrument of this kind, are its convenient size, and the ease and expedition with which the observations can be made. Nevertheless I think it right to remark that, in the present instance, it is probable there is a constant error of a few seconds affecting the results, arising from the indistinctness of the borders of the sun and moon, which prevented me from determining the exact point of contact. But as the junction was always made under the same apparent circumstances, the proportion between the results will not be affected thereby. This inconvenience may probably be overcome in any new telescope on this construction. The following are the observations which were made with this instrument.

Mean time at the place.	Distance of the borders.	Mean time at the place.	Distance of the borders.	Mean time at the place.	Distance of the borders.
h ′ ″	′ ″	h ′ ″	′ ″	h ′ ″	′ ″
2 17 18	11 8	2 33 14	17 4	2 45 11	21 22
21 3	12 27	34 44	17 30	48 36	22 37
24 13	13 41	36 34	18 11	50 58	23 30
26 10	14 25	39 7	19 4	52 47	24 10
28 2	15 5	40 47	19 43	54 0	24 38
31 22	16 20	43 20	20 40		

At 2^h 54′ I left this instrument in order to prepare for observing the termination of the eclipse as above stated: and I cannot but consider it as extremely fortunate that the sun was entirely free from clouds both at the beginning and end of the eclipse.

All these observations were made from a large window; near which was suspended a barometer, with a thermometer attached: and I had also suspended another thermometer in the open air, in the shade. At noon these instruments stood as follow:

Barometer	= 29.86
Thermometer, within . .	= 65°
———— without .	= 67°

During the progress of the eclipse, I watched the state of them, but *could not observe any alteration* in either of them. As soon as the eclipse was ended, I again noticed them more particularly, and they stood as follow:

Barometer	= 29.87
Thermometer, within . .	= 66°
———— without .	= 68°

The diminution of light was very trifling, and would scarcely have been perceptible, had not my attention been called to it. It by no means appeared so great as the diminution which took place in November 1816: although in that eclipse only .78 of the sun's disc was obscured, whereas in the present one .87 was obscured. But the former eclipse I observed through the dark atmosphere of London, where the abstraction of a small portion of light is easily perceptible. And I understand that, in the present eclipse, the diminution of light during the middle of the eclipse was very perceptible in the metropolis, and at places where the sun was obscured by clouds. Venus was seen by thousands of spectators with the naked eye: and I am informed that Mars also was visible to many.

Mr. Dollond informed me that he took the horizontal diameters of the sun and moon, at Greenwich, with one of his divided object-glass micrometers; and that they were in the proportion of 3.351 to 3.103. Therefore, assuming the semidiameter of the sun, as deduced from Delambre's tables, as a standard, the apparent semidiameter of the moon will be 14′ 44″,14; being about one second less than my own observation. The times at which Mr. Dollond and Mr. Taylor observed the commencement and end of the eclipse, at the Royal Observatory, were as follow:

	Dollond.	Taylor.	
Beginning	= 0^h 22′ 37″	0^h 22′ 33″,6	mean time at Greenwich.
End	= 3 14 40	3 14 44 ,5	
Duration	= 2 52 3	2 52 10 ,9	

Mr. GROOMBRIDGE has favoured me with the following observations of the eclipse at Blackheath: N. Lat. 51° 28′ 2″, E. Long. 0″,67 in time from Greenwich. End of the eclipse at 3^h 14′ 34″ mean time at the place: the beginning not accurately observed.

Vertical distance of the Cusps.

Mean time at the place.	Rev. of Microm.	Mean time at the place.	Rev. of Microm.	Mean time at the place.	Rev. of Microm.
h ′ ″		h ′ ″		h ′ ″	
0 42 12	12.61	2 17 16	35.94	2 43 22	27.84
1 23 44	20.81	20 30	34.78	45 20	26.89
47 4	8.11	22 38	34.16	48 4	25.84
50 36	15.89	25 32	33.45	53 18	23.57
2 1 40	39.28	29 8	32.38	56 46	21.85
5 6	41.11	31 26	31.64	59 0	20.50
9 33	37.01	34 28	30.47	3 0 36	19.39
13 5	37.05	40 8	28.87	9 41	7.28

Each revolution of the micrometer was equal to 44″,982.

The Rev. Dr. PEARSON measured the diameters of the sun and moon not only with one of DOLLOND's divided object-glass micrometers, but also with one of TROUGHTON's line-micrometers. By means of the former he made the moon's semidiameter equal to 14′ 44″,6; and with the latter, equal to 14′ 44″,7: the semidiameter of the sun being considered as the standard for the scale. These measures correspond with my own, prior to their reduction. Dr. PEARSON also measured the luminous portion of the sun, when most obscured, by means of one of his compound prismatic eye-pieces with variable powers, attached to a $2\frac{1}{2}$ feet achromatic telescope, and found it to be 3′ 58″,24. This measure would indicate an error in the lunar tables; as the eclipse ought not to have been of this magnitude even at Greenwich; and much less ought it to be so at the place where the observation was made. The distance between the cusps at 1^h 53′ was 28′ 53″,8 by DOLLOND's micrometer: and the distance of the cusps was exactly equal to the diameter of the moon, on its leaving the sun's disc at 2^h 2′ 25″. The end of the eclipse took place at 3^h 13′ 20″ mean time at the place: the beginning was not observed. Dr. PEARSON's observatory is situated at East Sheen, in N. Lat. 51° 27′ 35″,7, W. Long. 1′ 3″,7 in time from Greenwich.

Mr. William Allen observed the eclipse at Stoke Newington N. Lat. 51° 33′ 40″, W. Long. 22″ in time from Greenwich.

Beginning	=	0^h	22′	31″
End	=	3	13	59
Duration	=	2	51	28

Mr. Isaac Wiseman wrote to me from Norwich (N. Lat. 52° 38′, E. Long. 5′ 10″ in time from Greenwich), stating that the eclipse began there at 0^h 28′ 45″, and ended at 3^h 21′ 40″, mean time at the place. The observation was made with a three feet reflecting telescope, with a power of 180; and the time was deduced from a meridian of his own construction. This gentleman has also sent me the result of some experiments on the power of the burning lens on different substances, during the time of the eclipse. Having procured a piece of pasteboard, he affixed thereto four equal pieces of different coloured cloths; viz. black, blue, yellow, and red; and placed them successively in the focus of a burning lens, *on the day preceding the eclipse*. The following are the periods at which they respectively took fire: viz.

Black	in	7″
Blue		7
Red		8
Yellow		16

He also on the same day submitted the bulb of a thermometer (which then stood at 66°) to the focus of the lens; and in $1\frac{1}{4}$ minute it rose to 94°, and probably would have risen higher, had he not been apprehensive that the glass would have been broken by the heat. These experiments were made at about 2 o'clock in the afternoon, in order that they might correspond with the time of the eclipse at its greatest obscuration. On the following day, about half an hour after the commencement of the eclipse, he applied the cloths in succession to the focus of the lens, and found the periods, at which they respectively took fire, to be as follow: viz.

Black	in	20″
Blue		20
Red		16
Yellow		40

At about half an hour before the end of the eclipse he again submitted them to the focus of the lens, and found their periods of ignition to be as under: viz.

Black	in	17″
Blue		18
Red		14
Yellow		24

But during the time of the greatest obscuration he could not produce any effect on them whatever. The thermometer at the commencement of the eclipse was at 66°; and by 2 o'clock had fallen to 61$\frac{3}{4}$. This was about the middle of the eclipse: and Mr. WISEMAN assures me that at this time he *held the bulb in the focus of the burning lens for upwards of four minutes, but without producing any sensible effect.* At a quarter past 2, he repeated the same experiment, and with the same result, although the sun was free from clouds. At the termination of the eclipse the thermometer rose to 64°. Mr. WISEMAN also states, that he fitted up a prism in a darkened room, and that he made several observations on the coloured rays, which were thrown on a screen of white paper. He says that, during the continuance of the eclipse, the yellow and blue rays were generally increased in brilliancy, whilst the red became exceedingly faint, and did not occupy more than half their usual breadth. As I am not aware that any experiments of a similar kind were made during this eclipse, and as the results are somewhat singular, although anticipated by Mr. WISEMAN, I have thought it right to state them here in order that the attention of the public may be excited thereto in any future eclipse.

Mr. SLOANE of Belfast informs me that the eclipse commenced there at $11^h\ 47'\ 38''$ mean solar time: the observation was made with one of DOLLOND'S achromatic telescopes, magnifying about 75 times.

At Bury in Lancashire, the eclipse commenced at $0^h\ 9'\ 10'',5$ apparent time at that place, as observed with an achromatic telescope of five feet focus. The latitude of the place was 53° 35′ 30″, and its longitude west of Greenwich 9′ 8″ in time.

Most of the letters which I have received from the country remark that the diminution of light was not so great as was expected. The fall of the thermometer towards the middle of the eclipse, was various in various places. I have already stated that, as far as my own observations extended, I could not

perceive any diminution: the inspection of the instruments was made at intervals during the eclipse. In some places, I am informed, the fall was as much as 10°; and, where the thermometer was placed in the sun, as much as 15°. It appears that the power of a lens to ignite gunpowder, was suspended from 10 to 15 minutes, during the middle of the eclipse: and it has been already stated, that for about the same period the lens was incapable of producing any effect on the thermometer:—an experiment which I believe is new, and which is certainly worthy of repetition, whenever another eclipse of any considerable magnitude may present itself.

From the continent I have received some communications, which tend to confirm the observations made by former astronomers on this singular and rare phænomenon.

At Frankfort on the Maine, Mr. J. V. Albert observed the eclipse, as follows:

Beginning . . .	1h 14″,0	Apparent time.
Do. of the Annulus	2 37 ,0	
Middle of do.	2 39 ,45	
End of do.	2 42 ,30	

At the observatory of the Grand Duke of Baden at Manheim, M. Nicolai observed the eclipse as follows:

Beginning of the Annulus	2h 37′ 37″,8	Apparent time.
End of do.	2 42 32 ,0	
End of the eclipse . . .	4 0 50 ,0	

The actual formation of the annulus was very remarkable: for, about a second before it occurred, the fine curve of the moon's disc, then immediately in contact with the edge of the sun, appeared broken into several parts: and in a moment these parts flowed together like drops of water or quicksilver placed near each other. At the dissolution of the annulus, a similar appearance presented itself: for the delicate thread of light then formed by the annulus, instead of being broken in *one* place only, was in an instant divided in *several* places at once. The thermometer (reduced to Fahrenheit's scale) was at the commencement of the eclipse at 66½, and fell towards the middle to 63, but afterwards rose again to 66½.

At Augsburg, Professor Stark observed that the duration of the annulus was 5′ 47″,5; but neither the beginning nor end of the eclipse could be ob-

served on account of the unfavourable state of the atmosphere: REAUMUR's thermometer fell $3\frac{1}{2}$ degrees (equal to 8° of FAHRENHEIT).

At Spire, Professor SCHWERD made the following observations:

Beginning of the Annulus	2^h	37′	55″,5	Apparent time.
End of do.	2	42	43 ,5	
End of the eclipse . . .	4	0	57 ,1	

About six seconds before the formation of the annulus, a bright spot was seen on the point of one of the horns, which shortly after appeared to flow into it. About half a second before the complete junction of the two horns, there appeared a row of bright points. A similar appearance was observed at the dissolution of the annulus. The barometer stood at 28,1 inches: and the thermometer (reduced to FAHRENHEIT's scale) fell from $69\frac{1}{2}$ to 64. A burning-glass, six inches diameter, which immediately set wood in a flame, did not ignite tinder during the time of the middle of the eclipse; nor would it turn paper, in the least, brown.

At Munich (in the middle of the city) the formation of the annulus was observed at 2^h 53′ 23″ mean time. The barometer stood at 26,78 inches. And the thermometer (reduced to FAHRENHEIT's scale) fell about 3 degrees.

From the island of Zante I have received communications from two observers; differing in some trifling points from each other. But as the results are stated to the nearest *minute* only (omitting the seconds), I do not think it necessary to quote them in this place. It appears however that the atmosphere was not perceptibly darkened, till nearly the time of the formation of the annulus, which lasted about five minutes: and that the thermometer fell only $2\frac{1}{2}$ degrees during the whole time of the eclipse; viz. from $88\frac{1}{2}$ to 86.

X. *On the Solar Eclipse which took place on September 7, 1820. Communicated in a Letter to J. F. W. Herschel, Esq., Foreign Secretary, from Professor Moll of Utrecht.*

Read May 11, 1821.

Sir,

When I had the pleasure of meeting you at Slough, I promised to lay before the newly-erected Astronomical Society such information as I could collect on the great solar eclipse, which was to take place the next day, it being then the 6th of September. Immediately after my return to Holland, I began to collect what had been done in that country, the result of which I beg leave to lay before the Astronomical Society. In the first place, I mean to state what calculations were made previous to the observation; next, what observations were made; and lastly, what calculations were made afterwards.

As far back as 1806, the late Mr. Lambertus Nieuwenhuis made a graphic projection of this eclipse, calculated for Amsterdam; according to which, the eclipse was there to be annular. Mr. Bourjé of Middelburg calculated the circumstances of the eclipse, and gave delineations of its appearances for Middelburg, Groningen and Amsterdam. This gentleman's computations were made by Mr. De Kanter's ecliptic tables, which are an abridgement of those of Mayer, and intended to sacrifice some part of the accuracy to the shortness of the calculus. According to this computation the eclipse was *not* to be annular at Amsterdam. Mr. Baily, in his memoir on this eclipse, gives an approximation of the time of the commencement. The commencement and the end were also calculated by Mr. Keyser, of the Royal Institute of the Netherlands, and by Mr. Littrow, Director of the Imperial Observatory at Vienna. From the diagram subjoined to Mr. Baily's memoir, one would suppose him to believe the eclipse *not* annular at Amsterdam: he observes, however, that as to its being annular or not, some uncertainty may exist respecting the places near the borders. The eclipse was also calculated by Mr. Greve, Conrector of the

Gymnasium at Amsterdam: and I subjoin the calculations of the beginning and the end, as made by divers persons.

AMSTERDAM.

	Commencement.	End.
Mr. Bourjé . . .	$12^h\ 43'\ 30''$	$3^h\ 53'$
Nieuwenhuis .	12 43 30	3 34
Baily . . .	12 46 0	——
Littrow . .	12 47 0	3 38
Greve . . .	12 42 51	3 33 14″
Keyser . .	12 48 10 ,6	3 37 38

GRONINGEN.

Mr. Bourjé . . .	$12^h\ 53'\ 0''$	$3^h\ 41'\ 30''$
Greve . . .	12 50 56	3 41 21

MIDDELBURG.

Mr. Bourjé . . .	$12^h\ 38'$	$3^h\ 31'$

Observations at Amsterdam.

The following are the observations actually made at Amsterdam:

1. Professor Van Swinden, Member of the Royal Institute of the Netherlands, wished to confine himself to the appearance only of the eclipse, and whether it would be annular or not. He looked at it with a very good achromatic telescope: *it was* annular, and the annulus could even be distinguished with the naked eye. Just before the annulus was formed, he perceived above the yet obscurated part of the sun, A B, plate II, fig. 1, a very small arch of light, which slowly went from B to C. The illuminated arch was no part of the sun's disc, and the space between it and the moon's dark limb B D A was *not* illuminated. This arch appeared like a thin reddish thread of light: it might have been compared, as to colour and appearance, with the end of the flame of an Argand lamp, projecting beyond the chimney or glass tube. A few moments afterwards the annulus was actually formed, as it is attempted to be represented in the 2d figure. The appearance was striking beyond description. In the superior annular part some slight undulation was observed, as if a thin cloud of smoke was passing. Professor Van Swinden expected that the moon's disc passing the sun, would have remained a perfect circular orb, leaving

an annulus of pure light; but before the annulus was completely formed, a very curious phænomenon presented itself. Between the limbs of the sun and moon several dark threads, lines or belts, were observed, as if attaching the sun's and moon's discs together. These belts or threads were lacerated, and disappeared soon after, leaving the annulus completely formed. But when the annulus was about again to disappear, and when the moon's disc approached again the sun's limb, these belts or threads formed themselves again; then drew shorter and shorter till the annulus vanished entirely. At the end of the eclipse some black threads were perceived again when the moon's disc left the sun, but they were less distinct. This singular illuminated arc is represented in fig. 1, B C A. Mr. VAN SWINDEN quotes some former observations, when something similar appeared: for example, the transit of Venus on the 3d of June 1769; of which Mr. DUNN gave a delineation in the *Phil. Trans.*, vol. 60, page 58; by Mr. HIRST, vol. 69, page 228; by Dr. HORSLEY, ibid. page 184; by Dr. MASKELYNE, an observation, vol. 68, page 355, but no delineation. M. HIRST observed also something analogous in India, at the transit of Venus, the 6th July 1761: and the same was observed by some others in Europe. BERGMAN says, *Phil. Trans.* vol. 52, page 227, "*Citra contactum interiorem, seu dum limbus Veneris a Sole interiore separatur, hoc non momento evenit; sed, haud aliter ac binæ guttæ aquæ separandæ, inter se ligamentum formant, ita quoque e Venere ad marginem Solis tuberculum nigrum extendebatur. Hoc vero ligamento tandem in medio rumpente, apparuit, &c.* . . . *Circa emersionem eadem fere adparentia occurrebant, quamvis non adeo distincta, et in contactu ultimo, ligamento quasi Soli adhærebat Venus.*"— MASON also relates (*Phil. Trans.* vol. 60, page 467) of the transit of 1769, "*the limb of Venus appeared attached to the Sun by an eminence like a dark shade.*" To these quotations Professor VAN SWINDEN adds many others; as, the Observation of the Solar Eclipse, 1st March, N. S. 1739, by MACLAURIN, *Phil. Trans.* vol. 40; of the Solar Eclipse of the 24th of June 1778, by DON ANTONIO DE ULLOA, printed separately, and reprinted in *Journal de Physique*, 1781; of the great Solar Eclipse of the 14th of July, O. S. 1748, by SHORT in *Phil. Trans.* vol. 45; of the Eclipse of 1st of April, 1764, by Mr. DARQUIER at Toulouse, in *Observat. Astronom.* par DARQUIER, 1777, p. 84, &c.

Professor VAN SWINDEN did not perceive any spots on the sun, nor were there any indentures in the moon's disc. Mr. VAN SWINDEN declines offering any hypothesis in explanation of what he saw: he wishes only to state plain matter of fact.

2. The next Amsterdam observation is that of Messrs. Keyser, Member of the Royal Institute, and Buis. They were at the observatory of the society, whose motto is *Felix Meritis,* in latitude 52° 22′ 2″, longitude 19′ 33″ in time E. from Greenwich. They were furnished at the observatory with a good astronomical clock, by Janvier of Paris. Mr. Keyser in his own house has one by Lepaute of Paris. Their time was determined by equal altitudes with Lenoir's repeating circle, and the two clocks were frequently compared by means of a time-keeper. They observed with an excellent four-feet reflector by Dollond, and had besides several achromatic telescopes. The following was the rate of Mr. Keyser's clock several days previous to the observation :

			Correction of clock by mean time.
1820, Sept.	4	at noon	+ 6′ 44″,57
——	4—5	midnight	6 45 ,89
——	5	noon	6 45 ,27
——	6	noon	6 45 ,94
——	6—7	midnight	6 45 ,42

Commencement of the eclipse . . .	0^h	48′	43″	Apparent time.
The annulus was formed between .	{ 2	16	0	
	2	17	0	
The end of the eclipse	3	37	50	

They lost the exact observation of the annulus, by the annoyance of some indiscreet persons who attended upon the occasion.

3. Mr. E. H. Greve, *Conrector Gymnasii Amstelodamensis,* is the next observer. I do not believe his time could be as exactly kept as that of MM. Keyser and Buis; but he is the only person who actually did observe *the duration of the annulus,* and in this respect his observation deserves particular notice. In the morning of the 7th of September, about 8 o'clock, he took some altitudes of the sun, to ascertain the going of a chronometer : next day these observations were repeated. He does not say with what instrument his altitudes were taken, but I presume with a sextant and artificial horizon; nor does he mention the rate of his time-keeper; he believes that he is certain of his time within one or two seconds.

Commencement of the eclipse .	0^h	49′	47″	Apparent time.
The annulus is formed . . .	2	16	30	
The annulus disappears . .	2	17	14	
The end of the eclipse . . .	3	38	48	

A difference of 1′ will be observed between Mr. Keyser's and Mr. Greve's observation. I suspect the error lies with Mr. Greve. Mr. Keyser has a good deal of practice in astronomy; he was furnished with good instruments; he ascertained the rate of his clock several days before the observation. The observation of Mr. Keyser agrees very well with his own, and Professor Littrow's calculations; whilst that of Mr. Greve agrees with no calculation whatever. The eclipse was observed at Gottingen by Professor Encke; and from this and his own observation, Mr. Keyser calculated the meridian difference between Gottingen and Amsterdam, which agrees perfectly well with what was found before. Mr. Greve got his chronometer only on the day of the eclipse; so that he was ignorant of its daily rate. He took, it is said, some altitudes, without mentioning with what instrument, and without giving any particulars. I believe I may assert, that those who are not in the daily habit of the sextant, will find it rather difficult to be certain of their time by simple altitudes within a few seconds. I am thus decidedly of opinion, that Mr. Keyser's observation deserves credit.

Mr. Greve adds some remarks as to the appearance of the eclipse. He made use of an English reflector, made by Mann. The formation of the annulus appeared to Mr. Greve the most beautiful phænomenon which he ever beheld. The sky was very bright; its blue colour by degrees became darker, as at the approach of evening. About the middle of the eclipse the westerly cusp of the sun grew obtuse; from this cusp downwards was formed an indenture, which very soon cut out from the dark limb of the moon a semicircular space, equal in diameter to Jupiter: this illuminated semicircle showed in that place of the moon's superficies an excavation of extraordinary depth; it is represented fig. 4, at *b*. The easterly cusp was now rough in its inferior part, with some slight undulations and inflexions. Soon afterwards, beyond the apex of the cusp, sprung up a number of illuminated spots and lines of greater or less magnitude, and of different forms, with dark spaces between; these are seen at *a*, fig. 4. The same appearance was seen soon after in the westerly cusp; and, on a sudden, an illuminated arch, of a clear purple colour, was formed between both cusps, the extreme points of which were at a distance from each other of about 40° of the sun's circumference. This arch lasted for about 2′ of time; it is represented fig. 5: then darted forward from the illuminated cusps many luminous spots and lines, with darker places between them, which gradually grew brighter and brighter; they are seen fig. 6. Then the cusps drew nearer and nearer; and these luminous spots

and lines had the appearance of some crystallization of a bright salt when viewed with the microscope. They at length melted together, and the cusps of the sun could not be any longer distinguished. About 7 or 8 seconds after, the annulus was seen completely formed. The scars in the moon's disc disappeared entirely; the annulus grew broader, till it was again suddenly dissolved in many spots and belts of different forms. Then could the cusps again be distinguished, and the phænomenon ended as it began. Fig. 7 represents the annulus completely formed.

Observation at Leyden.

Latitude 52° 9′ 23″. Longitude 17′ 58″ in time, East from Greenwich.

Professor Ekama could not observe the commencement sufficiently accurate on account of a thin cloud passing the sun. The Leyden Observatory is furnished with astronomical clocks, with a repeating circle of Lenoir, two reflectors, and various telescopes with micrometers. The eclipse was not annular at Leyden.

Beginning of the eclipse . .		0h	43′	51″	Mean time.
Obscuration of the sun,	1 digit	0	50	50	
	2 digits	0	59	34	
	5 digits	1	23		
	6 digits	1	31		
	9 digits	1	54		
Greatest obscuration		2	12	10	
	6 digits	2	53		
	4 digits	3	6		
End of the eclipse		3	34	9	

Near the time of the greatest obscuration, the very quick motion of the cusps made it almost impossible to measure their distances. The very extremes of the cusps were not well defined, but joined together by a very thin luminous thread, which was not very well limited, and formed a very thin arch of light. The illuminated part of the sun was smaller than it is represented to be in Mr. Bourjé's figure for Amsterdam. Venus was visible to the naked eye.

Observation at Utrecht.

Latitude 52° 5′ 14″. Longitude 20′ 37″ in time, East from Greenwich.

The observatory is furnished with astronomical clocks, quadrants, sextants, transits, reflectors, and achromatic telescopes. In my absence, Professor Schröder, Dr. Van Rees, and Dr. Schröder observed the eclipse.

Beginning of the eclipse . .	12h 48′ 50″	Mean time.
The eclipse was annular . .	2 16	
The end of the eclipse . .	3 37 48,2	

The thermometer, which stood at 68° at the commencement, sunk to 64° at the greatest obscuration.

Observation at Groningen.

Latitude 53° 13′ 13″. Longitude 26′ 17″ in time, East from Greenwich.

Professor Uilkens and Mr. Beertaa do not mention what means they made use of to ascertain their time. I consider this observation very doubtful.

Commencement of the eclipse	0h 51′	Apparent time.
Middle of do.	2 17 30″	
End of do.	3 37 30	

The wind N. N. E.

The altitudes of the barometer and thermometer were as follow:

Time.	Therm. Fah.	Barometer. Rhinld Inches.
7h A. M.	56°	29i 2l,75
12	66	29 3,25
1 P. M.	67	29 4
2 20′	62	29 4
3 41	62	29 3,5

Observation at Burgst, near Breda.

Latitude of Breda 51° 35′ 23″. Longitude 19′ 6″ in time, E. from Greenwich.

Mr. Nahuys had several watches set right at noon, according to a meridian line, which he represents to be drawn with very great accuracy.

Commencement of the eclipse .	$0^h\ 49'\ 54''$	Apparent time.
The end	$3\ \ 38$	

Greatest obscuration $11\frac{1}{16}$ digits.

Shortest distance of the cusps $6\frac{3}{8}$ digits.

The appearance of the sun at its greatest obscuration was very nearly as is represented in Mr. BOURJÉ's figure for Amsterdam. Venus was visible to the naked eye till half an hour after the greatest obscuration.

Observation at Middelburg.

Latitude 51° 30′ 6″. Longitude 14′ 30″ East from Greenwich.

It appears that neither Mr. BOURJÉ nor DE KANTER was furnished with means of ascertaining the time of their clock any other way than by an accurately drawn meridian line. According to this their clock was set. An achromatic telescope was put in the shutter of a dark room; there was an apparatus to make the telescope follow the sun's motion. The sun's image, as formed by the telescope, was received on a white screen.

Commencement of the eclipse .	$0^h\ 39'\ 20''$	Apparent time.
The end	$3\ \ 32\ \ 15$	
The greatest obscuration	$2\ \ 9\ \ 30$	

The eclipse of course was *not* annular at Middelburg. At the time of the greatest obscuration, the blue of the sky was tinged with a strange darkish hue. Venus visible to the naked eye. The means of ascertaining the time both at Breda and Middelburg being rather coarse, no great accuracy can be expected from these observations.

These are the observations in Holland, of which accounts came to my knowledge. I do not mean any further comments on their comparative value; but I cannot abstain from mentioning, that I think those of Mr. KEYSER particularly worthy of notice.

Calculations made after the Observation.

Mr. NIEUWENHUIS, by his graphic construction, found the eclipse annular at Amsterdam. Mr. BOURJÉ, computing by Mr. DE KANTER's tables, found it *not* annular at Amsterdam. By actual observation it was proved annular at Amsterdam and Utrecht, not annular at Middelburg, Leyden and Breda.

Thus was confirmed, what Mr. BAILY says in his memoir, *that some uncertainty may exist with respect to those towns which are situated near the borders of the umbra.* Mr. NIEUWENHUIS being dead, we only know his projection, but not how he made it. Mr. DE KANTER, soon after the eclipse was over, calculated its circumstances again for Amsterdam, not by his own tables, as did Mr. BOURJÉ before, but by the method of the *nonagesimus ;* taking the elements from the *Connoissance des Tems.* According to this computation, the shortest distance of the centres of the sun and moon happened at $2^h\ 16'$. This shortest distance, neglecting the earth's complanation, was $1'\ 10'',4$: but, employing the earth's complanation, $1'\ 12'',9$.

The sun's semidiameter was	15′ 54″,8
Moon's horizontal diameter	14 41 ,1
Augmentation for 36° altitude	8 ,2
Apparent semidiameter of the moon . . .	14 49 ,3
Distance of the centres of sun and moon, neglecting the complanation	1 10 ,4
	15 59 ,7
Sun's semidiameter	15 54 ,8
Excess :	5 ,1

Thus the eclipse was *not* to be annular at Amsterdam. And, accounting for the complanation,

Apparent semidiameter of the moon . . .	14′ 49″,3
Shortest distance of the centres	1 12 ,9
	16 2 ,2
Sun's semidiameter	15 54 ,8
	7 ,4

Thus, taking the complanation in consideration, the eclipse is not found annular for Amsterdam, contrary to what was observed.

Mr. T. M. C. VAN UTENHOVE, Member of the States General of the Royal Institute, went through the same computation, following another method, employing Dr. OLBERS's formula, in BODE's *Astronom. Jahrbuch* 1808, page 196, and computing directly from BURCKHARDT's lunar tables. Thus the shortest

distance of the centres was found 1′ 9″,7. Accordingly there were (employing the complanation) 4″,1 wanted to close the ring on the superior part of the moon: and (neglecting the complanation) there was, according to Mr. VAN UTENHOVEN, one-fourth of a second wanting to close the ring.

Thus it appears that, computing from the best tables, the eclipse is not found annular at Amsterdam, whilst observation proved it to be so. This is however nothing but what might have been anticipated, from what has been observed long ago. The diameter of the sun in former eclipses appears somewhat too large; that of the moon too small. The cause of this phænomenon, and its extent, seems not perfectly ascertained, but its reality appears confirmed by this eclipse. Astronomers dispute, says LALANDE (in *Connoissance des Tems pour l'an* vii.), on the diameters of the sun and moon in eclipses. They agree, it is true, that the sun appears too large, and the moon too small; but they dissent as to the quantity of this difference. DUSÉJOUR subtracted 7″ from the moon's diameter; LEMONNIER wished still to augment this diminution: LALANDE himself, from the observation of the eclipse of 1791, which was annular in North America, and of 1793, which was annular in Norway and part of Germany, concludes that this diminution is much less. According to what was observed and calculated, somewhat more than 4″ ought to be deducted from the moon's diameter, or added to that of the sun, to have it annular at Amsterdam. The same occurrence which took place at Amsterdam, happened at Strasburg. Mr. LITTROW, Director of the Imperial Observatory at Vienna, computing from BURCKHARDT's lunar tables, found the eclipse of the 7th September *not* annular in Strasburg; but Messrs. KRAMP and WALZ found it was annular. Observation proved it to be annular, and thus confirmed Messrs. KRAMP's and WALZ's result. Vide *Zeitschrift für Astronomie,* September and October 1818; *Annales de Mathématiques,* May and October 1818, and September 1820. See also *LALANDE's Astronomy,* sect. 1991, *DELAMBRE's Astronomy,* vol. ii. chap. 26, sect. 197.

If the Astronomical Society finds this communication worthy of their approbation, I will with pleasure furnish them with such information as they might wish, and I may be able to collect, as to continental astronomy.

I am very respectfully yours,

G. MOLL.

Utrecht, January 12, 1821.

X

XI. *On the Comet discovered in the Constellation Pegasus, in 1821. Communicated in a Letter to J. F. W. Herschel, Esq., Foreign Secretary, from M. Nicollet of Paris.*

Read April 13, 1821.

Sir,

I HAVE perused with great satisfaction the regulations of the Astronomical Society of London, instituted for the promotion of astronomy. It will become a rallying point, and a great motive of encouragement and emulation to every one capable of contributing to its advancement. The appeal you make to astronomers, who may have arrived at results interesting to science, is the reason, Sir, that I take the liberty of writing to you to communicate the elements of the parabolic orbit of the comet I discovered on the 21st of January last at the Royal Observatory of Paris. I have computed these elements from the observations made by Messrs. Bouvard, Arago and myself from the 21st of January to the 1st of March inclusive. They verify the observations with as great a degree of exactness as can be hoped from a first approximation.

Perihelion passage, Mar. 21, 1821 .	$9^h\ 33'\ 7''$	in the evening, mean time at Paris.
Perihelion distance . . . . =	0.091113	
Longitude of the ascending node =	$48°\ 32'\ 12''$	
Long. of the perihel. on the orbit =	239 18 37	
Inclination of the orbit . . . =	74 10 53	

Heliocentric Motion, retrograde.

The geocentric motion of the comet was very slow, and nearly uniform. It is no longer visible in the evening; but, by reason of the small perihelion distance, the motion accelerates, and it will soon become visible again in the morning before sun-rise, when we may observe it again, and perfect the foregoing elements. Should my results prove of any interest to the Astronomical Society, I shall be highly gratified, Sir, by opportunities of giving further

proofs of my zeal and ardour in the cause of science, by addressing to it any future ones my labours may produce.

* * * * * * * * *

Accept, Sir, the offer I venture to make you, of every service in my power to render either to the Astronomical Society, or to yourself in particular, and believe me,

With respect and high consideration,

Your very humble and obedient Servant,

NICOLLET.

Royal Observatory,
Paris, March 17, 1821.

XII. *On the Comet discovered in the Constellation Pegasus in* 1821: *and on the luminous appearance observed on the dark side of the Moon on February* 5, 1821. *Communicated in a Letter to J. F. W. Herschel, Esq., Foreign Secretary, from Dr. Olbers of Bremen.*

Read April 13, 1821.

My dear Sir,

I LEARNT with great satisfaction from your obliging letter of the 9th of February, that the highly respectable Astronomical Society of London has honoured me with admission, on the 12th of January, as one of its Foreign Associates. I consider myself as greatly honoured by this distinction, so far beyond my merits; and request you will communicate to the society my sincere thanks, with the assurance that I will do all my slender powers will enable me in furtherance of their objects.

On the 30th of January of the present year, about 7 o'clock in the evening, I observed a small comet near the star γ *Pegasi**, not knowing that the same had been discovered on the 21st of January by M. Nicollet in Paris, and on the same day by M. Pons in la Marlia. I annex my observations of this comet up to the present time.

* *Note by the Foreign Secretary.*—I observed this comet on February 27, 1821, at Slough. My observation stands as follows:

"Observed the comet.—I found it easily in the night-glass near γ *Pegasi*, about four-fifths the field of the night-glass from the star, and almost exactly preceding it. At this time it was not visible to the naked eye, and γ could barely be seen. When grown somewhat darker, I set the seven- and ten-feet reflectors on it: in neither could any central star-like point be seen. (In the comet of 1819 I distinctly saw such an appearance, and pointed it out to some friends, who corroborated the fact.) It appeared a mere misty mass, but it was not above 8°, or at most 10° high. The tail was pretty considerable; it filled half the field of the night-glass, or about $2\frac{1}{2}$°. I imagined the tail to be somewhat less bright along its axis, but the dark space certainly was not very decided. The head seemed rather obtuse, and appeared, I thought, to have lateral portions of light, which seemed to go off at a greater angle than the tail."

1821.	Mean time at Bremen.	Apparent — Æ in Arc.	Apparent — North D.
Jan. 30	7h 17′ 51″	359° 27′ 4″	16° 5′ 1″
	8 29 3	26 24	4 24
Feb. 2	7 40 50	8 45	15 50 14
5	7 11 50	358 54 3	37 56:
7	6 50 6	44 41	28 55:
8	7 2 15	40 24	24 55
9	6 54 52	36 16	21 20
10	7 9 3	32 24	17 34
11	7 16 21	28 21	14 18
12	7 7 32	24 49	10 55
13	7 3 30	20 59	7 58:
14	7 27 44	17 23	4 31
19	6 49 20	357 59 48	14 48 10
March 1	7 5 2	18 28	8 48
5	6 58 39	356 54 7	13 42 53
6	6 56 20	46 33	34 21

The following determinations of the path of this comet have been transmitted to me: they approach very near to each other, though the elements will require some corrections when applied to the *ensemble* of the observations.

1821.	Prof. Encke of Seeberg.	Prof. Nicolai of Manheim.	H. Von Staudt of Göttingen.	Mean time at the place.
Time of the perihelion passage	March 21.405	March 21.6016	March 21.6026	
Longitude of the perihelion .	239° 20′ 45″	239° 34′ 5″	239° 36′ 0″	
Log. perihelion distance . .	8.95966	8.96466	8.9641627	
Longitude of ☊	48° 34′ 37″	48° 43′ 34″	48° 45′ 44″	
Inclination	74 5 0	73 23 15	73 16 33	

Motion retrograde.

On the 5th of February 1821, I observed the remarkable luminous appearance on the dark part of the moon; respecting which, Captain Kater (as the newspapers inform me) has read a notice to the Royal Society. As I under-

stand it, Captain Kater is convinced that it was actually a volcano in a state of eruption. I am not in possession of the grounds on which this conviction is founded; but must confess that I cannot yet bring myself to believe in the existence of any fiery volcanos in the moon; and I think this highly remarkable phænomenon is to be satisfactorily explained in another manner, more consistent with what we know of the physical construction of the moon. This luminous appearance seemed to me to be situated in or near the spot marked *Aristarchus*. This *Aristarchus* (as is well known) is always enlightened by the earth, in the dark portion of the moon, when three or four days old; and it is distinguishable from all the other spots in the moon by its brightness. But the luminous appearance of the 5th of February was entirely different from the usual appearance of *Aristarchus*, with which I was well acquainted; and in my five-feet achromatic telescope by Dollond, was equal to a star of the 6th magnitude. I shall ere long make public my ideas on these so called lunar volcanos, and shall have the honour, as soon as they are printed, to transmit them to you.

I shall consider it as a duty in future to communicate to you every thing of any importance pertaining to astronomy, either the result of my own observations, or which may come to my knowledge; and shall continue to address to you as at present, unless I hear from you to the contrary.

I am, with the greatest esteem and respect,

Your obedient

Bremen, March 11, 1821. William Olbers.

XIII. *On a luminous appearance seen on the dark part of the Moon in May 1821. Communicated in a Letter to the* REV. DR. PEARSON, *from the* REV. M. WARD.

Read May 11, 1821.

DEAR SIR,

I HAVE this moment laid aside my telescope from an examination of the moon. The atmosphere was more favourable for the purpose than I have observed it to be for many weeks; and as it so happened, that at about the same age of the last moon, I had carefully examined the part in obscurity to look for a volcano, and had not in any part observed a remarkable appearance, I was greatly surprised to find a paragraph in the public papers, giving a detailed account of a volcano near *Aristarchus,* seen on the very night I had satisfied myself that there was not even an appearance which could be mistaken for a volcano. I resumed the attempt this evening; and having passed the enlightened part of the moon from the field, and carefully avoided looking at it, to have my eye in the best state to discover any more conspicuously illuminated spot in the unenlightened part, I soon saw *Aristarchus* very clearly, having very much the appearance of a small comet, on the moon's surface*. It was then half-past nine: the moon 15° high, and 40° 16′ west

* Would it not be possible for the makers of telescopic eye-pieces to introduce a half-inch mother-of-pearl micrometer (such as are usually divided into 100 equal parts) across the focus and field of the eye-glass, when the planets are the objects under examination? This would answer two valuable purposes. An observer might arrange that the planet should traverse the field entirely within the mother-of-pearl, and thus be enabled to prepare his eye by keeping it in darkness: perhaps he might thus observe a satellite of Saturn which he had never before seen; or by using this method with Venus (whose light is far too brilliant to allow a satellite to be seen), a more certain opinion would be obtained on the subject of her having or not having one. It may be applied even to the light which Mars diffuses over the field. But this method of viewing the planets is, I am aware, in direct contradiction to an assertion I have lately heard, that a very faint light is rendered visible by being near to, and perhaps within the diffusion of a superior one. I have not seen the arguments by which this opinion is supported, or I should not perhaps have suggested this mode of searching for satellites. The other use of the micrometer alluded to

of the sun, I could perceive the shape to be extended towards *Grimaldus*, appearing in diameter equal to one of Jupiter's moons. I continued observing it till the moon was about 11° only high; when it extended itself to right and left horizontally, and became so very faint for the last degree as to be scarcely distinguishable: and having observed the occultation of a fixed star very near to it, at three minutes before ten I discontinued all further attention to it. The star at the instant before its occultation, from the then state of the atmosphere, appeared of about equal magnitude to, but far better defined than, *Aristarchus* did at its most perfect appearance. My telescope magnifies about 80 times. The star which was occulted was 136 *Tauri:* and came in contact with the limb of the moon, as nearly as I could ascertain, at the advancing pole of libration; and the instant of occultation was at $10^h\ 5'\ 55'',9$ P. M., Greenwich time, estimating Tamworth $6'\ 40'',8$ in time west of Greenwich.

I had written thus far, when I recollected that, as the following day was not a post-day, I could not call your attention to it, and that I should lose nothing in point of time by observing the moon on the Saturday night; but it proved cloudy*.

Sunday night, a quarter before ten.—I have again examined the appearance, and find it about as distinct as it was about ten minutes before I discontinued observing on the night of the 4th. The spot is certainly Aristarchus; but it is now much more difficult to observe on account of the moon throwing much more light down the tube of the telescope, and the luminous advancing edge being much nearer the spot, and my telescope having a large aperture: but I should imagine, if my 42-inch tube were inclosed in one which projected five or six feet beyond the object-glass, that the spot might be seen one night at least longer. When I first examined on the 4th the proportion of light thrown on the moon by the earth, and consequently on Aristarchus, was 1.777 out of 2000; to-night it was only 1.422, a diminution of .355; consequently exactly one-fifth less light is reflected by the spot, to say nothing of the inconvenience arising from the addition of one-fifth to the light of the moon. He-

---

is that, as Saturn moves 5 seconds in an hour, the micrometer would measure any separation of a planet from every star supposed to be a satellite; and thus, after a few hours' motion of Saturn, put the inquiry beyond doubt.—M.W.

* *Note by one of the Secretaries.*—On the night here alluded to, when this phænomenon was invisible at Tamworth, on account of the clouds, it was distinctly seen by me in the neighbourhood of London, through a 3½ feet refracting telescope. Its appearance was nearly similar to that described by Mr. Ward.—F. Baily.

VELIUS describes Aristarchus under the name *Mons Porphyrites,* as *aut ex rupe rubrâ, aut sabulo* (this, by the by, is impossible; for the moon's attraction of gravity to its centre would not admit of a cavity of sand (loose sand) similar to Aristarchus) *sive terrâ rubicundâ constare, aut prorsus ardere, sive perpetuo igne exundare.* Its colour must therefore have greatly changed since 1644, for it is singularly *white* when illustrated by the sun; and when the other parts of the moon are yellow, or faintly red, this preserves its predominant whiteness; and its appearance on the 4th and 6th instants was similar to the light of the glow-worm. Could any light, such as we read is occasionally seen on the mountains of Asia Minor, or the phosphoric fire near Derbend, be peculiar to this cavity of the moon? and if so, has it changed, and does it change the colour of its flame?

Your polite attention to me when in town has occasioned my taking the liberty of troubling you with these hasty observations, which I would have put into a more regular form, but I am going to the philosophical lectures at Birmingham this evening; and, in order to save this day's post, I must now conclude with begging you to accept my esteem and thanks.

I am, dear Sir,

Yours very sincerely,

Tamworth, May 4, 1821. MICHAEL WARD.

XIV. *On the Occultations of Fixed Stars by the Moon: on the Repeating Circle: on the Perturbations, &c. of the new Planets: and Observations of the late Comet and of the Planet Vesta. Communicated in a Letter to the Rev. T. Catton, F.R.S., from Professor Littrow of Vienna.*

Read June 8, 1821.

Sir,

YOU mention in your letter to M. Carrighan that you observed, on the 6th of February, the immersion of δ *Piscium* at St. John's College, and that Mr. South likewise observed it at his observatory. It is with much pleasure that I communicate to you my corresponding observation of the same immersion made at the observatory at Vienna.

Immersion of δ *Piscium* at $7^h\ 29'\ 2''$ mean time*.

On the same day I observed the immersion of another star of the 6th magnitude at $7^h\ 5'\ 8''$ mean time.

The right ascension of this second star is $9°\ 45'$, and the declination $6°\ 20'$ north. The month of February, and the latter part of January of this year were very favourable for this species of observations, which are of so much use both to geography and navigation; and as they derive the greatest part of their value from being simultaneously observed, I flatter myself that I may be of some service by communicating to you the other observations which I have received, and to which you will doubtless find corresponding ones in England. They will be more important from the circumstance of the longitude of my observatory being well established; it being $0^h\ 56'\ 10''$ from the Royal Observatory of Paris.

* *Note by one of the Secretaries.*—This occultation of δ *Piscium* was also observed by me, at Gray's-Inn, at $6^h\ 14'\ 30''$ mean time at the place. The other star, here alluded to, is probably 62 *Piscium*.—F. Baily.

1821.	Star.	Mean time at Vienna.	
Jan. 12	Anon. 8^m	13ʰ 36′ 18″,4	Immersion.
	Anon. $\begin{cases} Æ = 43° \ 29' \\ D = 20 \ \ 46 \end{cases}$	14 3 4,0	——
13	Electra	5 3 11,3	——
	Taygeta	5 30 16,0	——
	Anon. 8^m	5 51 9,7	——
	Maia	5 54 41,3	——
	Asterope	5 55 20,4	——
	Anon. 8^m	6 9 50,9	——
20	47 β Leonis	18 3 22,2	——
		19 12 31,2	Emersion.
Feb. 5	988 Piscium	6 22 14,7	Immersion.
8	Anon. $\begin{cases} Æ = 37° \ 8' \\ D = 18 \ \ 57 \end{cases}$	8 20 21,8	——
	34 μ Arietis	10 13 26,3	——
	Anon. 7^m	10 40 59,9	——
9	Anon. $\begin{cases} Æ = 50° \ 22' \\ D = 23 \ \ 2 \end{cases}$	6 0 3,3	——
10	Anon. $\begin{cases} Æ = 66 \ \ 51 \\ D = 26 \ \ 34 \end{cases}$	9 55 20,0	——

It was extremely interesting to me to observe by your letter, that Mr. Troughton, whose opinion is of such great weight, is against the repeating circle; and I should be glad, at any expense, to procure his memoir which was read at the meeting of the Astronomical Society. The idea of T. Mayer, who first produced instruments of multiplication, is without doubt excellent in theory; but there are many things good in theory, which are bad, or at least difficultly applicable to practice. Since I have had occasion to use these instruments, I became of Mr. Troughton's opinion. Several times I have proposed to myself to combat this abuse, which throws us back; but these ideas are so inveterately rooted in Germany, that I feared too much resistance, notwithstanding all the experience and the facts I could cite on the subject. It is a kind of malady which has got possession of all my countrymen: and I believe that the

memoir of Mr. TROUGHTON (which I would willingly translate into German) is the only mèdicine that can cure it.

M. GAUSS, of Göttingen, whom you know by his *Disquisitiones Arithmeticæ*, and by his *Theoria Motus Corp. Cœl.*, two excellent and classical works, has occupied himself during several years with the perturbations of the four new planets. For this purpose he has given a new and direct method: and the other solutions of this problem of the three bodies, given by EULER, LAGRANGE, and LAPLACE, were indirect; or at least approximations, as they employed infinite series more or less convergent.

One principal result of this immense labour of M. GAUSS is a more exact determination of the mass of those planets which exert a sensible influence on these asteroids. Of these masses, that of Jupiter was supposed best known. But M. GAUSS shows clearly that the mass of Jupiter, given by LAPLACE, is wrong by more than a tenth part. The perturbations of Pallas, produced by Jupiter, amount to *several degrees*; and consequently afford a very certain means of determining the mass of the latter. This change in the mass of the principal planet of our system is doubtless of great consequence, and the planetary tables will soon feel its effects. It is to be regretted that M. GAUSS withholds his labour so long from the public.

The comet, discovered by M. PONS at the observatory of Marlia, was observed by me by means of a circular micrometer, as under:

1821.	Mean time at Vienna.	AR.	D. North.
Feb. 9	6h 52′ 38″	358° 36′ 7″	15° 22′ 47″
10	6 41 9	32 40	
11	7 9 28	28 46	15 15 14
12	6 28 54	25 7	15 11 49
13	6 43 4	21 18	
14	6 54 14	18 3	15 3 46
15	6 51 53	14 0	15 0 24
16	6 52 23	10 30	14 57 15
17	6 52 33	7 10	14 54 8
20	7 20 2	357 57 45	14 45 18
21	7 17 42	53 6	14 41 59
26	7 19 36	33 24	— — —

I have received some good observations of the opposition of the planet Vesta, which are so much the more valuable, as I understand it has been totally neglected at several observatories.

1821.	Mean time at Vienna.	Æ.	D. North.
	h ′ ″	° ′ ″	° ′ ″
Jan. 2	13 5 6,6	118 33 42,7	
12	12 15 17,0	115 55 42,4	23 24 34,7
13	12 10 15,3	115 39 10,0	23 29 34,2
15	12 0 11,7	115 6 9,6	23 40 58,6
19			24 2 26,9
21			24 12 17,0
26	11 5 10,0	112 8 49,2	24 37 14,5
28	10 55 18,9	111 39 6,6	
Feb. 3	10 26 12,5	110 16 9,4	25 10 22,7
6	10 11 58,2	109 39 24,4	
7	10 7 16,6	109 27 38,0	
9	9 57 59,0	109 6 28,3	

To the books I have mentioned, I add a trifle of my own on the late solar eclipse. In two months I shall have finished a work more worthy of your attention, and which I shall have the honour of presenting to the society.

Vienna, March 18, 1821. (Signed) LITTROW.

XV. *On the places of* 145 *new Double Stars. By Sir* WILLIAM HERSCHEL, *President of this Society.*

Read June 8, 1821.

AN account of the places of 145 new double stars, which were intended to be arranged like those of my two catalogues printed in the *Philosophical Transactions*, as soon as the four particulars of their comparative magnitudes and colour, mutual distance and angle of position could be ascertained. Some of the places of these double stars are taken from a first and second review of the stars with my seven-feet Newtonian telescope; the former being made with a magnifying power of 227, the latter with 460. Some of these double stars are also collected from a review of the ecliptic with a very high magnifier. The rest are taken from my sweeps of the heavens, with the twenty-feet telescope, and a power of 157.

The places of the double stars will be found sufficiently accurate for finding them, as these objects, by their singular appearance, will be easily discovered in a field of view of a large diameter. It should be mentioned, that 15 of these objects were communicated to the late Rev. Mr. FRANCIS WOLLASTON, and inserted by him in his catalogue, distinguished by (M. S.).

It will be seen, by my observations of these double stars, that few of them contain more than one or two of the required particulars, and that the distance and position of the two stars when given, are only in terms of general estimations: so that any lover of astronomy, furnished with a proper telescope and micrometers, who wishes to undertake the work of completing these observations, will find sufficient employment in this interesting pursuit. If this should be the case, it will be an apology for my laying the following observations, in their imperfect state, before this Astronomical Society.

WILLIAM HERSCHEL.

Slough, near Windsor,
February 1, 1821.

Observations of the new Double Stars.

(1.) 125 Sweep, Jan. 24, 1784. A very pretty treble star, making an equilateral triangle, all equal and w. 3d class far, or 4th near. 93 (τ) *Virginis* f. . . . n. 0° 5′, R. A. 14^h 4′ ±″, P. D. 87° 24′.

(2.) 162 Sw. March 11, 1784. A double star preceding the head of *Monoceros*, not in Fl., a very considerable star. 15 *Monocerotis* p. 10′ 30″, n. 1° 12′, R. A. 6^h 18′ 43″, P. D. 78° 43′.

682 Sw. Jan. 11, 1787. Double. 75 (l) *Orionis* f. 14′ 3″, n. 1° 24′, R. A. 6^h 19′ 22″, P. D. 78° 35′.

(3.) 183 Sw. March 21, 1784. A very pretty treble star, making an isosceles triangle, the vertex preceding, and the base in the same meridian. All equal stars w. of the 4th class, near I suppose. 48 *Serpentis* f. 20′ 15″ n. 0° 19′, R. A. 16^h 22′ 0″, P. D. 72° 28′.

(4.) 210 Sw. May 9, 1784. About 18′ south of 51 (ξ) *Libræ* double 3d class far.

(5.) 218 Sw. May 16, 1784. Suspected an extended nebulosity between two stars, but 240 showed two double stars, making a parallelogram without nebulosity. 58 (ε) *Herculis* f. 22′ 42″, n. 1° 20′, R. A. 17^h 15′ 16″, P. D. 57° 27′.

(6.) 236 Sw. July 12, 1784. Between three nebulæ (10, 11, 12, V class) is a double star of the 2d or 3d class. 5 (i) *Sagittarii* f. 2′ 42″, n. 0° 49″, R. A. 17^h 49′ 39″, P. D. 113° 27′.

(7.) 236 Sw. July 12, 1784. A very close treble star, making a triangle, whose vertex is following. 16 (ψ) *Capricorni* p. 11′ 48″, s. 0° 25′, R. A. 20^h 21′ 28″, P. D. 116° 26′.

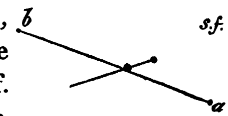
(8.) 240 Sw. July 18, 1784. A double star 33 : : *Vulpeculæ* f. 11′ 18″, n. 0° 8′, R. A. 20^h 59′ 53′, P. D. 68° 22′.

301 Sw. Oct. 20, 1784. Double, equal, 4th class near. 33 *Vulpeculæ* f. 12′ 18″, n. 0° 7′, R. A. 21^h 0′ 52″, P. D. 68° 23′.

963 Sw. Oct. 3, 1790. Double 4th class, equal, both considerably large. 33 *Vulpeculæ* f. 12′ 21″, n. 0° 3′, R. A. 21^h 1′ 12″, P. D. 68° 26′.

(9.) 243 Sw. July 22, 1784. 5 *Aquilæ*, treble, the 3d excessively small. Position following the other two, the line bending a little towards the south. Distance almost the same from the 2d, as the 2d from the 1st.

Review, August 5, 1796. 5 *Aquilæ*, treble. Distance of the largest and next to it 0 rev. 24.5 parts + $4\frac{1}{2}$ for zero = 11″,9. Position 2 rev. −61.3 parts + 1.1 for zero = 31° 27′,3, considerably unequal. The 2d and 3d very unequal. The 1st and 3d extremely unequal. s. f. The 3d is more s. f. still, and requires some attention to be seen. Lw. S. dr. 3d very obscure; 460 shows it better than a lower power.



(10.) 247 Sw. August 10, 1784. Double, 3d class. 19 *Capricorni* p. 6′ 30″, s. 0° 14′, R. A. 20ʰ 36′ 7″, P. D. 108° 58′.

40 feet Journal, 1st Sw. eclip. Sept. 21, 1791. Double 19 *Capricorni* p. 6′ 30″. s. 0° 15′, R. A. 20ʰ 36′ 31″, P. D. 108° 57″.

Rev. of eclip. Oct. 18, 1792. Double, a little unequal, 3d or 4th class. Position p. It is the preceding of two pretty L. stars; they are near 2 degrees following 15 (υ) in a line parallel to 12 (ο) and 43 (κ) *Capricorni*.

Rev. Oct. 12, 1801. Double, the preceding of 2 p. L. stars, about the middle between 12 (ο) and θ *Capricorni*. 3d class, a little unequal. The preceding is the smallest.

(11.) 258 Sw. Sept. 6, 1784. Double 3d class near, 7 m., both taken together in time and number. 64 *Pegasi* p. 14′ 18″, n. 1° 9′, R. A. 22ʰ 57′ 2″, P. D. 58° 13′.

(12.) 264 Sw. Sept. 10, 1784. Double. 20 *Arietis* f. 14′ 36″, s. 0° 30′, R. A. 2ʰ 17′ 56″, P. D. 65° 44′.

(13.) 269 Sw. Sept. 13, 1784. Double. 21 (η) *Cygni* p. 10′ 12″, n. 1° 7′, R. A. 19ʰ 37′ 55″, P. D. 54° 22′.

(14.) 275 Sw. Sept. 16, 1784. Two large stars, the time and number taken between them; the second is double. 5 *Pegasi* f. 16′ 6″, n. 0° 19′, R. A. 21ʰ 43′ 41″, P. D. 71° 19′.

(15.) 279 Sw. Sept. 20, 1784. Double. 63 (χ) *Aquarii* f. 19′ 18″, n. 0° 57′, R. A. 22ʰ 45′ 52″. P. D. 94° 24′.

(16.) 313 Sw. Nov. 12, 1784. 57 (m) *Pegasi* double. Position about 20 or 30° sp. L. r. S. b., considerably unequal, 4th class.

(17.) 316 Sw. Nov. 16, 1784. A double star of the 2d class. 65 (1st κ) *Tauri* p. 16′ 35″, n. 0° 45′, R. A. 3ʰ 55′ 52″, P. D. 67° 28′.

(18.) 312 Sw. Nov. 17, 1784. Double 2d class, near: sp. perhaps 1°; a large star, followed by two more. 11 *Eridani* f. 15′ 0″ n..., R. A. 3ʰ 8′ 41″:: P. D. 113° 38′::

(19.) 326 Sw. Nov. 20, 1784. Double 2d class, both L. 11 (e) *Navis*

p. 22′ 23″, s. 0° 43′, R. A. 7^h 25′ 12″, P. D. 113° 1′. Is n *Argus* in *Puppi*, L. c. 656.

(20.) 326 Sw. Nov. 20, 1784. 6 m. double, 6th class. Position directly preceding, considerably unequal. 15 *Navis* f. 1^h 32′ 3″, n. 1° 2′, R. A. 9^h 30′ 22″, P. D. 112° 38′.

(21.) 340 Sw. Dec. 13, 1784. 17 (1st ρ) *Orionis*. Double, 2d class. Position n. f. unequal.

Rev. Jan. 17, 1809. Distance between 3 and 4 diameters of L. A pretty object.

(22.) 353 Sw. Jan. 6, 1785. Double, 3d class, equal; position nearly in the meridian, 8. 8 m. 87 (c) *Leonis* f. 47′ 38″, s. 0° 59′, R. A. 12^h 6′ 57″, P. D. 92° 48′.

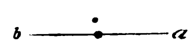
674 Sw. Dec. 29, 1786. Double, of the 4th class, near, equal, nearly in the meridian; but the most south is the preceding. Position about 82°, 8. 8m. 29 (ν) *Virginis* p. 23′ 41″, s. 2° 29′, R. A. 12^h 7′ 13″, P. D. 92° 46′.

Rev. of eclip. March 14, 1793. Double, 4th class, $3\frac{1}{2}$ degrees south of 15 (η) *Virginis*; a large star.

(23.) 360 Sw. Jan. 29, 1785. Double, equal 8. 8 m., nearly in the meridian; 3d class near, or 9. 9m. 42 (ψ) *Tauri* p. 26′ 12″, s. 0° 20′, R. A. 3^h 27′ 25″, P. D. 61° 56′.

(24.) 362 Sw. Jan. 31, 1785. 39 (A) *Eridani* has a very small star to the south. 2d class very near. Position in the meridian.

Rev. Jan. 17, 1809. Extremely unequal. I see it best with the double eye-piece. Very unequal will be more proper. With 240 it will not bear light enough to see the wires: it is, however, about 85° sp. With 160 the distance is about between 2 and 3 diameters of L.



(25.) 368 Sw. Feb. 7, 1785. Double, nearly equal, both w. about 9 m. 3d class far. Position nearly in the parallel. 6 (3d b) *Hydræ* and *Crat.* p. 49′ 7″, n. 1° 53′, R. A. 9^h 53′ 51″, P. D. 107° 5′. (Is 40 *Felis* of BODE's Cat.)

(26.) 379 Sw. March 5, 1785. 7.6m. has a star about 8.9 m. following 6th class. 20 *Sextantis* f. 65′ 29″, n. 0° 18′, R. A. 11^h 8′ 23″, P. D 96° 1′.

(27.) 380 Sw. March 5, 1785. 72 (1st L) *Virginis*, double, extremely unequal. Position about 30° n. f. 4th class near. L. w. S. r.

913 Sw. March 20, 1789. 72 (1st L) *Virginis*, double.

(28.) 383 Sw. March 10, 1785. Double, very unequal; 3d class. 24 *Libræ* p. 15′ 2″, s. 1° 28′, R. A. 14^h 45′ 0″, P. D. 110° 26′.

(28.) 1008 Sw. May 25, 1791. Double, considerably unequal. Position sp. but near the parallel; 3d class, 7.8m. Mayer's 575 z. f. 22′ 10″, s. 0° 58′, R.A. 14h 45′ 17″, P.D. 110° 28′.

(29.) 385 Sw. March 12, 1785. 7 m. has a very small star just preceding, about 4th or 5th class. 72 (τ) *Cancri* f. 25′ 36″, s. 1° 5′, R.A. 9h 20′ 39″, P.D. 60° 34′. (Is 29 *Leonis* of Bode's Cat.)

(30.) 386 Sw. March 13, 1785. A very small and close double star (with 240); the sweeping power made me suspect it to be nebulous. 72 (τ) *Cancri* f. 1′ 6″, n. 1° 15′, R.A. 8h 57′ 3″, P.D 58° 18′.

(31.) 393 Sw. April 6, 1785. Double, equal, 3d class, nearly in the same parallel. 12 (e) *Comæ Ber*. p. 1′ 54″, n. 1° 12′, R.A. 12h 9′ 47″, P.D. 61° 45′.

(32.) 405 Sw. May 1, 1785. Double, 4th or 5th class. 7 (ζ) *Coronæ* f. 7′ 6″, s. 0° 13′, R.A. 15h 38′ 36″, P.D. 58° 52′.

(33.) 411 Sw. May 28, 1785. Double, 4th class near, equal. 50 *Libræ* p. 22′ 8″, s. 0° 17′, R.A. 15h 27′ 9″, P.D. 98° 5′.

Rev. May 22, 1797. My double star of 411 sweep is about 1° 40′ n. f. 37 *Libræ*. (Fl. star, observed page 45, north of 37, is in its place.)

(34.) 430 Sw. Sept. 1, 1785. Double. 19 *Piscis Austr*. p. 11′ 37″, n. 1° 11′, R.A. 22h 27′ 47″, P.D. 119° 28′.

(35.) 478 Sw. Nov. 27, 1785. 6 m. double, very unequal. Position ... following. 74 *Aquarii* f. 44′ 16″, s. 1° 30′, R.A. 23h 26′ 25″, P.D. 104° 16′.

(36.) 521 Sw. Feb. 2, 1786. 35 *Sextantis*. Double, 3d class near. Position south preceding.

675 Sw. Dec. 30, 1786. 35 *Sextantis*, double, 3d class far, a little unequal. Position south preceding.

(37.) 521 Sw. Feb. 2, 1786. Double, both 8m; 3d class near. 64 *Virg*. f. 1h 42′ 7″, n. 0° 3′, R.A. 14h 53′ 27″, P.D. 83° 41′.

557 Sw. April 29, 1786. Double, equal, 3d class, 8.8 m. 3 *Serpentis* p. 11′ 2″, n. 0° 37′, R.A 14h 53′ 31″, P.D 83° 40′.

(38.) 548 Sw. March 27, 1786. Double, equal 1½ diameter, 7.7 m. 24 (ι) *Crateris* f. 1h 2′ 24″, n. 0° 11′, R.A. 12h 30′ 11″, P.D. 101° 50′. (Is 58 *Corvi* in Bode's Cat., a star of Hev.).

(39.) 559 Sw. April 30, 1786. Double, 3d class near, a little unequal. Position almost in the meridian 23 (τ) *Scorpii* p. 11′ 22″, s. 1° 24′, R.A. 16h 11′ 11″, P.D. 119° 10′. (It is Mayer's 644 z. L. c. 1366.)

(40.) 566 Sw. May 26, 1786. A double star within neb. IV. 41. 14 *Sagittarii* p. 11′ 58″, s. 1° 15′, R.A. 17h 49′ 30″, P.D. 113° 1′.

(41.) 595 Sw. Sept. 20, 1786. 53 *Aquarii,* double (cloudy).

1050 Sw. Sept. 6, 1793. 53 *Aquarii,* double, equal; 2d class, or 3d class near. Position about 15° from n. p. to s. f.

(42.) 613 Sw. Oct. 17, 1786. 13 *Lacertæ* has an extremely small star following, 3d class.

(43.) 616 Sw. Oct. 18, 1786. 10 (κ) *Pegasi,* double, extremely unequal, the small star almost n. but a little preceding; 3d class near I suppose.

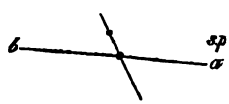
(44.) 619 Sw. Oct. 18, 1786. Double, equal, 3d class. 4 (ω) *Aurigæ* p. 28′ 0″, n. 2° 1′, R. A. 4^h 16′ 33″, P. D. 50° 26′.

Rev. Oct. 16, 1795. 1° 40′ sp. 58 (e) *Persei* in a line parallel to β and ι *Aurigæ,* double; 3d class, equal.

(45.) 621 Sw. Oct. 24, 1786. Double. 41 (δ) *Andromedæ* p. 7′ 55″, n. 0° 46′, R. A. 0^h 47′ 51″, P. D. 46° 26′.

Jour. Sept. 18, 1794. About 1° 45′ n. p. 41 *Andromedæ,* double, nearly equal, in a line parallel to 57 (γ) and 42 (ϕ) nearly; a considerable star, 2d or 3d class. The southmost is the smallest. Position not far from the meridian; 7 feet. 41 *Andromedæ* p. 7′ 55″, n. 0° 46′.

Rev. August 5, 1796. The double star 7′ 55″, p. 41 *Andromedæ.* Position 3 rev. + 31.5 parts — 1.1 for zero = 74° 20′.4 sp. Considerably unequal. Distance 0 rev. 13.9 parts + $2\frac{1}{2}$ for zero = 7″.2 L. w. S. w. rather pretty unequal.



(46.) 654 Sw. Dec. 19, 1786. Double. 69 (λ) *Eridani* p. 0° 48′, n. 0° 5′, R. A. 4^h 58′ 47″, P. D. 98° 56′.

Rev. Jan. 17, 1809. 69 (λ) *Eridani* $\frac{1}{2}$° preceding the nearest of two. Considerably unequal. L. w. S. r. Position with 240 0 rev., 27.2 parts + 2.5 for zero = 6°.683 or 6° 41′.2.

It is IV. 43 of my first catalogues; λ 69 is a single star.

(47.) 692 Sw. Jan. 17, 1787. 7 m. double L. r. S. b., extremely unequal. 14 *Trianguli* f. 3′ 43″, n. 1° 11′, R. A. 2^h 23′ 16″, P. D. 53° 38′.

(48.) 700 Sw. Feb. 13, 1787. Double. 1 *Lupi* p. 21′ : : 43″, n. 0° 44′, R. A. 14^h 39′ : : 45″, P. D. 119° 58′.

(49.) 704 Sw. Feb. 22, 1787. 8 *Sextantis* 5 m. Fl. 6 m. Double, 4th or 5th class, extremely unequal. Position n. p.

(50.) 710 Sw. March 15, 1787. δ *Ant. Pneum,* L. C. 933. Woll. Cat. zone 119° 10^h .... Double, very unequal; 2d class. Position about 40° sp.

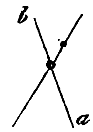
(51.) 711 Sw. March 15, 1787. 4 (h) *Centauri.* Double, very unequal. Position 80° sp.; 3d or 4th class.

(52.) 714 Sw. March 17, 1787. Double, very unequal: 2d class, about 80° sp. L. r. S. b. 7 m. 6 *Canum Ven.* p. 5′ 37″, s. 1° 7′, R.A. 12^h 9′ 45″, P.D. 50° 54′.

(53.) 722 Sw. March 20, 1787. 54 (ν) *Ursæ*, double, very unequal, 60 or 70° s. f.; 2d class.

(54.) 748 Sw. July 10, 1787. 7 m. Double, 3d class. Position preceding, a very little south L. w. S. d. 37 (k) *Aquilæ* p. 1^h 3′ 46″, n. 0° 7′, R.A. 18^h 19′ 27″, P.D. 100° 55′.

Rev. Aug. 6, 1796. Of 748 Sw. South of 1 *Aquilæ*, double, L. w. S. d. very unequal. Distance 0 rev. 27.0 parts + 2.5 for zero = 12″,9. It is difficult to measure on account of the position. Position 0 rev. + 65.4 parts − 1.1 for zero = 14° 28′,1. It is the preceding of two large stars near 3° south of 1 *Aquilæ*.



(55.) 752 Sw. August 19, 1787. Double, 2d class, equal, nearly in the meridian. 17 (θ) *Sagittæ* f. 4′ 16″, n. 1° 19′, R.A. 20^h 4′ 46″, P.D. 68° 25′.

(56.) 754 Sw. Sept. 11, 1787. 41 *Aquarii*, double, 2d class near, very unequal. Position s. f.

(57.) 765 Sw. Oct. 14, 1787. Double 7.7 m. 3 *Lacertæ* p. 30′ 30″, n. 3° 38′, R.A. 21^h 44′ 42″, P.D. 35° 11′.

768 Sw. Oct. 16, 1787. Double. 14 *Cephei* p. 10′ 15″, s. 2° 11′, R.A. 21^h 44′ 22″, P.D. 35° 11′.

(58.) 794 Sw. Dec. 13, 1787. Double 7 m., 2d class, near, equal. 25 (σ) *Andromedæ* p. 17′ 42″, s. 3° 7′, R.A. 23^h 49′ 19″, P.D. 57° 31′.

981 Sw. Nov. 26, 1790. Double, 2d class, nearly equal, not far from the meridian. 73 *Pegasi* f. 24′ 30″, n. 0° 12′, R.A. 23^h 48′ 44″, P.D. 57° 28′.

Journal, Sept, 18, 1794. Sp. 25 (σ) *Andromedæ*, a pretty considerable star; the largest of two. A pretty double star, 1st or 2d class, very nearly equal. Position not much from the meridian. 25 *Andromedæ* p. 17′ 42″, s. 3° 7′.

(59.) 806 Sw. Feb. 3, 1788. Double, unequal. 49 (π) *Hydræ* p. 10′ 16″, n. 1° 7′, R.A. 13^h 43′ 29″, P.D. 114° 33′.

(60.) 813 Sw. March 4, 1788. Double, 4th class, equal from sp. to nf. 50 *Aurigæ* p. 2′ 21″, s. 1° 23′, R.A. 6^h 21′ 49″, P.D. 48° 42′.

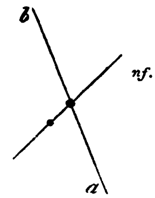
(61.) 815 Sw. March 9, 1788. 20 *Lyncis*, double, equal, sp. to nf. 8.8m. 4th class.

(62.) 834 Sw. April 27, 1788. Double, equal, 4th or 5th class, nearly in the meridian. 37 (ξ) *Bootis* f. 21′ 37″, n. 0° 4′, R.A. 15^h 3′ 10″, P.D. 69° 57′.

835 Sw. April 28, 1788. Double, that to the north is a very little smaller, and a little following the meridian of the other. 37 (ξ) *Bootis* f. 21′ 44″, n. 0° 6′, R.A. 15^h 3′ 17″, P.D. 69° 55′.

1006 Sw. May 24, 1791. Double. 37 (ξ) *Bootis* f. 21′ 41″, n. 0°, 5′, R.A. 15^h 3′ 23″, P.D. 69° 57′.

Rev. July 25, 1796. The most north of three that form an arch; double. Position 3 rev. + 40 parts + 8.4 for zero = 78° 23′,4. It is the double star following (ξ) *Bootis* of the 834 Sweep. A little unequal.



Rev. August 6, 1796. The most north of three, double. Distance 0 rev. 54.6 parts + 2.5 for zero = 25″,0. A little unequal. L. r. S. dr.

(63.) 842 Sw. May 5, 1788. Double, 5th or 6th class, equal, 7.7 m. Nebula observed in this sweep at 15^h 1′ 36″, p. 10′ 38″, s. 3° 8′, R.A. 14^h 52′ 51″, P.D. 35° 23′.

927 Sw. April 24, 1789. Double, 7.7 m.

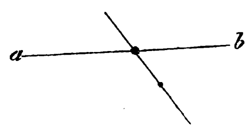
(64.) 872 Sw. Oct. 30, 1788. Double, of the 2d class, nearly in the parallel. 2 *Lacertæ* f. 2′ 16″, s. 1° 13′, R.A. 22^h 14′ 30″, P.D. 45° 44′.

(65.) 880 Sw. Nov. 4, 1788. 35 *Cassiopeæ Hevelii*, double, 2d class, very unequal. (In zone 23° . . . 2^h 11′, &c. WOLL. Cat.)

(66.) 894 Sw. Dec. 18, 1788. 4 *Camelopardalis Hevelii*, double, very unequal, 3d or 4th class. (WOLL. Cat., zone 35° . . 3^h . .)

(67.) 894 Sw. Dec. 18, 1788. 1 *Camelopardalis*, double, 3d class, a little unequal. Position n. p.

Rev. Mar. 22, 1795. 1st *Camelopardalis*, double, considerably unequal. Position with 164 n. p. 2 Rev. − 26.7 parts + 3.5 for zero = 39° 46′,8: 2d measure 2 rev. − 26.1 parts + 3.5 for zero = 39° 54′,9.



(68.) 919 Sw. April 12, 1789. Double, pretty unequal. 64 (γ) *Ursæ* p. 38′ 46″, s. 0° 56′, R.A. 11^h 3′ 59″, P.D. 36° 4′.

(69.) 940 Sw. March 10, 1790. f. *Hydræ* 1153 L. c. double, a little unequal, 3d class, pretty near. Position about 75° sp. (WOLL. Cat. zone 115° . . . 13^h . . .)

(70.) 953 Sw. March 19, 1790. Double, 3d class. Position a little n. f. a little unequal. 76 *Ursæ* p. 1^h 13′ 7″, s. 3° 5′, R.A. 11^h 18′ 12″, P.D. 29° 13′.

1039 Sw. April 9, 1793. Double, equal, 3d class, near. Position nearly in the parallel. 42 *Ursæ* f. 41′ 4″, n. 0° 26′, R.A. 11^h 19′ 29″, P.D. 29° 9′.

(71.) 956 Sw. May 3, 1790. Double, unequal. 9 *Draconis* f. 1^h 24′ 8″, n. 1° 4′, R.A. 14^h 15′ 53″, P.D. 21° 12′.

(72.) 958 Sw. Sept. 7, 1790. Double, very unequal, with a third at no great distance preceding. 20 *Cygni* f. 23′ 8″, n. 0° 1′, R.A. 20^h 8′ 29″, P.D. 37° 31′.

(73.) 959 Sw. Sept. 11, 1790. 50 (α) *Cygni,* 2 m. It has a very small star directly following, about 1′ distance.

Rev. Jan. 17, 1809. The small star is extremely small, and in the 10 feet with 240 will bear no illumination for seeing the wires. Its position is a few degrees from the parallel, on the following side.

(74.) 970 Sw. Oct. 9, 1790. Double, very unequal, 3 or 4° n. f., 3d or 4th class, 6 m. 5 *Pegasi* f. 13′ 48″, n. 0° 27′, R.A. 21^h 41′ 40″, P.D. 71° 10′.

(75.) 980 Sw. Nov. 13, 1790. Double, equal, both 7 m. 26 *Aurigæ* f. 2′ 47″, s. 1° 0′, R.A. 5^h 27′ 59″, P.D. 60° 39′.

(76.) 983 Sw. Dec. 2, 1790. 7 m., double, extremely unequal. Position about 80° s. f., 2d class, very near. 65 (i) *Piscium* p. 16′ 53″, n. 0° 34′, R.A. 0^h 21′ 41″, P.D. 62° 52′.

(77.) 989 Sw. Dec. 28, 1790. Double, equal, 3d class. 41 *Persei Hevelii* f. 35′ 50″, s. 0° 37′, R.A. 4^h 38′ 25″, P.D. 40° 51′.

(78.) 999 Sw. March 24, 1791. Treble, the two largest equal, 3d class. The first star very small, north, preceding the other two; a little further from the preceding of the two, than they are from each other. (R.A. By the sweep 11^h 37′ : : 23″, P.D. 121° : : 1′, no star in the sweep to settle its place.)

(79.) 1000 Sw. April 2, 1791. Double, 4th class, near, equal. 14 (τ) *Ursæ* f. 2′ 6″, s. 1° 52′, R.A. 8^h 55′ 38″, P.D. 27° 30′.

(80.) 1008 Sw. May 25, 1791. Double, equal, 7.7 m. Distance about 1′, or a little more. Mayer's 575 z. p. 9′ 16″, n. 0° 30′, R.A. 14^h 13′ 51″, P.D. 109° 0′.

(81.) 1014 Sw. May 28, 1791. Double. Position sp., extremely unequal; not in Woll. 20 (1st ν) *Coronæ* p. 0′ 8″, n. 0° 8′, R.A. 16^h 14′ 26″, P.D. 55°, 34′.

Rev. March 20, 1795. Fl. 20 (ν) *Coronæ,* consists of two equal stars 6 m. 6 m. The most north and preceding of them has a very small star on the preceding side.

Rev. March 22, 1795. The preceding and the most north of the two stars 6 m. 6 m. has its little star about 50° sp., which is also nearer to the star than the small one of the former double star is to its larger one. (See VI. 18.)

(82.) 1021 Sw. April 20, 1792. Double, equal, 4th or 5th class. 8 (η) *Bootis* f. 7′ 24″, n. 0° 58′, R.A. 13^{h} 52′ 10″, P.D. 69° 34′.

Rev. August 6, 1796. I cannot find the double star of the 1021 sweep 7′ 24″ following ♄ *Bootis*.

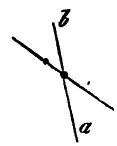
(83.) 1024 Sw. August 22, 1792. Double, extremely unequal. Position directly preceding 7 m. 4 (ϵ) *Sagittæ* p. 2′ 57″, s. 0° 11′, R.A. 19^{h} 24′ 52″, P.D. 74° 11′.

Rev. Oct. 17, 1795. I cannot see the small star of the double star in the 1024 sweep observed at 19^{h} 25′ 33″.

Rev. August 7, 1796. I cannot see the small star of the double star in the 1024 sweep observed at 19^{h} 25′ 33″, with the 7-feet telescope. I see a very small star following 6th class, but the star I look for should be preceding.

(84.) 1024 Sw. August 22, 1792. Double, considerably unequal. Position about 25° np. 5 α *Sagittæ* p. 0° 46′, s. 1° 24′, R.A. 19^{h} 30′ 0″, P.D. 73° 51′.

Rev. Oct. 17, 1795. The double star observed in 1024 sweep at 19^{h} 30′ 37″. Of the 5th or 6th class, very unequal. L. deep red: S. blueish or dusky. Position np.



Rev. August 6, 1796. 1° south of 6 *Sagittæ*. Double, very unequal. Position np. 2 Rev. — 59.8 parts + 1.1 for zero = 31° 47′,6 L. r. S. b.

Rev. August 7, 1796. 1° south of 6 *Sagittæ*, in a line parallel to 5 and 4 *Sagittæ*; a pretty small star. Distance 0 rev. 59.6 parts + 2.5 for zero = 27″,2.

(85.) 1025 Sw. August 23, 1792. Double, very unequal L. r. S. b. Position nf. 5 *Vulpeculæ* f. 0′ 9″, s. 0° 12′, R.A. 19^{h} 17′ 16″, P.D. 70° 30′.

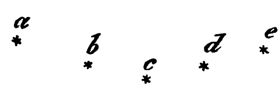
(86.) 1027 Sw. Sept. 15, 1792. 8 m. Double. Neb. IV. 72, joins to it. 34 *Cygni* p. 5′ 10″, n. 0° 23′, R.A. 20^{h} 4′ 53″, P.D. 52° 13′. (Time not accurate.)

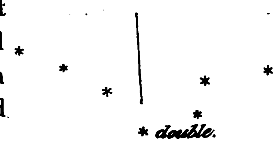
(87.) 1027 Sw. Sept. 15, 1792. Double, of the 2d class, unequal. 34 *Cygni* f. 21′ 5″, n. 0° 28′, R.A. 20^{h} 31′ 8″, P.D. 52° 8′.

(88.) Rev. of eclip. Sept. 15, 1792. Double (between 87 and ϕ 90 *Aquarii*).

(89.) 1028 Sw. Sept. 16, 1792. Double, a little unequal, 4th or 5th class. 3 *Cephei Hevelii* f. 13′ 10″, n. 0° 3′, R.A. 20^{h} 21′ 44″, P.D. 33° 58′.

(90.) Rev. of eclip. Oct. 16, 1792. Double: *c* is the double star, and *a b c d e* are about the 80 *Aquarii*.





(91.) Rev. of eclip. Oct. 16, 1792. Double, 1st class, very near, a little unequal. It is a very small star, about 2 degrees south of 18 (λ) *Piscium*. With 900 I saw them very well. The line goes to 18 and 17 (λ & ι) *Piscium*.

(92.) Rev. of eclip. Oct. 21, 1792. Double, a pretty object, a little unequal, less than a diameter asunder. Position nf., a third star following at some distance. It is the preceding of two in a line between 98 (μ) and 110 (ο) *Piscium*, and about half way between them. The line from μ, in which the two stars are (of which it is the preceding), passes a little north of 110 (ο).

Rev. Oct. 5, 1801. Double, 1st class. A beautiful minute object with 400. It is a star sp. 110 (ο) towards μ, the largest of two.

Rev. Dec. 9, 1801. Double, 1st class, extremely close, equal. It is a star 1° 40′ nf. μ, the first of two in that line. It is a very beautiful object. A third large star in view. They are less than half a diameter asunder. Position about 80° nf. The northern star is rather the smallest.

Rev. Dec. 10, 1801. The double star nf. μ *Piscium*; as described last night.

(93.) Rev. of eclip. Jan. 4, 1793. Double, 2d class, a little unequal np. 36 (A.) *Tauri* 1½° in a line parallel to 54 (ν) and *Pleïades*.

(94.) Rev. Jan. 8, 1793. 55 (δ) *Geminorum*, 6 m. One double towards 43 (ζ).

Rev. March 25, 1795. Sp. δ *Geminorum*, near 2° in a line parallel to 60 and 27 (ε) *Geminorum*. Double, with a third star near. About the 4th class.

(95.) Rev. of eclip. Jan. 8, 1793. Double, 3d class, s.f. 55 (δ) *Gemin.*

Rev. Dec. 14, 1795. S.f. δ *Geminorum* towards r, and about 25′ from r. Double, 3d class, a little unequal.

(96.) 1033 Sw. March 4, 1793. ε *Pixidis Naut.* L. c. 831 6 m. Double, very unequal, 5th or 6th class. Position about 50 or 60 degrees S.f. L. r. S. dr. (WOLL. Cat. Zone 119° ... 9h ...)

(97.) 1038 Sw. April 8, 1793. Double, considerably unequal. Position nf. 39 *Ursæ* f. 16′ 14″, n. 1° 43′, R.A. 10h 46′ 51″, P.D. 30° 0′.

1039 Sw. April 9, 1793. Double, as described last night. 42 *Ursæ* f. 8′ 42″, s. 0° 26′, R.A. 10h 47′ 7″, P.D. 30° 1′.

(98.) 1042 Sw. May 12, 1793. Double, 1st class, equal. Position directly in the meridian, 1½ diameter asunder. *Bootis* 19 *Hevelii* p. 10′ 50″, s. 0° 22′, R.A. 14h 3′ 5″, P.D. 83° 35′.

(99.) 1042 Sw. May 12, 1793. Double, 1st class, very unequal. Position directly preceding: 1 diameter of L. asunder. *Bootis* 19 *Hevelii* p. 7′ 52″, n. 0° 16′, R.A. 14h 6′ 3″, P.D. 82° 57′.

(100.) 1047 Sw. August 25, 1793. Double, equal. Position from np. to s.f. 2d class. 3 *Vulpeculæ* f. 3′ 47″, s. 0° 46′, R.A. 19h 18′ 6″, P.D. 64° 54′.

(101.) 1053 Sw. Sept. 27, 1793. Double, 3d class. Position from np. to s.f., equal. 1st *Piscis Austr.* p. 18′ 59″, n. 1′ 46″, R.A. 20h 29′ 30″, P.D. 121° 17′.

(102.) 1054 Sw. Sept. 28, 1793. Double, equal, 3d class. Position from np. to s.f. but nearer the parallel. 31 (*o*) *Aquarii* f. 12′ 41″, s. 0° 47′, R.A. 22h 5′ 17″, P.D. 93° 56′.

(103.) Journal, Feb. 25, 1794. ½ degree south of the 15th *Monocerotis;* double, a pretty considerable star, very unequal, 3d class far.

(104.) 1058 Sw. April 19, 1794. Double, 3d class, a little unequal, a few degrees np. 12 (δ) *Hydræ Crat.* p. 3′ 37″, s. 1° 33′, R.A. 11h 5′ 27″, P.D. 105° 14′.

(105.) Rev. of eclip. Jan. 13, 1795. Double, the middle one of an arch, almost in the meridian: 2d class, unequal; the southern one is the smallest. It is near 2 degrees south of 19 *Arietis.*

Journal, Jan. 15, 1795. Double: it is the most south but one of four small stars in a crooked row, which is nearly in the meridional direction, and it is about 1° 50′ south of the 19th *Arietis.* 1st class, unequal.

(106.) Rev. of eclip. Jan. 13, 1795. Double, 3d class, the middle one of three in the meridian nearly, the most south of which I suppose to be 29 *Arietis*; or ¾° north of 29 *Arietis.*

Journal, Jan. 15, 1795. Double; it is the middle one of an arch of three stars, that are nearly in a meridional direction, the most south of which is the 27 *Arietis.* Or it is about ¾ degree north of, and a little following, the 27 *Arietis.* 2d class, unequal.

(107.) Rev. of eclip. Jan. 13, 1795. Double, very unequal, 3d class, 1° 25′ sp. 37 (*o*) *Arietis.*

Journal, Jan. 15, 1795. Double, 1° 25′ sp. 37 (*o*) *Arietis*; 2d class, very unequal.

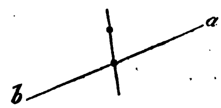
(108.) Rev. March 20, 1795. 2° 40′ s. f. 54 (λ) *Geminorum* towards β *Cancri*, double, 1st class, pretty unequal.

(109.) Rev. Oct. 16, 1795. About 10′ south of 17 (χ) *Cygni* in a line pa-

rallel to 58 and 21, is a very small star, which is double, 1st class, nearly equal; the preceding however is the largest: 1 diameter of S.

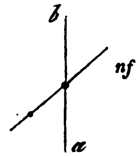
(110.) Rev. Oct. 16, 1795. About $\frac{4}{5}$° or 50′ south of 17 (χ) *Cygni* in a line parallel to 6 and 10. A considerable star, double, 5th class, very unequal.

(111.) Rev. Oct. 16, 1795. About 25 or 30′ nf. 18 (ν) *Geminorum.* A very small star, double, 5th class. L. r. S. d., very unequal, or rather extremely unequal. Position 3 rev. + 20 − 7 for zero = 77° 12′ s. f.



(112.) Rev. Oct. 30, 1795. 1° 40′ north, following 93 *Aquarii.* A considerable star, double, pretty unequal. The preceding is the smallest. It is in a line parallel to ν and ω *Piscium.* 3d class I believe.

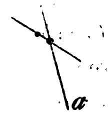
(113.) Journal, Nov. 9, 1795. A small telescopic star nf. 15 *Cygni,* double, 2d class, very unequal. It is about 5′ or 6′ from 15 *Cygni,* and its position with 15 is 4 rev. − 37 parts − 23.2 for zero = 71° 55′.



Journal, Dec. 30, 1795. The small double star north following 15 *Cygni* follows it 17″,5 in time: 7-feet reflector, power 115.

(114.) Journal, April 5, 1796. 7-feet reflector, power 460. (ζ) *Bootis,* double, 1st class. Very nearly in contact; I can however see a small division. A little unequal, the preceding is the smallest.

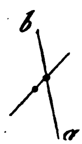
Rev. August 6, 1796. ζ *Bootis,* double. Position 2 rev. − 14.5 parts + 1.1 for zero = 41′ 59″,1 n. p. With 460 a division is but barely visible $\frac{1}{4}$ of S. Both w. A little, or pretty unequal.



Rev. July 12, 1807. ζ *Bootis.* They are fine, equal, whitish stars: the interval between their apparent disks with 460 is $\frac{1}{3}$ of the diameter of either.

(115.) Rev. July 26, 1796. 2° 50′ n. p. *Arcturus.* Double, 2d class I believe. Considerably unequal. In a line parallel to ϵ and ρ: it is a very small telescopic star.

Rev. August 6, 1796. 2° 40′ n. p. *Arcturus.* Double. Distance 4 diameters of L. In a line parallel to 32 and α *Bootis.* Position n.f. 2 rev. + 78.2 parts − 1.1 for zero = 62° 20′,9.



(116.) Rev. August 10, 1796. About 4 degrees n.f. 23 (θ) *Bootis,* the second star in the line from θ to this, double, 3d or 4th class, considerably

unequal. Almost directly following I believe, instead of n. f.; but the evening is bad.

(117.) Rev. Oct. 25, 1797. The star most south of my double star VI. 119, is double of the 1st class. Considerably unequal. Position n.f. 1st class. The angle is such, that a line continued and met by one from the other star, so as to make an isosceles triangle, would meet the line of position at a little more than twice the distance of the two large stars. I verified it with 460, after having looked a considerable time with 920, by way of getting the eye in order. A division can hardly be perceived. But the situation is so low, that certainly the greatest difficulty of seeing the stars arises from that cause. Both together might conveniently stand between the two stars of ζ *Aquarii*, and leave a considerable interval on each side.

(118.) 1066 Sw. Dec. 10, 1797. Double, close of the 2d class, considerably unequal. Distance 1 diameter of L. L. r. S. dr. (See 1068 Sweep.)

1068 Sw. Dec. 12, 1797. The double star of 1066 sweep. 5 *Draconis Hevelii* f. 13′ 54″, s. 0° 23′, R. A. 12^h 24′ 12″, P. D. 14° 4′.

(119.) 1076 Sw. Sept. 5, 1798. 7 m. Double, extremely unequal. Position s.f. 52 (h^2) *Sagittarii* p. 7′ 2″, s. 2° 1′, R. A. 19^h 17′ 34″, P. D. 117° 20′, 1st class. (It is q. L. C. 1600. And MAYER's 786 z.)

(120.) 1078 Sw. Sept. 13, 1798. Double, considerably unequal. The small star is blue, 3d or 4th class. 66 *Draconis* p. 12′ 16″, n. 2° 10′. R. A. 19^h 50′ 15″, P. D. 26° 25′.

(121.) 1081 Sw. Oct. 7, 1798. Double, considerably unequal, 3d or 4th class. Position preceding, or a few degrees sp. 16 *Cephei* f. 8′ 30″, s. 3° 0′, R. A. 22^h 4′ 31″, P. D. 20° 46′.

(122.) 1082 Sw. Oct. 7, 1798. 21 *Cassiopeæ*, double, 6th class, very unequal. Position s.f.

(123.) 1089 Sw. Jan 30, 1799. 8 m. Double, a very small star. Position directly north. 2d class, extremely unequal. 15 (π 1st) *Canis* f. 2′ 1″, n. 0° 5′, R. A. 6^h 46′ 47″, P. D. 109° 55′.

(124.) 1093 Sweep, January 21, 1800. Double, 9.9 m., 2d or 3d class. 15 (y 2d) *Orionis* f. 22′ 24″, n. 1° 33′, R. A. 5^h 20′ 38″, P. D. 73° 9′.

(125.) Rev. Sept. 4, 1801. The 2d of two nf. 22 (λ) *Sagittarii*, probably double; or has a larger diameter. It is about 25′ from λ towards the stars 23, 24, 25. I am pretty sure it is double.

Rev. Sept. 12, 1801. 20′ nf. 22 (λ) *Sagittarii*, double, 1st class; both very small. The smallest of 2 stars.

(126.) Rev. Sept. 4, 1801. About 10′ n p. 39 (o) *Sagittarii*, double, very close. (It is N° 191 in Cat. of omitted stars.)

(127.) Rev. Sept. 4, 1801. The middle one of 3 nf. α *Capricorni* is double, 2d or 3d class.

(128.) Rev. Sept. 7, 1801. 67 *Aquarii*, double, 1st class.

(129.) Rev. Sept. 12, 1801. 1 degree south of 39 (o) *Sagittarii*, double, 2d class near, considerably unequal.

(130.) Rev. Sept. 12, 1801. Double, 1st class. It is a small star, equally distant from d and μ *Capricorni*, but a little more south than either. It is a little nearer μ than d.

(131.) Rev. Sept. 12, 1801. Within the triangle δ μ *Capricorni* and ι *Aquarii*, 18 more. 1 double, 3d class, very unequal.

(132.) Rev. Sept. 12, 1801. Double, 2d class, near. It is between α and ε *Tauri*, rather nearer α, and it is a little following the line that joins α and ε; a considerable star.

(133.) Rev. Sept. 15, 1801. Double, 1st class, 2 degrees sp. 73 (λ) *Aquarii* towards σ.

Rev. Sept. 16, 1801. The double star 2 degrees sp. λ *Aquarii* is very unequal. Position n.p. Distance 1 diameter of L. It is not towards σ, but rather in a line between 57 (σ) and 43 (θ). The third star in view is north of the double star, or a little nf. The distance of the D. star, after long looking at it, is nearly 2 diameters of L.

(134.) Rev. Oct. 2, 1801. Double, 2 degrees n.p. 14 *Capricorni*, in a line parallel to α and β. It is the middle one of three small telescopic stars in that line; 2d or 3d class, considerably unequal. There is a star very near it in WOLLASTON's Catalogue.

(135.) Rev. Oct. 5, 1801. Double, 1st class, both very small. One-third from 74 *Aquarii* towards 93 (ψ). In the finder it appears to be double, owing to a star very near it.

(136.) Rev. Oct. 6, 1801. Double, 2d class, equal. It is a star 35′ nf. 17 (ι) *Piscium*, in a line from $\varkappa$ through ι.

(137.) Rev. Oct. 6, 1801. Double, 2d class, equal. It is south, and a little following θ *Piscium*; about 1° 10′ from it, in a line towards 16.

(138.) Rev. Oct. 12, 1801. Double, 1st class, very near. Very small stars. It is the angular star of a triangle of very small stars: $1\frac{1}{2}$° n.p. 11 (ϱ) *Capricorni* towards 63 *Sagittarii*. Considerably unequal. The preceding is the smallest.

(139.) Rev. Oct. 12, 1801. Double, 1st class, very minute stars. It is a very small star south of 2 that appear coarsely double in the finder. It follows 29 *Capricorni* $\frac{3}{4}$° towards ♌; and forms a triangle with 29 and the above-mentioned very coarse double star of the finder.

(140.) Rev. Nov. 27, 1801. Double, 2d class, unequal. The south-preceding star is the smallest. It is 1° 40′ s. f. $\varkappa$ *Aquarii* towards ψ.

(141.) Rev. Dec. 7, 1801. Double, 2d class. It is 1° 20′ nf. 18 (ν) *Geminorum*, in a line parallel to γ and ϵ. Equal; or the preceding perhaps the smallest.

(142.) Rev. Dec. 7, 1801. Double, 1st class, very near. 1$\frac{1}{2}$° s.f. 70(θ) *Leonis*, in a line from b through θ continued.

(143.) Rev. Jan. 29, 1802. 1$\frac{3}{4}$ or 1$\frac{1}{4}$ degree north of 32 and 31 *Virginis*, double, 1st class, extremely near, less than half a diameter of either; nearly equal. Position sp. The most south is the smallest.

(144.) 1112 Sw. Sept. 30, 1802. Double, 2d class. 32 *Ursæ* of Bode's Catalogue p. 2′ 3″, n. 0° 8′, R.A. 8h 36′ 34″, P.D. 18° 29′.

(145.) 1112 Sw. Sept 30, 1802. Double 7m. 8m: the 8m. about $\frac{3}{4}$ of a minute s.f. the 7m. 133 *Ursæ* of Bode's Catalogue p. 3′ 53″, n. 2° 17′, R.A. 10h 2′ 47″, P.D. 17° 59′.

XVI. *Universal Tables for the reduction of the fixed Stars. By S. Groombridge, Esq., F.R.S. and S.R.A. Nap.*

Read November 10, 1820.

THE science of astronomy having within the last twenty years received considerable improvements both in theory and practice, (the former from the researches of many able mathematicians, and the latter from the skill of modern artists in the construction of instruments,) the patronage of several public bodies, and the labour of individuals, have been called forth to an extensive cultivation of the science. When a course of observations has been made, a small portion only of the task has been accomplished. In order to render these of use, it will be necessary to reduce the same to a certain epoch; whereby the places thereof may be compared with the observations of other astronomers, and their several stations be correctly determined. This operation will require a tedious calculation, and occupy a large portion of time. To abridge the labour, tables have been computed to facilitate the various corrections of the fixed stars; but in the present more perfect theory of the solar system, it has become necessary to extend these tables, not only that greater accuracy may be obtained, but also to render the operation familiar to persons who may not be acquainted with spherical trigonometry.

The tables herewith presented to the society are computed with the proper signs, to apply the sum of the equations to the mean place of the star, at any given epoch; thence to find the apparent place. Consequently, by reversing the sign of the sum of the equations, we reduce the observed or apparent place, to the mean place at the epoch required. The general practice is to reduce all the observations, both of the stars and planets, to the first of January as a mean epoch; but this being an indefinite period, variable in each year, both in the longitude of the sun, and the situation of the observer on the earth, I should propose to fix the vernal equinox, as the more proper pe-

riod; this being an universal epoch. The tables for the mean diurnal precession of the equinoxes are thus computed, and adapted to the longitude of the sun at the time of observation, which is used as the argument, instead of the day of the month: yet should the reduction be required to the first of January preceding such vernal equinox, the same is effected by the following factors into the annual precession.

	For Æ.	For N.P.D.
1st Year of 365 days	−0.206	−0.215
2d —— of 365 —	−0.207	−0.216
3d —— of 365 —	−0.208	−0.216
4th —— of 366 — (or Bissextile) . .	−0.209	−0.217

To reduce the fixed stars, there are required four elements for the right ascension, and three for the north polar distance. Those for right ascension are; 1°, the annual number; 2°, the maximum of aberration; 3°, the annual mean precession; 4°, the natural tangent of declination (divided by 15, if for the right ascension in time). The three elements for the north polar distance are; 1°, the annual number; 2°, the maximum of aberration; 3°, the annual mean precession. The arguments to find these quantities, with which to enter the tables, are the right ascension and declination of the star. And it may be remarked, that whether the place is mean or apparent, from which the Æ and N.P.D. are assumed, the computation will not be sensibly affected; unless the reduction be required for a period of several years; when these quantities would be more accurately obtained by estimating the Æ and N.P.D. at the half of the period.

The following is an example of the star 17 (Hev.) *Cephei.*

Right ascension in time . . .	21ʰ.48′.46″.5
———— in space . . .	327°.11′.37″,6 = 327°.11′,6
Declination north	55 .44 . 54 = 55 .44 ,9

For Æ in time.		For N. P. D.	
Annual number, by Tab. A .	10ˢ.24°. 54′	Annual number, by Tab. F .	7ˢ.17°. 34′
Maximum aber. ——— B .	2″,261	Maximum aber. ——— H .	19″,060
Mean precession ——— C .	+2 ,029	Mean precession ——— K .	−16 ,818
Nat. tangent of dec. div. by 15	0 ,098		

The annual number for the right ascension is the longitude of the ecliptic, intersected by the circle of declination passing through the star; and is therefore the reduction of the equator to the ecliptic, which is given in table A.

But the annual number in north polar distance, being the longitude of the ecliptic intersected by a perpendicular to the circle of declination drawn from the star, the operose computation of the spherical triangles is avoided by table F, with a less probability of error. Having taken out the four quantities from the table F (which include within their area the Æ and Dec.), the correction of the differences may be found by proportion, or more directly by proportional logarithms, whose radius is two degrees; and which for convenience should be copied on a separate sheet, in order to have it at the same view as the table F. Such table is here given, and is called the *table of proportional logarithms.*

For Right Ascension.

To find the several equations, enter table I with the difference of the sun's longitude and the annual number, on the side, and the maximum of aberration at the top (always taking the difference less than six signs), and the true aberration is found by inspection. Should the maximum of aberration exceed 25″, (which may happen in the right ascension of stars near the pole) enter with a half, third, or fourth part, to bring the same within the extent of the tables, and the true quantity will be the double, &c. For the precession in right ascension, multiply the mean annual precession in table C by the decimal in table D, if the reduction is to an earlier epoch; or the complement thereof, with the negative sign, if to a subsequent epoch; and carefully notice the effect of the algebraic sign of the product. The deviation, or lunar nutation of the earth's axis, is taken from the double tables M, according to the directions therein given; and the equation of the equinoxes from table E.

For North Polar Distance.

Enter table I with the difference of the sun's longitude and the annual number, on the side, and the maximum of aberration at the top, and the true aberration is found by inspection. Table G gives the semi-annular solar equation. Table L contains the decimal factors for multiplying the mean annual precession and their complements, to be used as above, according as the epoch precedes or follows the observation; and the deviation, or lunar nutation of the earth's axis, is given in the double tables M.

The sum of these equations, applied to the mean place of the star, at the given epoch, will be the apparent place at the time required; or should the apparent place be known from observation, the star may be reduced to the mean epoch, changing the algebraic sign of the sum of the equations.

The following is a synopsis of the Tables.

Table A = the annual number in Æ.

$$B = \log. \left(\frac{20'',255}{15} \cos P\right)$$

$$C = 3'',0599 + \frac{20'',0095}{15} \sin Æ . \tan D.$$

$$D \begin{cases} N^{\circ}\ 1. = \dfrac{\odot}{365.24} - \text{solar nutation.} \\ N^{\circ}\ 2. = \dfrac{\odot}{360} - \text{solar nutation.} \end{cases}$$

$$E = \frac{16'',5168}{15} \sin ☊ .$$

$$F = (\alpha + \beta) = \text{the annual number in N.P.D.}$$

$$\text{where, } \tan \alpha = \frac{\sin \frac{1}{2}(D \backsim \omega) \tan \frac{1}{2} Æ.}{\sin \frac{1}{2}(D + \omega)}$$

$$\tan \beta = \frac{\cos \frac{1}{2}(D \backsim \omega) \tan \frac{1}{2} Æ.}{\cos \frac{1}{2}(D + \omega)}$$

$$G = 0'',4345 \sin (2 \odot - Æ)$$

$$H = 20'',255 \frac{\sin P}{\sin \theta} \qquad \text{where, } \tan \theta = \frac{\tan P}{\sin \lambda}$$

$$I = G . \cos (\odot \backsim a)$$

$$K = 20'',0095 \cos Æ$$

$$L \begin{cases} N^{\circ}\ 1. = \dfrac{\odot}{365.24} \\ N^{\circ}\ 2. = \dfrac{\odot}{360} \end{cases}$$

$$M = \begin{Bmatrix} 1'',23 \sin (Æ + ☊) \\ 8\ ,40 \sin (Æ - ☊) \end{Bmatrix}$$

In the above tables, the following quantities have been assumed: viz. the obliquity of the ecliptic $= \omega = 23^{\circ}\,27'.\,50''$; the precession of the equinoxes (lunisolar) in longitude $= 50'',255$; the aberration of the sun $= 20'',255$; the lunar nutation $= 9'',63$; the solar nutation $= 0'',4345$. The annual number a is determined by table A or F. In order to determine the angle of position P, and the latitude of the star λ, let us assume $\phi = 90^{\circ} \backsim Æ$, or $270^{\circ} \backsim Æ$; and make $\tan \delta = \cos \phi . \tan \omega$: then we shall have

$$\cot P = \frac{\sin (\text{N.P.D.} \backsim \delta) . \cot \phi}{\sin \delta}$$

$$\cot \lambda = \frac{\sin \omega . \sin \phi}{\sin P}$$

Examples of the use of the Tables.

Observed the above star 11th of October 1819.

Reduced to the vernal equinox . . . . 1820.

For right ascension in time.

		s ° ′			
Annual number	=	10 24 54	Aberration (Tab. I.)	=	+ 1″,36
Sun's longitude	=	6 17 41			
Diff.	=	4 7 13			
Precession (Tab. C.)	=	+ 2″,029	Precession	=	− 0 ,92
Decimal no. (Tab. D.)	=	− ,453			
		s ° ′			
Æ of star + 3 signs	=	1 27 12	Nutation (Tab. M.)		
Longitude of moon's node	=	0 10 45			
Sum	=	2 7 57	−1.140		
Diff.	=	1 16 27	−6.087		
			−7.227 × .098	=	− 0 ,71
Equation of the equinoxes by Tab. E.				=	− 0 ,21
					− 0 ,48

For north polar distance.

		s ° ′			
Annual number	=	7 17 34	Aberration (Tab. I.)	=	−16″,52
Sun's longitude	=	6 17 41			
Diff.	=	0 29 53			
Semi-annual solar nutation			(Tab. G.)	=	+ 0 ,40
Precession (Tab. K.)	=	−16″,818	Precession	=	+ 7 ,40
Decimal no. (Tab. L.)	=	− ,440			
		s ° ′			
Æ of the star	=	10 27 12	Nutation (Tab. M.)		
Longitude of moon's node	=	0 10 45			
Sum	=	11 7 57	+0,462 Sum	=	+ 6 ,25
Diff.	=	10 16 27	+5,787		
					− 2 ,47

Here, the reduction being required, from the observed place to the mean epoch, the algebraic signs must be changed; therefore the correction for the Æ will be +0″,48, and for the N.P.D. +2″,47 to be applied to the observed place of the star.

η Eridani, mean place at the vernal equinox 1810.

The elements are taken for 1815, being half the period.

For Æ in time.	
Mean Æ	2ʰ 47′ 8″,95
in space	41° 47′ 14″,2
Annual number, by Tab. A.	1ˢ 14° 19′
Maxim. Aber. ——— B.	1″,308
Mean Precession ——— C.	+2″,909
Nat. Tangent, div. by 15	0 ,011

For N. P. D.	
Mean N.P.D.	99° 39′ 28″,6
or Declin. South	9 39 28 ,6
Annual number, by Tab. F.	5ˢ 15° 52′
Maximum Aber. ——— H.	10″,339
Mean Precession ——— K.	−14″,905

Required the apparent place 8th January 1820.

Right ascension in time.

ˢ ° ′ 1 14 19			
9 17 31	Aber. (Tab. I.)	=	+ 0″,59
3 26 48			
+ 2″,909	Preces.(Tab.C.)	=	+28 ,55
9 ,816			
ˢ ° ′ 10 11 47			
0 6 2	Nutation (Tab. M.)		
10 17 49	+0″,826		
10 5 45	+6″,816		
	+7 ,642×,011	=	+ 0 ,08
Equat. of Equinoxes, (Tab. E.)		=	− 0 ,12
			+29 ,10
Mean Æ.		=	2 47 8 ,95
Apparent Æ.		=	2 47 38 ,05

North polar distance.

ˢ ° ′ 5 15 52			
9 17 31	Aber. (Tab. I.)	=	+ 5″,42
4 1 39			
Semi-ann. sol. Nut. (Tab. G.)		=	+ 0 ,05
−14″,905	Preces.(Tab.K.)	=	−146 ,11
9 ,803			
ˢ ° ′ 1 11 47			
0 6 2	Nutation (Tab. M.)		
1 17 49	−0″,911		
1 5 45	−4 ,907	Sum =	− 5 ,82
			−2 26 ,46
Mean N. P. D.		=	99 39 28 ,60
Apparent N.P.D.		=	99 37 2 ,14

When several observations have been made of the same star, it will be sufficient to compute the equations for each 8th or 10th day; and to take the proportion of their respective sums for the intermediate days: since with a very little practice it will easily be discovered whether the second differences are increasing or decreasing.

Table A.—For the annual number of a Star in Right Ascension; or the Reduction of the Equator to the Ecliptic.

Argument.—Right Ascension of the Star.

Rt. Ascension.				Equation.	Rt. Ascension.				Equation.
+	−	+	−	° ′ ″	+	−	+	−	° ′ ″
0	180	180	360	0 0 0					
1	179	181	359	0 5 24	46	134	226	314	2 27 51
2	178	182	358	0 10 48	47	133	227	313	2 27 22
3	177	183	357	0 16 11	48	132	228	312	2 26 42
4	176	184	356	0 21 33	49	131	229	311	2 25 51
5	175	185	355	0 26 53	50	130	230	310	2 24 50
6	174	186	354	0 32 11	51	129	231	309	2 23 39
7	173	187	353	0 37 26	52	128	232	308	2 22 18
8	172	188	352	0 42 38	53	127	233	307	2 20 46
9	171	189	351	0 47 46	54	126	234	306	2 19 4
10	170	190	350	0 52 51	55	125	235	305	2 17 13
11	169	191	349	0 57 51	56	124	236	304	2 15 13
12	168	192	348	1 2 46	57	123	237	303	2 13 3
13	167	193	347	1 7 36	58	122	238	302	2 10 43
14	166	194	346	1 12 21	59	121	239	301	2 8 15
15	165	195	345	1 17 0	60	120	240	300	2 5 38
16	164	196	344	1 21 32	61	119	241	299	2 2 52
17	163	197	343	1 25 58	62	118	242	298	1 59 58
18	162	198	342	1 30 17	63	117	243	297	1 56 56
19	161	199	341	1 34 28	64	116	244	296	1 53 46
20	160	200	340	1 38 32	65	115	245	295	1 50 28
21	159	201	339	1 42 28	66	114	246	294	1 47 3
22	158	202	338	1 46 15	67	113	247	293	1 43 31
23	157	203	337	1 49 54	68	112	248	292	1 39 52
24	156	204	336	1 53 25	69	111	249	291	1 36 6
25	155	205	335	1 56 46	70	110	250	290	1 32 14
26	154	206	334	1 59 58	71	109	251	289	1 28 16
27	153	207	333	2 3 1	72	108	252	288	1 24 12
28	152	208	332	2 5 54	73	107	253	287	1 20 2
29	151	209	331	2 8 37	74	106	254	286	1 15 47
30	150	210	330	2 11 10	75	105	255	285	1 11 27
31	149	211	329	2 13 33	76	104	256	284	1 7 3
32	148	212	328	2 15 45	77	103	257	283	1 2 34
33	147	213	327	2 17 47	78	102	258	282	0 58 1
34	146	214	326	2 19 39	79	101	259	281	0 53 24
35	145	215	325	2 21 20	80	100	260	280	0 48 44
36	144	216	324	2 22 50	81	99	261	279	0 44 1
37	143	217	323	2 24 9	82	98	262	278	0 39 14
38	142	218	322	2 25 17	83	97	263	277	0 34 26
39	141	219	321	2 26 15	84	96	264	276	0 29 35
40	140	220	320	2 27 1	85	95	265	275	0 24 42
41	139	221	319	2 27 37	86	94	266	274	0 19 47
42	138	222	318	2 28 2	87	93	267	273	0 14 52
43	137	223	317	2 28 15	88	92	268	272	0 9 55
44	136	224	316	2 28 18	89	91	269	271	0 4 58
45	135	225	315	2 28 10	90	90	270	270	0 0 0

Table B.—For the Maximum of Aberration of a Star, in Right Ascension in Time.

Argument.—Right Ascension of the Star.

To the Logarithm found in the Table add the Logarithmic Secant of the Star's Declination.

Signs.	O.	VI.	I.	VII.	II.	VIII.	Signs.
D	Log.	Dif. 100′	Log.	Dif. 100′	Log.	Dif. 100′	D
0	0.09296	0.00002	0.10295	0.00100	0.12166	0.00089	30
1	0.09297	0.00006	0.10355	0.00100	0.12219	0.00088	29
2	0.09301	0.00010	0.10415	0.00103	0.12272	0.00085	28
3	0.09307	0.00015	0.10477	0.00104	0.12323	0.00082	27
4	0.09316	0.00019	0.10539	0.00105	0.12372	0.00080	26
5	0.09327	0.00022	0.10602	0.00105	0.12420	0.00078	25
6	0.09340	0.00027	0.10665	0.00107	0.12467	0.00075	24
7	0.09356	0.00031	0.10729	0.00108	0.12512	0.00073	23
8	0.09375	0.00035	0.10794	0.00108	0.12556	0.00069	22
9	0.09396	0.00039	0.10858	0.00110	0.12597	0.00068	21
10	0.09419	0.00042	0.10924	0.00109	0.12638	0.00064	20
11	0.09444	0.00046	0.10989	0.00109	0.12676	0.00061	19
12	0.09472	0.00050	0.11054	0.00110	0.12713	0.00058	18
13	0.09502	0.00053	0.11120	0.00109	0.12748	0.00055	17
14	0.09534	0.00057	0.11185	0.00110	0.12781	0.00052	16
15	0.09568	0.00060	0.11251	0.00109	0.12812	0.00050	15
16	0.09604	0.00065	0.11316	0.00108	0.12842	0.00045	14
17	0.09643	0.00067	0.11381	0.00107	0.12869	0.00043	13
18	0.09683	0.00070	0.11445	0.00107	0.12895	0.00039	12
19	0.09725	0.00073	0.11509	0.00106	0.12918	0.00036	11
20	0.09769	0.00076	0.11573	0.00105	0.12940	0.00033	10
21	0.09815	0.00079	0.11636	0.00104	0.12960	0.00029	9
22	0.09862	0.00083	0.11698	0.00103	0.12977	0.00026	8
23	0.09912	0.00084	0.11760	0.00102	0.12993	0.00022	7
24	0.09962	0.00087	0.11821	0.00100	0.13006	0.00020	6
25	0.10014	0.00090	0.11881	0.00098	0.13018	0.00015	5
26	0.10068	0.00092	0.11940	0.00097	0.13027	0.00013	4
27	0.10123	0.00094	0.11998	0.00095	0.13035	0.00009	3
28	0.10179	0.00096	0.12055	0.00093	0.13040	0.00005	2
29	0.10237	0.00097	0.12111	0.00091	0.13043	0.00002	1
30	0.10295		0.12166		0.13044		0
Signs.	V.	XI.	IV.	X.	III.	IX.	Signs.

Table C.—For the Annual Precession of a Star, in Right Ascension in Time.

Argument.—Right Ascension of the Star.

Multiply the Precession found in the Table, by the natural Tangent of the Star's Declination; the Product added to the constant quantity 3″.0599, according to the algebraic sign, gives the annual Precession, if the Declination is North: but if the Declination is South, change the sign in the Table.

Signs.	O. VI.	+ −	I. VII.	+ −	II. VIII.	+ −	Signs.
D	″	Dif. 100′	″	Dif. 100′	″	Dif. 100′	D
0	0.0000	0.0389	0.6670	0.0334	1.1552	0.0191	30
1	0.0233	0.0388	0.6870	0.0332	1.1667	0.0185	29
2	0.0466	0.0387	0.7069	0.0327	1.1778	0.0180	28
3	0.0698	0.0388	0.7265	0.0323	1.1886	0.0174	27
4	0.0931	0.0387	0.7459	0.0320	1.1990	0.0167	26
5	0.1163	0.0385	0.7651	0.0316	1.2090	0.0160	25
6	0.1394	0.0386	0.7841	0.0312	1.2186	0.0155	24
7	0.1626	0.0385	0.8028	0.0308	1.2279	0.0149	23
8	0.1857	0.0384	0.8213	0.0303	1.2368	0.0143	22
9	0.2087	0.0382	0.8395	0.0300	1.2454	0.0135	21
10	0.2316	0.0382	0.8575	0.0295	1.2535	0.0130	20
11	0.2545	0.0382	0.8752	0.0290	1.2613	0.0123	19
12	0.2774	0.0379	0.8926	0.0286	1.2687	0.0116	18
13	0.3001	0.0377	0.9098	0.0281	1.2757	0.0110	17
14	0.3227	0.0377	0.9267	0.0276	1.2823	0.0103	16
15	0.3453	0.0374	0.9433	0.0271	1.2885	0.0097	15
16	0.3677	0.0372	0.9596	0.0266	1.2943	0.0091	14
17	0.3900	0.0370	0.9756	0.0262	1.2998	0.0084	13
18	0.4122	0.0368	0.9913	0.0258	1.3048	0.0078	12
19	0.4343	0.0365	1.0068	0.0252	1.3095	0.0070	11
20	0.4562	0.0365	1.0219	0.0247	1.3137	0.0064	10
21	0.4781	0.0360	1.0367	0.0241	1.3175	0.0058	9
22	0.4997	0.0358	1.0512	0.0236	1.3210	0.0050	8
23	0.5212	0.0356	1.0654	0.0230	1.3240	0.0045	7
24	0.5426	0.0353	1.0792	0.0225	1.3267	0.0037	6
25	0.5638	0.0350	1.0927	0.0220	1.3289	0.0030	5
26	0.5848	0.0347	1.1059	0.0215	1.3307	0.0024	4
27	0.6056	0.0345	1.1188	0.0209	1.3321	0.0018	3
28	0.6263	0.0340	1.1313	0.0202	1.3332	0.0010	2
29	0.6467	0.0338	1.1434	0.0196	1.3338	0.0004	1
30	0.6670		1.1552		1.3340		0
Signs.	V. XI.	+ −	IV. X.	+ −	III. IX.	+ −	Signs.

Table D. (No. 1.)—**Decimal numbers of each degree of the Sun's Longitude, (and their complements) for multiplying the annual Precession of a Star, in Right Ascension in Time, including the semi-annular Solar Nutation.**

Degr.	O.		I.		II.		III.		IV.		V.		VI.		VII.		VIII.		IX.		X.		XI.		Degr.
	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	
0	.000	1.000	.065	.935	.150	.850	.255	.745	.360	.640	.445	.555	.511	.489	.575	.425	.656	.344	.756	.244	.855	.145	.936	.064	0
1	.002	.998	.067	.933	.153	.847	.258	.742	.363	.637	.447	.553	.513	.487	.577	.423	.659	.341	.759	.241	.858	.142	.939	.061	1
2	.004	.996	.070	.930	.156	.844	.262	.738	.366	.634	.450	.550	.515	.485	.580	.420	.663	.337	.763	.237	.861	.139	.941	.059	2
3	.006	.994	.072	.928	.160	.840	.266	.734	.369	.631	452	.548	.517	.483	.582	.418	.666	.334	.766	.234	.864	.136	.943	.057	3
4	.008	.992	.075	.925	.163	.837	.269	.731	.372	.628	.455	545	.519	.481	.584	.416	.669	.331	.770	.230	.867	.133	.946	.054	4
5	.010	.990	.077	.923	.166	.834	.273	.727	.375	.625	.457	.543	.521	.479	.587	.413	.672	.328	.773	.227	.870	.130	.948	.052	5
6	.012	.988	.080	.920	.170	.830	.276	.724	.379	.621	.459	.541	.523	.477	.589	.411	.675	.325	.777	.223	.873	.127	.950	.050	6
7	.014	.986	.083	.917	.173	.827	.280	.720	.382	.618	.462	.538	.525	.475	.592	.408	.678	.322	.780	.220	.876	.124	.952	.048	7
8	.016	.984	.085	.915	.177	.823	.284	.716	.385	.615	.464	.536	.527	.473	.594	.406	.682	.318	.783	.217	.879	.121	.955	.045	8
9	.018	.982	.088	.912	.180	.820	.287	.713	.388	.612	.466	.534	.529	.471	.597	.403	.685	.315	.787	.213	.882	.118	.957	.043	9
10	.020	.980	.091	.909	.183	.817	.291	.709	.391	.609	.468	.532	.531	.469	.600	.400	.688	.312	.790	.210	.885	.115	.959	.041	10
11	.022	.978	.093	.907	.187	.813	.294	.706	.394	.606	.471	.529	.533	.467	.602	.398	.691	.309	.794	.206	.888	.112	.961	.039	11
12	.024	.976	.096	.904	.190	.810	.298	.702	.397	.603	.473	.527	.535	.465	.605	.395	.695	.305	.797	.203	.890	.110	.963	.037	12
13	.027	.973	.099	.901	.194	.806	.302	.698	.400	.600	.475	.525	.537	.463	.607	.393	.698	.302	.800	.200	.893	.107	.965	.035	13
14	.029	.971	.102	.898	.197	.803	.305	.695	.402	.598	.477	.523	.539	.461	.610	.390	.701	.299	.804	.196	.896	.104	.967	.033	14
15	.031	.969	.104	.896	.201	.799	.309	.691	.405	.595	.479	.521	.541	.459	.613	.387	.705	.295	.807	.193	.899	.101	.970	.030	15
16	.033	.967	.107	.893	.204	.796	.312	.688	.408	.592	.482	.518	.544	.456	.616	.384	.708	.292	.810	.190	.901	.099	.972	.028	16
17	.035	.965	.110	.890	.208	.792	.316	.684	.411	.589	.484	.516	.546	.454	.618	.382	.711	.289	.814	.186	.904	.096	.974	.026	17
18	.037	.963	.113	.887	.211	.789	.319	.681	.414	.586	.486	.514	.548	.452	.621	.379	.715	.285	.817	.183	.907	.093	.976	.024	18
19	.040	.960	.116	.884	.215	.785	.323	.677	.417	.583	.488	.512	.550	.450	.624	.376	.718	.282	.820	.180	.909	.091	.978	.022	19
20	.042	.958	.119	.881	.219	.781	.326	.674	.419	.581	.490	.510	.552	.448	.627	.373	.722	.278	.824	.176	.912	.088	.980	.020	20
21	.044	.956	.122	.878	.222	.778	.329	.671	.422	.578	.492	.508	.554	.446	.630	.370	.725	.275	.827	.173	.914	.086	.982	.018	21
22	.046	.954	.125	.875	.226	.774	.333	.667	.425	.575	.494	.506	.556	.444	.632	.368	.728	.272	.830	.170	.917	.083	.984	.016	22
23	.048	.952	.128	.872	.229	.771	.336	.664	.427	.573	.496	.504	.559	.441	.635	.365	.732	.268	.833	.167	.920	.080	.986	.014	23
24	.051	.949	.131	.869	.233	.767	.340	.660	.430	.570	.498	.502	.561	.439	.638	.362	.735	.265	.837	.163	.922	.078	.988	.012	24
25	.053	.947	.134	.866	.237	.763	.343	.657	.432	.568	.500	.500	.563	.437	.641	.359	.739	.261	.840	.160	.924	076	.990	.010	25
26	.055	.945	.137	.863	.240	.760	.346	.654	.435	.565	.502	.498	.565	.435	.644	.356	.742	.258	.843	.157	.927	.073	.992	.008	26
27	.058	.942	.140	.860	.244	.756	.350	.650	.438	.562	.505	.495	.568	.432	.647	.353	.746	.254	.846	.154	.929	.071	.994	.006	27
28	.060	.940	.144	.856	.247	.753	.353	.647	.440	.560	.507	.493	.570	.430	.650	.350	.749	.251	.849	.151	.932	.068	.996	.004	28
29	.063	.937	.147	.853	.251	.749	.356	.644	.443	.557	.509	.491	.572	.428	.653	.347	.752	.248	.852	.148	.934	.066	.998	.002	29
30	.065	.935	.150	.850	.255	.745	.360	.640	.445	.555	.511	.489	.575	.425	.656	.344	.756	.244	.855	.145	.936	.064	1.000	.000	30

Table D. (No. 2.)—Decimal numbers of each Day (and their complements) for multiplying the Annual Precession of a Star, in Right Ascension in Time; including the semi-annular Solar Nutation.

Days.	January.		February.		March.		April.		May.		June.	
	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.
1	.008	.992	.107	.893	.175	.825	.238	.762	.307	.693	.400	.600
2	.011	.989	.109	.891	.178	.822	.240	.760	.310	.690	.403	.597
3	.015	.985	.112	.888	.180	.820	.242	.758	.303	.687	.407	.593
4	.018	.982	.115	.885	.182	.818	.245	.755	.315	.685	.410	.590
5	.022	.978	.118	.882	.184	.816	.247	.753	.318	.682	.413	.587
6	.025	.975	.120	.880	.186	.814	.249	.751	.321	.679	.417	.583
7	.028	.972	.123	.877	.188	.812	.251	.749	.323	.677	.420	.580
8	.032	.968	.126	.874	.190	.810	.253	.747	.326	.674	.423	.577
9	.035	.965	.128	.872	.192	.808	.255	.745	.329	.671	.427	.573
10	.038	.962	.131	.869	.194	.806	.257	.743	.332	.668	.430	.570
11	.042	.958	.133	.867	.196	.801	.260	.740	.335	.665	.434	.566
12	.045	.955	.136	.864	.198	.802	.262	.738	.338	.662	.437	.563
13	.048	.952	.138	.862	.200	.800	.264	.736	.341	.659	.440	.560
14	.052	.948	.141	.859	.202	.798	.266	.734	.343	.657	.444	.556
15	.055	.945	.143	.857	.204	.796	.268	.732	.346	.654	.447	.553
16	.058	.942	.146	.854	.206	.794	.271	.729	.349	.651	.451	.549
17	.061	.939	.148	.852	.208	.792	.273	.727	.352	.648	.454	.546
18	.065	.935	.151	.849	.210	.790	.275	.725	.355	.645	.458	.542
19	.068	.932	.153	.847	.212	.788	.278	.722	.358	.642	.461	.539
20	.071	.929	.155	.845	.214	.786	.280	.720	.362	.638	.465	.535
21	.074	.926	.158	.842	.216	.784	.282	.718	.365	.635	.468	.532
22	.077	.923	.160	.840	.218	.782	.285	.715	.368	.632	.471	.529
23	.080	.920	.162	.838	.220	.780	.287	.713	.371	.629	.475	.525
24	.083	.917	.165	.835	.222	.778	.290	.710	.374	.626	.478	.522
25	.086	.914	.167	.833	.224	.776	.292	.708	.377	.623	.482	.518
26	.089	.911	.169	.831	.226	.774	.295	.705	.380	.620	.485	.515
27	.092	.908	.171	.829	.228	.772	.297	.703	.384	.616	.489	.511
28	.095	.905	.173	.827	.230	.770	.300	.700	.387	.613	.492	.508
29	.098	.902			.232	.768	.302	.698	.390	.610	.496	.504
30	.101	.899			.234	.766	.305	.695	.393	.607	.499	.501
31	.104	.896			.236	.764			.397	.603		

Days.	July.		August.		Sept.		October.		Nov.		December.	
	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.
1	.503	.497	.602	.398	.681	.319	.742	.258	.812	.188	.901	.099
2	.506	.494	.605	.395	.683	.317	.744	.256	.814	.186	.904	.096
3	.509	.491	.608	.392	.685	.315	.746	.254	.817	.183	.907	.093
4	.513	.487	.611	.389	.687	.313	.748	.252	.820	.180	.910	.090
5	.516	.484	.614	.386	.689	.311	.750	.250	.822	.178	.914	.086
6	.520	.480	.616	.384	.692	.308	.752	.248	.825	.175	.917	.083
7	.523	.477	.619	.381	.694	.306	.754	.246	.828	.172	.921	.079
8	.526	.474	.622	.378	.696	.304	.757	.243	.830	.170	.924	.076
9	.530	.470	.625	.375	.698	.302	.759	.241	.833	.167	.927	.073
10	.533	.467	.627	.373	.700	.300	.761	.239	.836	.164	.931	.069
11	.536	.464	.630	.370	.702	.298	.763	.237	.839	.161	.934	.066
12	.540	.460	.633	.367	.704	.296	.765	.235	.842	.158	.938	.062
13	.543	.457	.635	.365	.706	.294	.767	.233	.844	.156	.941	.059
14	.546	.454	.638	.362	.708	.292	.769	.231	.847	.153	.945	.055
15	.550	.450	.640	.360	.710	.290	.772	.228	.850	.150	.948	.052
16	.553	.447	.643	.357	.712	.288	.774	.226	.853	.147	.952	.048
17	.556	.444	.645	.355	.714	.286	.776	.224	.856	.144	.955	.045
18	.559	.441	.648	.352	.716	.284	.778	.222	.859	.141	.959	.041
19	.562	.438	.650	.350	.718	.282	.780	.220	.862	.138	.962	.038
20	.566	.434	.653	.347	.720	.280	.783	.217	.865	.135	.966	.034
21	.569	.431	.655	.345	.722	.278	.785	.215	.868	.132	.969	.031
22	.572	.428	.658	.342	.724	.276	.787	.213	.871	.129	.973	.027
23	.575	.425	.660	.340	.726	.274	.790	.210	.875	.125	.976	.024
24	.578	.422	.662	.338	.728	.272	.792	.208	.878	.122	.980	.020
25	.581	.419	.665	.335	.730	.270	.794	.206	.881	.119	.983	.017
26	.584	.416	.667	.333	.732	.268	.797	.203	.884	.116	.987	.013
27	.587	.413	.670	.330	.734	.266	.799	.201	.887	.113	.990	.010
28	.590	.410	.672	.328	.736	.264	.802	.198	.890	.110	.994	.006
29	.593	.407	.674	.326	.738	.262	.804	.196	.894	.106	.997	.003
30	.596	.404	.676	.324	.740	.260	.807	.193	.897	.103	1.001	$\overline{.001}$
31	.599	.401	.679	.321			.809	.191			1.004	.004

Table E.—For the Equation of the Equinoxes in Right Ascension in Time, common to all the Stars.

Argument.—Longitude of the Moon's Ascending Node.

N.B. Mr. Bessel makes the maximum of this table = 1″,026, or about $\frac{1}{15}$ less than the quantities here given: Table M will also be slightly affected by the corrections which he has recently introduced.

Signs.	O. VI.	− +	I. VII.	− +	II. VIII.	− +	Signs.
D	″	Dif. 100′	″	Dif. 100′	″	Dif. 100′	D
0	0.000	0.032	0.551	0.027	0.954	0.016	30
1	0.019	0.032	0.567	0.028	0.963	0.015	29
2	0.038	0.033	0.584	0.027	0.972	0.015	28
3	0.058	0.032	0.600	0.027	0.981	0.015	27
4	0.077	0.032	0.616	0.027	0.990	0.014	26
5	0.096	0.032	0.632	0.026	0.998	0.014	25
6	0.115	0.032	0.647	0.026	1.006	0.013	24
7	0.134	0.032	0.663	0.025	1.014	0.012	23
8	0.153	0.032	0.678	0.025	1.021	0.012	22
9	0.172	0.032	0.693	0.025	1.028	0.011	21
10	0.191	0.032	0.708	0.024	1.035	0.010	20
11	0.210	0.032	0.722	0.025	1.041	0.010	19
12	0.229	0.032	0.737	0.024	1.047	0.010	18
13	0.248	0.031	0.751	0.024	1.053	0.009	17
14	0.266	0.031	0.765	0.023	1.058	0.009	16
15	0.285	0.031	0.779	0.022	1.064	0.008	15
16	0.304	0.030	0.792	0.022	1.068	0.008	14
17	0.322	0.030	0.805	0.022	1.073	0.007	13
18	0.340	0.030	0.818	0.022	1.077	0.006	12
19	0.358	0.031	0.831	0.021	1.081	0.006	11
20	0.377	0.030	0.844	0.020	1.084	0.006	10
21	0.395	0.029	0.856	0.020	1.088	0.005	9
22	0.412	0.030	0.868	0.019	1.090	0.005	8
23	0.430	0.030	0.879	0.020	1.093	0.004	7
24	0.448	0.029	0.891	0.019	1.095	0.004	6
25	0.465	0.030	0.902	0.018	1.097	0.003	5
26	0.483	0.029	0.913	0.017	1.098	0.003	4
27	0.500	0.029	0.923	0.018	1.100	0.001	3
28	0.517	0.028	0.934	0.017	1.100	0.001	2
29	0.534	0.028	0.944	0.017	1.101	0.000	1
30	0.551		0.954		1.101		0
Signs.	V. XI.	− +	IV. X.	− +	III. IX.	− +	Signs.

Table F.—For the annual Argument of a Star in North Polar Distance.

Enter with the Right Ascension on the left side if the Declination is North, on the right side if South.

Subtract from 180°	Add to 180°	Declination of the Star. 0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	30°	Add to 180°	Subtract from 180°
		° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		
270	270	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	90	90
268	272	0 0	0 9.7	0 18.2	0 25.5	0 32.0	0 37.8	0 43.0	0 47.7	0 52.0	0 56.0	0 59.7	1 3.1	1 6.2	1 9.2	1 12.0	1 14.7	92	88
266	274	0 0	0 19.5	0 36.3	0 51.0	1 4.0	1 15.6	1 26.0	1 35.4	1 44.1	1 52.0	1 59.3	2 6.1	2 12.5	2 18.4	2 24.0	2 29.4	94	86
264	276	0 0	0 29.2	0 54.4	1 16.4	1 35.9	1 53.3	2 8.9	2 23.1	2 36.1	2 48.0	2 58.9	3 9.2	3 18.7	3 27.6	3 36.0	3 44.0	96	84
262	278	0 0	0 38.9	1 12.5	1 41.8	2 7.8	2 31.0	2 51.8	3 10.8	3 28.0	3 43.9	3 58.5	4 12.2	4 24.9	4 36.8	4 48.0	4 58.6	98	82
260	280	0 0	0 48.5	1 30.5	2 7.2	2 39.6	3 8.6	3 34.7	3 58.4	4 20.0	4 39.8	4 58.1	5 15.2	5 31.1	5 46.0	6 0.0	6. 13.2	100	80
258	282	0 0	0 58.1	1 48.4	2 32.4	3 11.4	3 46.2	4 17.5	4 45.9	5 11.8	5 35·6	5 57.6	6 18.1	6 37.2	6 55.1	7 11.9	7 27.8	102	78
256	284	0 0	1 7.6	2 6.2	2 57.6	3 43.0	4 23.7	5 0.2	5 33.3	6 3.6	6 31.4	6 57.1	7 21.0	7 43.3	8 4.2	8 23.8	8 42.4	104	76
254	286	0 0	1 17.1	2 24.0	3 22.7	4 14.6	5 1.0	5 42.8	6 20.7	6 55.3	7 27.1	7 56.5	8 23.8	8 49.3	9 13.2	9 35.7	9 56.9	106	74
252	288	0 0	1 26.5	2 41.7	3 47.6	4 46.0	5 38.2	6 25.3	7 8.0	7 47.0	8 22.8	8 55.9	9 26.6	9 55.3	10 22.2	10 47.5	11 11.4	108	72
250	290	0 0	1 35.9	2 59.2	4 12.4	5 17.3	6 15.3	7 7.6	7 55.1	8 38.5	9 18.3	9 55.1	10 29.3	11 1.2	11 31.1	11 59.2	12 25.8	110	70
248	292	0 0	1 45.1	3 16.6	4 37.0	5 48.4	6 52.3	7 49.8	8 42.1	9 29.9	10 13.7	10 54.3	11 31.9	12 7.0	12 40.0	13 10.9	13 40.2	112	68
246	294	0 0	1 54.2	3 33.8	5 1.5	6 19.3	7 29.0	8 31.9	9 29.0	10 21·1	11 9.0	11 53.3	12 34.5	13 12.8	13 48.8	14 22.6	14 54.5	114	66
244	296	0 0	2 3.2	3 50.8	5 25.7	6 50.1	8 5.6	9 13.8	10 15.7	11 12.3	12 4.2	12 52.3	13 36.9	14 18.5	14 57.5	15 34.2	16 8.8	116	64
242	298	0 0	2 12.1	4 7.7	5 49.8	7 20.6	8 42.0	9 55.5	11 2.2	12 3.2	12 59.3	13 51.1	14 39.2	15 24.1	16 6.1	16 45.7	17 23.0	118	62
240	300	0 0	2 20.9	4 24.4	6 13.6	7 50.9	9 18.1	10 37.0	11 48.6	12 54.0	13 54.2	14 49.8	15 41.4	16 29.6	17 14.7	17 57.1	18 37.1	120	60
238	302	0 0	2 29.5	4 40.9	6 37.2	8 20.9	9 54.0	11 18.2	12 34.7	13 44.7	14 49.0	15 48.4	16 43.5	17 35.0	18 23.2	19 8.2	19 51.2	122	58
236	304	0 0	2 38.0	4 57.1	7 0.5	8 50.7	10 29.7	11 59.2	13 20.6	14 35.1	15 43.6	16 46.8	17 45.5	18 40.3	19 31.5	20 19.8	21 5.2	124	56
234	306	0 0	2 46.3	5 13.1	7 23.5	9 20.2	11 5.1	12 40.0	14 6.3	15 25.3	16 38.0	17 45.1	18 47.4	19 45.5	20 39.8	21 31.0	22 19.2	126	54
232	308	0 0	2 54.4	5 28.8	7 46.3	9 49.4	11 40.2	13 20.5	14 51.8	16 15.3	17 32.2	18 43.2	19 49.1	20 50.5	21 48.0	22 42.1	23 33.0	128	52
230	310	0 0	3 2.4	5 44.3	8 8.7	10 18.2	12 14.9	14 0.7	15 37.0	17 5.1	18 26.2	19 41.1	20 50.7	21 55.5	22 56.1	23 53.1	24 46.8	130	50
228	312	0 0	3 10.2	5 59.5	8 30.8	10 46.7	12 49.3	14 40.6	16 21.9	17 54.7	19 20.0	20 38.9	21 52.1	23 0.3	24 4.1	25 4.0	26 0.6	132	48
226	314	0 0	3 17.8	6 14.4	8 52.6	11 14.9	13 23.4	15 20.1	17 6.5	18 44.0	20 13.6	21 36.5	22 53.3	24 5.0	25 12.0	26 14.9	27 14.2	134	46
224	316	0 0	3 25.2	6 28.9	9 13.9	11 42.6	13 57.2	15 59.3	17 50.8	19 33.0	21 7.0	22 33.8	23 54.4	25 9.5	26 19.8	27 25.7	28 27.8	136	44
222	318	0 0	3 32.4	6 43.1	9 34.9	12 10.0	14 30.5	16 38.2	18 34.8	20 21.7	22 0.1	23 31.0	24 55.4	26 13.9	27 27.4	28 36.4	29 41.3	138	42
220	320	0 0	3 39.3	6 57.0	9 55.5	12 36.9	15 3.4	17 16.6	19 18.4	21 10.1	22 52.9	24 28.0	25 56.1	27 18.2	28 35.0	29 47.0	30 54.8	140	40
218	322	0 0	3 46.1	7 10.6	10 15.7	13 3.4	15 35.8	17 54.7	20 1.7	21 58.2	23 45.5	25 24.7	26 56.7	28 22.4	29 42.4	30 57.5	32 8.2	142	38
216	324	0 0	3 52.6	7 23.7	10 35.4	13 29.4	16 7.8	18 32.3	20 44.6	22 46.0	24 37.8	26 21.2	27 57.0	29 26.4	30 49.8	32 8.0	33 21.6	144	36
214	326	0 0	3 58.8	7 36.4	10 54.6	13 54.9	16 39.4	19 9.5	21 27.1	23 33.4	25 29.8	27 17.4	28 57.2	30 30.2	31 57.0	33 18.4	34 34.9	146	34
212	328	0 0	4 4.8	7 48.8	11 13.3	14 19.9	17 10.4	19 46.2	22 9.1	24 20.4	26 21.5	28 13.4	29 57.2	31 33.9	33 4.1	34 28.7	35 48.1	148	32
210	330	0 0	4 10.6	8 0.7	11 31.5	14 44.4	17 40.8	20 22.4	22 50.7	25 7.1	27 12.8	29 9.1	30 57.0	32 37.4	34 11.1	35 38.9	37 1.4	150	30
208	332	0 0	4 16.1	8 12.2	11 49.2	15 8.3	18 10.8	20 58.1	23 31.9	25 53.4	28 3.9	30 4.6	31 56.6	33 40.8	35 18.0	36 49.1	38 14.6	152	28
206	334	0 0	4 21.3	8 23.2	12 6.4	15 31.6	18 40.1	21 33.3	24 12.5	26 39.2	28 54.5	30 59.8	32 55.9	34 44.0	36 24.8	37 59.2	39 27.8	154	26
204	336	0 0	4 26.2	8 33.8	12 22.9	15 54.2	19 8.8	22 7.8	24 52.7	27 24.6	29 44.8	31 54.6	33 55.0	35 47.0	37 31.5	39 9.3	40 41.0	156	24
202	338	0 0	4 30.8	8 43.8	12 38.8	16 16.2	19 36.9	22 41.8	25 32.3	28 9.5	30 34.8	32 49.2	34 53.9	36 49.9	38 38.1	40 19.3	41 54.1	158	22
200	340	0 0	4 35.1	8 53.4	12 54.1	16 37.6	20 4.3	23 15.1	26 11.3	28 53.9	31 24.3	33 43.5	35 52.6	37 52.7	39 44.6	41 29.3	43 7.3	160	20
198	342	0 0	4 39.2	9 2.4	13 8.8	16 58.2	20 30.9	23 47.8	26 49.7	29 37.9	32 13.4	34 37.4	36 51.0	38 55.3	40 51.0	42 39.2	44 20.6	162	18
196	344	0 0	4 42.9	9 10.9	13 22.8	17 18.1	20 56.9	24 19.7	27 27.5	30 21.3	33 2.1	35 31.0	37 49.2	39 57.7	41 57.4	43 49.2	45 33.9	164	16
194	346	0 0	4 46.3	9 18.8	13 36.1	17 37.2	21 22.1	24 51.0	28 4.7	31 4.1	33 50.3	36 24.2	38 47.1	41 0.0	43 3.7	44 59.2	46 47.3	166	14

192	348	0 0	4 49.4	9 26.2	13 48.7	17 55.5	21 46.4	25 21.4	28 41.1	31 46.3	34 38.0	37 17.1	39 44.8	42 2.1	44 9.9	46 9.2	48 0.7	168	12
190	350	0 0	4 52.1	9 33.0	14 0.5	18 13.0	22 9.9	25 51.1	29 16.9	32 27.9	35 25.2	38 9.6	40 42.2	43 4.0	45 16.0	47 19.2	49 14.2	170	10
188	352	0 0	4 54.5	9 39.2	14 11.5	18 29.6	22 32.5	26 19.9	29 51.8	33 8.9	36 11.9	39 1.7	41 39.3	44 5.8	46 22.1	48 29.2	50 27.9	172	8
186	354	0 0	4 56.6	9 44.8	14 21.7	18 45.3	22 54.2	26 47.8	30 26.0	33 49.2	36 58.0	39 53.4	42 36.2	45 7.5	47 28.2	49 39.4	51 41.8	174	6
184	356	0 0	4 58.3	9 49.7	14 31.1	19 0.0	23 14.9	27 14.7	30 59.3	34 28.8	37 43.6	40 44.7	43 32.8	46 9.0	48 34.5	50 49.6	52 55.8	176	4
182	358	0 0	4 59.7	9 54.0	14 39.6	19 13.8	23 34.6	27 40.7	31 31.8	35 7.6	38 28.6	41 35.5	44 29.1	47 10.4	49 40.3	51 59.9	54 10.0	178	2
180	360	0 0	5 0.7	9 57.6	14 47.2	19 26.5	23 53.2	28 5.7	32 3.3	35 45.6	39 12.9	42 25.8	45 25.1	48 11.6	50 46.4	53 10.3	55 24.5	180	360
178	2	0 0	5 1.4	10 0.6	14 53.9	19 38.1	24 10.6	28 29.5	32 33.8	36 22.8	39 56.6	43 15.7	46 20.8	49 12.7	51 52.4	54 20.9	56 39.2	182	358
176	4	0 0	5 1.7	10 2.8	14 59.7	19 48.7	24 26.9	28 52.3	33 3.2	36 59.0	40 39.5	44 5.1	47 16.2	50 13.7	52 58.5	55 31.7	57 54.3	184	356
174	6	0 0	5 1.6	10 4.4	15 4.5	19 58.1	24 42.0	29 13.8	33 31.5	37 34.4	41 21.7	44 53.9	48 11.2	51 14.5	54 4.7	56 42.7	59 9.7	186	354
172	8	0 0	5 1.2	10 5.2	15 8.2	20 6.2	24 55.8	29 34.0	33 58.7	38 8.7	42 3.1	45 42.1	49 5.9	52 15.2	55 10.9	57 54.0	60 25.5	188	352
170	10	0 0	5 0.4	10 5.3	15 10.9	20 13.2	25 8.2	29 52.9	34 24.7	38 41.9	42 43.7	46 29.8	50 0.3	53 15.8	56 17.3	59 5.6	61 41.7	190	350
168	12	0 0	4 59.2	10 4.7	15 12.6	20 18.8	25 19.2	30 10.3	34 49.3	39 14.1	43 23.4	47 16.8	50 54.3	54 16.3	57 23.8	60 17.5	62 58.5	192	348
166	14	0 0	4 57.6	10 3.3	15 13.2	20 23.1	25 28.7	30 26.3	35 12.5	39 45.0	44 2.1	48 3.2	51 47.9	55 16.8	58 30.4	61 29.7	64 15.8	194	346
164	16	0 0	4 55.7	10 1.1	15 12.6	20 26.0	25 36.7	30 40.7	35 34.2	40 14.6	44 39.8	48 48.8	52 41.2	56 17.1	59 37.2	62 42.4	65 33.7	196	344
162	18	0 0	4 53.4	9 58.2	15 10.9	20 27.4	25 43.0	30 53.4	35 54.4	40 42.9	45 16.5	49 33.7	53 34.0	57 17.3	60 44.2	63 55.5	66 52.4	198	342
160	20	0 0	4 50.7	9 54.4	15 8.0	20 27.3	25 47.6	31 4.3	36 12.9	41 9.7	45 52.0	50 17.8	54 26.4	58 17.5	61 51.5	65 9.2	68 11.8	200	340
158	22	0 0	4 47.6	9 49.9	15 3.9	20 25.6	25 50.4	31 13.3	36 29.6	41 34.9	46 26.2	51 1.1	55 18.3	59 17.6	62 59.1	66 23.5	69 32.1	202	338
156	24	0 0	4 44.1	9 44.5	14 58.5	20 22.4	25 51.4	31 20.4	36 44.4	41 58.5	46 59.1	51 43.4	56 9.8	60 17.7	64 7.0	67 38.5	70 53.3	204	336
154	26	0 0	4 40.3	9 38.4	14 51.9	20 17.4	25 50.4	31 25.4	36 57.1	42 20.2	47 30.6	52 24.8	57 0.8	61 17.7	65 15.3	68 54.3	72 15.6	206	334
152	28	0 0	4 36.1	9 31.4	14 44.0	20 10.8	25 47.3	31 28.1	37 7.6	42 40.0	48 0.5	53 5.1	57 51.2	62 17.7	66 24.1	70 10.9	73 39.1	208	332
150	30	0 0	4 31.5	9 23.6	14 34.8	20 2.3	25 42.0	31 28.5	37 15.8	42 57.7	48 28.7	53 44.2	58 41.1	63 17.7	67 33.4	71 28.5	75 3.9	210	330
148	32	0 0	4 26.5	9 14.9	14 24.2	19 52.0	25 34.5	31 26.4	37 21.5	43 13.1	48 55.1	54 22.1	59 30.4	64 17.8	68 43.3	72 47.1	76 30.2	212	328
146	34	0 0	4 21.2	9 5.4	14 12.2	19 39.8	25 24.6	31 21.6	37 24.5	43 26.0	49 19.4	54 58.6	60 19.0	65 17.9	69 53.9	74 7.0	77 58.1	214	326
144	36	0 0	4 15.5	8 55.1	13 58.9	19 25.6	25 12.2	31 14.0	37 24.5	43 36.3	49 41.6	55 33.6	61 6.9	66 18.0	71 5.2	75 28.2	79 27.8	216	324
142	38	0 0	4 9.5	8 44.0	13 44.2	19 9.4	24 57.3	31 3.4	37 21.5	43 43.6	50 1.4	56 6.9	61 54.0	67 18.3	72 17.5	76 51.0	80 59.6	218	322
140	40	0 0	4 3.1	8 32.1	13 28.1	18 51.2	24 39.7	30 49.7	37 15.1	43 47.7	50 18.5	56 38.4	62 40.3	68 18.7	73 30.7	78 15.5	82 33.7	220	320
138	42	0 0	3 56.4	8 19.3	13 10.5	18 30.8	24 19.2	30 32.6	37 5.1	43 48.3	50 32.6	57 7.9	63 25.6	69 19.2	74 45.1	79 42.0	84 10.3	222	318
136	44	0 0	3 49.4	8 5.8	12 51.5	18 8.3	23 55.9	30 11.9	36 51.1	43 45.1	50 43.5	57 35.2	64 9.9	70 20.0	76 0.9	81 10.7	85 49.8	224	316
134	46	0 0	3 42.0	7 51.4	12 31.1	17 43.6	23 29.5	29 47.4	36 32.9	43 37.7	50 50.9	57 59.9	64 53.0	71 21.0	77 18.2	82 42.0	87 32.6	226	314
132	48	0 0	3 34.3	7 36.2	12 9.3	17 16.6	22 59.9	29 19.0	36 10.2	43 25.7	50 54.1	58 21.7	65 34.8	72 22.4	78 37.2	84 16.1	89 19.1	228	312
130	50	0 0	3 26.3	7 20.3	11 46.0	16 47.4	22 27.2	28 46.4	35 42.5	43 8.5	50 52.8	58 40.3	66 15.1	73 24.1	79 58.3	85 53.6	91 9.8	230	310
128	52	0 0	3 18.0	7 3.6	11 21.4	16 15.9	21 51.1	28 9.3	35 9.5	42 45.7	50 46.4	58 55.1	66 53.7	74 26.3	81 21.8	87 34.9	93 5.2	232	308
126	54	0 0	3 9.4	6 46.1	10 55.3	15 42.0	21 11.5	27 27.5	34 30.7	42 16.7	50 34.2	59 5.7	67 30.4	75 29.1	82 48.0	89 20.6	95 6.0	234	306
124	56	0 0	3 0.5	6 28.0	10 27.8	15 5.9	20 28.5	26 40.9	33 45.8	41 40.7	50 15.4	59 11.4	68 4.8	76 32.5	84 17.6	91 11.5	97 13.1	236	304
122	58	0 0	2 51.3	6 9.1	9 59.0	14 27.5	19 41.9	25 49.1	32 54.2	40 57.1	49 49.1	59 11.3	68 36.5	77 36.8	85 51.0	93 8.4	99 27.4	238	302
120	60	0 0	2 41.9	5 49.6	9 28.9	13 46.8	18 51.6	24 52.0	31 55.4	40 4.9	49 14.2	59 4.5	69 5.0	78 42.1	87 29.1	95 12.4	101 50.0	240	300
118	62	0 0	2 32.3	5 29.4	9 57.5	13 3.8	17 57.7	23 49.4	30 49.0	39 3.2	48 29.4	58 49.6	69 29.7	79 48.7	89 12.9	97 24.8	104 22.2	242	298
116	64	0 0	2 22.4	5 8.6	8 24.8	12 18.7	17 0.1	22 41.1	29 34.4	37 51.1	47 33.1	58 25.1	69 49.8	80 56.9	91 3.5	99 47.3	107 5.7	244	296
114	66	0 0	2 12.3	4 47.3	7 51.0	11 31.4	15 58.9	21 27.0	28 11.2	36 27.3	46 23.5	57 49.1	70 4.0	82 7.1	93 2.7	102 21.8	110 2.3	246	294
112	68	0 0	2 2.0	4 25.4	7 16.0	10 42.0	14 54.2	20 7.0	26 39.0	34 50.8	44 58.5	56 58.9	70 11.0	83 19.9	95 12.5	105 11.0	113 14.4	248	292
110	70	0 0	1 51.5	4 2.9	6 40.0	9 50.6	13 45.9	18 41.2	24 57.3	33 0.2	43 15.6	55 51.2	70 8.6	84 36.1	97 35.8	108 18.2	116 44.6	250	290
108	72	0 0	1 40.9	3 40.0	6 3.0	8 57.4	12 34.3	17 9.5	23 5.9	30 54.5	41 11.8	54 21.7	69 53.9	85 56.9	100 16.8	111 47.7	120 36.1	252	288
106	74	0 0	1 30.1	3 16.7	5 25.1	8 2.4	11 19.5	15 32.2	21 4.6	28 32.4	38 43.9	52 24.6	69 22.6	87 23.8	103 20.9	115 44.7	124 52.6	254	286
104	76	0 0	1 19.1	2 53.0	4 46.3	7 5.8	10 1.7	13 49.4	18 53.5	25 53.2	35 48.3	49 52.5	68 28.2	88 59.7	106 56.5	120 16.3	129 38.3	256	284
102	78	0 0	1 8.0	2 28.9	4 6.8	6 7.8	8 41.2	12 1.6	16 32.9	22 56.2	32 21.3	46 35.2	67 0.2	90 48.9	111 15.6	125 31.2	134 57.5	258	282
100	80	0 0	0 56.8	2 4.5	3 26.7	5 8.5	7 18.3	10 9.1	14 3.3	19 41.5	28 19.9	42 20.2	64 41.2	92 59.4	116 36.7	131 39.9	140 54.4	260	280
98	82	0 0	0 45.6	1 39.9	2 46.0	4 8.1	5 53.2	8 12.5	11 25.6	16 9.8	23 41.9	36 52.0	61 0.4	95 46.4	123 28.2	138 54.0	147 32.0	262	278
96	84	0 0	0 34.3	1 15.1	2 4.9	3 6.9	4 26.5	6 12.5	8 40.8	12 22.7	18 27.5	29 54.8	54 59.3	99 45.6	132 32.3	147 23.9	154 51.0	264	276
94	86	0 0	0 22.9	0 50.2	1 23.5	2 5.0	2 58.4	4 9.8	5 50.4	8 22.8	12 40.0	21 18.2	44 43.4	106 34.9	144 45.0	157 14.1	162 48.3	266	274
92	88	0 0	0 11.5	0 25.1	0 41.8	1 2.6	1 29.4	2 5.4	2 56.2	4 13.8	6 27.0	11 8.5	26 54.7	122 44.8	160 49.7	168 15.5	171 15.8	268	272
90	90	0 0	0 0	0 0	0 0	0 0.0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	180 0	180 0	180 0	180 0	270	270

Table F.—For the annual Argument of a Star in North Polar Distance.

Enter with the Right Ascension on the left side if the Declination is North, on the Right side if South.

Subtract from 180°	Add to 180°	Declination of the Star. 30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°	52°	54°	56°	58°	60°	Add to 180°	Subtract from 180°
		° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		
270	270	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	90	90
268	272	1 14.7	1 17.2	1 19.6	1 21.9	1 24.1	1 26.2	1 28.3	1 30.2	1 32.2	1 34.0	1 35.9	1 37.7	1 39.5	1 41.2	1 42.9	1 44.6	92	88
266	274	2 29.4	2 34.4	2 39.2	2 43.8	2 48.2	2 52.4	2 56.5	3 0.5	3 4.4	3 8.1	3 11.8	3 15.4	3 18.9	3 22.4	3 25.8	3 29.2	94	86
264	276	3 44.0	3 51.6	3 58.8	4 5.6	4 12.3	4 18.6	4 24.8	4 30.7	4 36.5	4 42.1	4 47.6	4 53.0	4 58.3	5 3.5	5 8.7	5 13.7	96	84
262	278	4 58.6	5 8.7	5 18.3	5 27.5	5 36.3	5 44.8	5 53.0	6 0.9	6 8.6	6 16.1	6 23.5	6 30.6	6 37.7	6 44.6	6 51.5	6 58.2	98	82
260	280	6 13.2	6 25.9	6 37.9	6 49.4	7 0.4	7 11.0	7 21.2	7 31.1	7 40.7	7 50.1	7 59.3	8 8.2	8 17.0	8 25.7	8 34.3	8 42.7	100	80
258	282	7 27.8	7 43.0	7 57.4	8 11.2	8 24.4	8 37.1	8 49.4	9 1.3	9 12.8	9 24.0	9 35.0	9 45.8	9 56.3	10 6.7	10 17.0	10 27.1	102	78
256	284	8 42.4	9 0.1	9 16.9	9 33.0	9 48.4	10 3.2	10 17.5	10 31.4	10 44.8	10 57.9	11 10.7	11 23.3	11 35.5	11 47.6	11 59.6	12 11.4	104	76
254	286	9 56.9	10 17.1	10 36.3	10 54.7	11 12.3	11 29.3	11 45.6	12 1.5	12 16.8	12 31.8	12 46.4	13 0.7	13 14.7	13 28.5	13 42.1	13 55.6	106	74
252	288	11 11.4	11 34.1	11 55.7	12 16.4	12 36.2	12 55.3	13 13.7	13 31.5	13 48.7	14 5.6	14 22.0	14 38.0	14 53.7	15 9.3	15 24.5	15 39.6	108	72
250	290	12 25.8	12 51.1	13 15.1	13 38.1	14 0.1	14 21.3	14 41.7	15 1.4	15 20.6	15 39.3	15 57.5	16 15.3	16 32.7	16 49.9	17 6.8	17 23.6	110	70
248	292	13 40.2	14 8.0	14 34.4	14 59.7	15 23.9	15 47.2	16 9.6	16 31.3	16 52.4	17 12.9	17 32.9	17 52.4	18 11.6	18 30.5	18 49.0	19 7.4	112	68
246	294	14 54.5	15 24.9	15 53.7	16 21.3	16 47.7	17 13.1	17 37.5	18 1.2	18 24.1	18 46.5	19 8.3	19 29.5	19 50.4	20 10.9	20 31.1	20 51.1	114	66
244	296	16 8.8	16 41.7	17 12.9	17 42.8	18 11.4	18 38.9	19 5.4	19 31.0	19 55.8	20 20.0	20 43.5	21 6.5	21 29.1	21 51.2	22 13.0	22 34.6	116	64
242	298	17 23.0	17 58.4	18 32.1	19 4.3	19 35.1	20 4.7	20 33.2	21 0.7	21 27.4	21 53.4	22 18.7	22 43.4	23 7.6	23 31.4	23 54.8	24 17.9	118	62
240	300	18 37.1	19 15.1	19 51.2	20 25.7	20 58.7	21 30.4	22 0.9	22 30.4	22 58.9	23 26.7	23 53.8	24 20.2	24 46.1	25 11.5	25 36.5	26 1.1	120	60
238	302	19 51.2	20 31.7	21 10.3	21 47.1	22 22.2	22 56.0	23 28.6	24 0.0	24 30.4	25 0.0	25 28.8	25 56.9	26 24.4	26 51.4	27 18.0	27 44.2	122	58
236	304	21 5.2	21 48.3	22 29.3	23 8.4	23 45.7	24 21.6	24 56.2	25 29.5	26 1.8	26 33.2	27 3.7	27 33.5	28 2.6	28 31.2	28 59.3	29 27.0	124	56
234	306	22 19.2	23 4.8	23 48.2	24 29.6	25 9.2	25 47.2	26 23.7	26 59.0	27 33.1	28 6.3	28 38.5	29 9.9	29 40.7	30 10.9	30 40.5	31 9.7	126	54
232	308	23 33.0	24 21.3	25 7.1	25 50.8	26 32.6	27 12.7	27 51.2	28 28.4	29 4.4	29 39.3	30 13.2	30 46.3	31 18.7	31 50.4	32 21.5	32 52.2	128	52
230	310	24 46.8	25 37.7	26 26.0	27 12.0	27 56.0	28 38.1	29 18.7	29 57.8	30 35.6	31 12.2	31 47.8	32 22.6	32 56.5	33 29.8	34 2.5	34 34.6	130	50
228	312	28 0.6	26 54.0	27 44.8	28 33.1	29 19.3	30 3.5	30 46.1	31 27.1	32 6.7	32 45.1	33 22.4	33 58.8	34 34.3	35 9.1	35 43.2	36 16.8	132	48
226	314	27 14.2	28 10.3	29 3.5	29 54.2	30 42.6	31 28.9	32 13.5	32 56.3	33 37.8	34 17.9	34 56.9	35 34.9	36 12.0	36 48.3	37 23.8	37 58.8	134	46
224	316	28 27.8	29 26.5	30 22.2	31 15.2	32 5.8	32 54.3	33 40.8	34 25.6	35 8.8	35 50.7	36 31.4	37 10.9	37 49.6	38 27.3	39 4.3	39 40.7	136	44
222	318	29 41.3	30 42.7	31 40.9	32 36.2	33 29.1	34 19.6	35 8.1	35 54.8	36 39.8	37 23.5	38 5.8	38 46.9	39 27.1	40 6.3	40 44.7	41 22.5	138	42
220	320	30 54.8	31 58.8	32 59.6	33 57.2	34 52.3	35 44.9	36 35.4	37 24.0	38 10.8	38 56.2	39 40.1	40 22.8	41 4.5	41 45.2	42 25.0	43 4.1	140	40
218	322	32 8.2	33 14.9	34 18.2	35 18.2	36 15.5	37 10.2	38 2.7	38 53.2	39 41.8	40 28.9	41 14.4	41 58.7	42 41.8	43 24.0	44 5.2	44 45.6	142	38
216	524	33 21.6	34 31.0	35 36.8	36 39.2	37 38.7	38 35.6	39 30.0	40 22.4	41 12.8	42 1.5	42 48.7	43 34.6	44 19.2	45 2.7	45 45.3	46 27.0	144	36
214	326	34 34.9	35 47.1	36 55.4	38 0.2	39 1.9	40 0.9	40 57.4	41 51.6	42 43.8	43 34.2	44 23.0	45 10.4	45 56.5	46 41.4	47 25.3	48 8.3	146	34
212	328	35 48.1	37 3.1	38 14.0	39 21.2	40 25.2	41 26.3	42 24.8	43 20.9	44 14.9	45 7.0	45 57.4	46 46.2	47 33.8	48 20.1	49 5.3	49 49.6	148	32
210	330	37 1.4	38 19.1	39 32.6	40 42.3	41 48.5	42 51.8	43 52.2	44 50.2	45 46.0	46 39.8	47 31.7	48 22.1	49 11.1	49 58.7	50 45.3	51 30.8	150	30
208	332	38 14.6	39 35.1	40 51.2	42 3.4	43 11.9	44 17.3	45 19.8	46 19.6	47 17.2	48 12.6	49 6.2	49 58.1	50 48.4	51 37.4	52 25.2	53 12.0	152	28
206	334	39 27.8	40 51.2	42 9.9	43 24.5	44 35.4	45 42.9	46 47.4	47 49.1	48 48.4	49 45.5	50 40.7	51 34.1	52 25.8	53 16.2	54 5.2	54 53.2	154	26
204	336	40 41.0	42 7.2	43 28.7	44 45.8	45 58.9	47 8.6	48 15.1	49 18.8	50 19.8	51 18.6	52 15.3	53 10.2	54 3.3	54 55.0	55 45.3	56 34.4	156	24
202	338	41 54.1	43 23.3	44 47.5	46 7.1	47 22.6	48 34.5	49 43.0	50 48.6	51 51.4	52 51.8	53 50.1	54 46.4	55 40.9	56 33.9	57 25.4	58 15.7	158	22
200	340	43 7.3	44 39.5	46 6.4	47 28.6	48 46.5	50 0.5	51 11.1	52 18.5	53 23.1	54 25.2	55 25.0	56 22.8	57 18.6	58 12.9	59 5.6	59 57.0	160	20
198	342	44 20.6	45 55.8	47 25.5	48 50.2	50 10.5	51 26.7	52 39.3	53 48.7	54 55.1	55 58.8	57 0.1	57 59.3	58 56.5	59 52.0	60 46.0	61 38.5	162	18
196	344	45 33.9	47 12.2	48 44.7	50 12.0	51 34.7	52 53.1	54 7.8	55 19.1	56 27.3	57 32.6	58 35.5	59 36.1	60 34.7	61 31.4	62 26.5	63 20.1	164	16
194	346	46 47.3	48 28.7	50 4.1	51 34.0	52 59.1	54 19.8	55 36.6	56 49.8	57 59.7	59 6.7	60 11.1	61 13.2	62 13.0	63 11.0	64 7.2	65 1.9	166	14

192	348	48 0.7	49 45.3	51 23.6	52 56.3	54 23.8	55 46.8	57 5.7	58 20.8	59 32.5	60 41.1	61 47.0	62 50.5	63 51.7	64 50.8	65 48.2	66 43.9	168	12
190	350	49 14.2	51 2.1	52 43.4	54 18.8	55 48.8	57 14.1	58 35.1	59 52.1	61 5.6	62 15.9	63 23.3	64 28.2	65 30.7	66 31.0	67 29.5	68 26.2	170	10
188	352	50 27.9	52 19.1	54 3.4	55 41.6	57 14.2	58 41.8	60 4.8	61 23.8	62 39.1	63 51.0	65 0.0	66 6.2	67 10.0	68 11.5	69 11.1	70 8.8	172	8
186	354	51 41.8	53 36.3	55 23.7	57 4.7	58 39.9	60 9.8	61 35.0	62 56.0	64 13.0	65 26.6	66 37.1	67 44.7	68 49.7	69 52.4	70 53.1	71 51.8	174	6
184	356	52 55.8	54 53.8	56 44.4	58 28.2	60 6.0	61 38.3	63 5.7	64 28.6	65 47.5	67 2.7	68 14.7	69 23.6	70 29.9	71 33.8	72 35.5	73 35.1	176	4
182	358	54 10.0	56 11.6	58 5.4	59 52.1	61 32.6	63 7.3	64 36.9	66 1.8	67 22.5	68 39.4	69 52.8	71 3.1	72 10.6	73 15.6	74 18.3	75 18.9	178	2
180	360	55 24.5	57 29.7	59 26.8	61 16.5	62 59.7	64 36.9	66 8.7	67 35.6	68 58.1	70 16.6	71 31.5	72 43.2	73 51.9	74 58.0	76 1.7	77 3.2	180	360
178	2	56 39.2	58 48.2	60 48.6	62 41.5	64 27.4	66 7.0	67 41.1	69 10.0	70 34.3	71 54.5	73 10.9	74 23.9	75 33.8	76 41.0	77 45.6	78 48.1	182	358
176	4	57 54.3	60 7.1	62 11.0	64 7.0	65 55.7	67 37.9	69 14.2	70 45.1	72 11.3	73 33.1	74 50.9	76 5.3	77 16.4	78 24.6	79 30.2	80 33.5	184	356
174	6	59 9.7	61 26.4	63 33.9	65 33.1	67 24.7	69 9.4	70 48.0	72 21.0	73 49.0	75 12.4	76 31.8	77 47.4	78 59.7	80 9.0	81 15.5	82 19.6	186	354
172	8	60 25.5	62 46.3	64 57.5	66 59.9	68 54.4	70 41.8	72 22.7	73 57.8	75 27.6	76 52.7	78 13.4	79 30.4	80 43.8	81 54.1	83 1.5	84 6.4	188	352
170	10	61 41.7	64 6.8	66 21.7	68 27.5	70 25.0	72 15.0	73 58.3	75 35.4	77 7.1	78 33.8	79 56.0	81 14.3	82 28.8	83 40.1	84 48.4	85 54.0	190	350
168	12	62 58.5	65 27.9	67 46.7	69 56.0	71 56.5	73 49.2	75 34.9	77 14.1	78 47.6	80 16.0	81 39.6	82 59.1	84 14.7	85 27.0	86 36.1	87 42.4	192	348
166	14	64 15.8	66 49.7	69 12.6	71 25.4	73 29.0	75 24.5	77 12.5	78 53.9	80 29.2	81 59.2	83 24.3	84 45.0	86 1.7	87 14.8	88 24.8	89 31.8	194	346
164	16	65 33.7	68 12.3	70 39.3	72 55.7	75 2.6	77 0.9	78 51.3	80 34.8	82 12.0	83 43.6	85 10.1	86 32.0	87 49.7	89 3.8	90 14.5	91 22.1	196	344
162	18	66 52.4	69 35.8	72 7.1	74 27.3	76 37.4	78 38.5	80 31.4	82 17.1	83 56.1	85 29.3	86 57.1	88 20.2	89 38.9	90 53.8	92 5.2	93 13.5	198	342
160	20	68 11.8	71 0.3	73 36.0	76 0.0	78 13.5	80 17.4	82 12.9	84 0.7	85 41.6	87 16.3	88 45.5	90 9.7	91 29.4	92 45.1	93 57.2	95 6.0	200	340
158	22	69 32.1	72 25.8	75 6.1	77 34.1	79 51.0	81 57.9	83 55.9	85 45.8	87 28.5	89 4.8	90 35.3	92 0.6	93 21.2	94 37.7	95 50.4	96 59.7	202	338
156	24	70 53.3	73 52.5	76 37.5	79 9.6	81 30.0	83 39.9	85 40.4	87 32.5	89 17.1	90 54.9	92 26.7	93 53.0	95 14.5	96 31.7	97 44.9	98 54.6	204	336
154	26	72 15.6	75 20.5	78 10.4	80 46.7	83 10.7	85 23.7	87 26.8	89 21.0	91 7.4	92 46.7	94 19.7	95 47.0	97 9.3	98 27.1	99 40.8	100 50.9	206	334
152	28	73 39.1	76 50.0	79 45.0	82 25.6	84 53.3	87 9.3	89 15.0	91 11.4	92 59.5	94 40.3	96 14.4	97 42.7	99 5.8	100 24.1	101 38.2	102 48.6	208	332
150	30	75 3.9	78 21.0	81 21.4	84 6.4	86 37.9	88 57.0	91 5.3	93 3.8	94 53.6	96 35.8	98 11.1	99 40.2	101 3.9	102 22.8	103 37.3	104 47.8	210	330
148	32	76 30.2	79 53.8	82 59.7	85 49.4	88 24.6	90 46.9	92 57.8	94 58.4	96 49.9	98 33.4	100 9.7	101 39.6	103 3.9	104 23.2	105 38.0	106 48.6	212	328
146	34	77 58.1	81 28.6	84 40.2	87 34.6	90 13.8	92 39.2	94 52.6	96 55.3	98 48.4	100 33.2	102 10.4	103 41.1	105 5.9	106 25.5	107 40.4	108 51.1	214	326
144	36	79 27.8	83 5.5	86 23.0	89 22.4	92 5.5	94 34.2	96 50.1	98 54.7	100 49.4	102 35.3	104 13.4	105 44.7	107 9.9	108 29.7	109 44.7	110 55.3	216	324
142	38	80 59.6	84 44.8	88 8.5	91 12.9	94 0.1	96 31.9	98 50.3	100 56.9	102 53.0	104 40.0	106 18.9	107 50.6	109 16.1	110 36.0	111 50.9	113 1.4	218	322
140	40	82 33.7	86 26.8	89 57.0	93 6.5	95 57.7	98 32.7	100 53.6	103 2.0	104 59.4	106 47.3	108 26.8	109 58.9	111 24.5	112 44.4	113 59.1	115 9.4	220	320
138	42	84 10.3	88 11.8	91 48.6	95 3.4	97 58.7	100 36.9	103 0.1	105 10.2	107 8.8	108 57.5	110 37.5	112 9.8	113 35.4	114 55.1	116 9.5	117 19.3	222	318
136	44	85 49.8	90 0.1	93 43.8	97 4.0	100 3.4	102 44.7	105 10.1	107 21.7	109 21.4	111 10.7	112 51.0	114 23.3	115 48.7	117 8.1	118 22.0	119 31.3	224	316
134	46	87 32.6	91 52.1	95 43.0	99 8.7	102 12.1	104 56.3	107 23.8	109 36.8	111 37.4	113 27.1	115 7.4	116 39.6	118 4.7	119 23.5	120 36.9	121 45.4	226	314
132	48	89 19.1	93 48.2	97 46.6	101 17.8	104 25.2	107 12.2	109 41.5	111 55.7	113 56.9	115 46.8	117 27.0	118 58.8	120 23.3	121 41.5	122 54.0	124 1.7	228	312
130	50	91 9.8	95 49.1	99 55.1	103 31.8	106 43.1	109 32.6	112 3.6	114 18.7	116 20.2	118 10.0	119 49.9	121 21.1	122 44.8	124 2.1	125 13.6	126 20.2	230	310
128	52	93 5.2	97 55.2	102 9.0	105 51.1	109 6.1	111 58.0	114 30.3	116 46.0	118 47.5	120 37.0	122 16.2	123 46.5	125 9.2	126 25.4	127 35.7	128 41.1	232	308
126	54	95 6.0	100 7.2	104 28.9	108 16.4	111 34.8	114 28.7	117 1.9	119 17.8	121 19.0	123 7.8	124 46.0	126 15.2	127 36.6	128 51.4	130 0.4	131 4.3	234	306
124	56	97 13.1	102 26.0	106 55.5	110 48.2	114 9.6	117 5.1	119 38.8	121 54.5	123 54.9	125 42.6	127 19.5	128 47.2	130 7.1	131 20.3	132 27.6	133 29.8	236	304
122	58	99 27.4	104 52.3	109 29.7	113 27.1	116 51.1	119 47.6	122 21.3	124 36.2	126 35.4	128 21.6	129 56.8	131 22.7	132 40.7	133 52.0	134 57.4	135 57.8	238	302
120	60	101 50.0	107 27.2	112 12.3	116 13.9	119 39.8	122 36.6	125 9.6	127 23.2	129 20.7	131 4.9	132 38.0	134 1.7	135 17.5	136 26.6	137 29.9	138 28.1	240	300
118	62	104 22.2	110 12.0	115 4.2	119 9.3	122 36.3	125 32.6	128 4.2	130 15.8	132 10.9	133 52.6	135 23.1	136 44.3	137 57.5	139 4.1	140 5.0	141 0.9	242	298
116	64	107 5.7	113 7.9	118 6.6	122 14.1	125 41.1	128 36.0	131 5.3	133 14.1	135 6.2	136 44.8	138 12.3	139 30.4	140 40.7	141 44.5	142 42.7	143 36.0	244	296
114	66	110 2.3	116 16.5	121 20.4	125 29.1	128 54.9	131 47.1	134 13.1	136 18.3	138 6.7	139 41.6	141 5.4	142 20.1	143 27.1	144 27.8	145 22.9	146 13.4	246	394
112	68	113 14.4	119 39.5	124 47.0	128 55.2	132 18.1	135 6.3	137 27.9	139 28.4	141 12.3	142 42.9	144 2.6	145 13.3	146 16.7	147 13.8	148 5.7	148 53.1	248	292
110	70	116 44.6	123 18.8	128 27.7	132 33.0	135 51.2	138 33.9	140 49.7	142 44.6	144 23.1	145 48.7	147 3.6	148 10.0	149 9.3	150 2.6	150 50.9	151 35.0	250	290
108	72	120 36.1	127 16.6	132 23.6	136 23.4	139 34.5	142 10.0	144 18.6	146 6.8	147 39.1	148 58.9	150 8.5	151 10.0	152 4.8	152 53.9	153 38.4	154 18.9	252	288
106	74	124 52.6	131 34.9	136 35.9	140 26.8	143 28.4	144 54.6	147 54.6	149 34.9	151 0.0	152 13.3	153 17.1	154 13.2	155 3.1	155 47.8	156 28.1	157 4.8	254	286
104	76	129 38.3	136 16.0	141 5.6	144 43.7	147 32.8	149 47.6	151 37.4	153 8.6	154 25.6	155 31.7	156 29.0	157 19.4	158 4.0	158 43.9	159 19.8	159 52.5	256	284
102	78	134 57.5	141 21.5	145 53.3	149 14.0	151 47.5	153 48.7	155 26.7	156 47.6	157 55.7	158 53.9	159 44.2	160 28.3	161 7.3	161 42.1	162 13.4	162 41.8	258	282
100	80	140 54.4	146 52.8	150 59.0	153 57.4	156 12.1	157 57.4	159 22.0	160 31.5	161 29.8	162 19.4	163 2.2	163 39.6	164 12.6	164 42.1	165 8.5	165 32.5	260	280
98	82	147 32.0	152 49.9	156 22.1	158 53.0	160 45.6	162 12.9	163 22.6	164 19.7	165 7.3	165 47.8	166 22.6	166 53.0	167 19.8	167 43.7	168 5.1	168 24.5	262	278
96	84	154 51.0	159 11.6	162 0.8	163 59.3	165 26.8	166 34.2	167 27.8	168 11.5	168 47.8	169 18.7	169 45.1	170 8.2	170 28.5	170 46.6	171 2.8	171 17.5	264	276
94	86	162 48.3	165 54.5	167 52.6	169 14.4	170 14.3	171 0.2	171 36.5	172 6.1	172 30.6	172 51.4	173 9.2	173 24.7	173 38.4	173 50.5	174 1.4	174 11.2	266	274
92	88	171 15.8	172 53.0	173 53.9	174 35.6	175 6.0	175 29.3	175 47.6	176 2.5	176 14.9	176 25.4	176 34.3	176 42.1	176 49.0	176 55.1	177 0.6	177 5.5	268	272
90	90	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	270	270

Table F.—For the annual Argument of a Star in North Polar Distance.

Enter with the Right Ascension on the left side if the Declination is North, on the right side if South.

Subtract from 180°	Add to 180°	Declination of the Star. 60°	62°	64°	66°	68°	70°	72°	74°	76°	78°	80°	82°	84°	86°	88°	90°	Add to 180°	Subtract from 180°
		° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′		
270	270	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	90	90
268	272	1 44.6	1 46.3	1 48.0	1 49.6	1 51.3	1 53.0	1 54.7	1 56.3	1 58.0	1 59.8	2 1.5	2 3.3	2 5.1	2 7.0	2 8.9	2 10.8	92	88
266	274	3 29.2	3 32.6	3 35.9	3 39.2	3 42.6	3 45.9	3 49.3	3 52.6	3 56.0	3 59.5	4 3.0	4 6.5	4 10.1	4 13.9	4 17.7	4 21.6	94	86
264	276	5 13.7	5 18.8	5 23.8	5 28.8	5 33.8	5 38.8	5 43.8	5 48.9	5 54.0	5 59.1	6 4.4	6 9.7	6 15.1	6 20.7	6 26.4	6 32.2	96	84
262	278	6 58.2	7 5.0	7 11.6	7 18.3	7 24.9	7 31.6	7 38.3	7 45.0	7 51.8	7 58.7	8 5.7	8 12.7	8 19.9	8 27.3	8 34.9	8 42.6	98	82
260	280	8 42.7	8 51.1	8 59.4	9 7.7	9 16.0	9 24.3	9 32.7	9 41.1	9 49.5	9 58.1	10 6.8	10 15.6	10 24.6	10 33.8	10 43.2	10 52.8	100	80
258	282	10 27.1	10 37.1	10 47.1	10 57.0	11 7.0	11 16.9	11 26.9	11 37.0	11 47.1	11 57.3	12 7.7	12 18.3	12 29.0	12 40.0	12 51.2	13 2.8	102	78
256	284	12 11.4	12 23.1	12 34.7	12 46.2	12 57.8	13 9.4	13 21.0	13 32.7	13 44.5	13 56.3	14 8.4	14 20.7	14 33.1	14 45.9	14 58.9	15 12.4	104	76
254	286	13 55.6	14 8.9	14 22.1	14 35.3	14 48.5	15 1.7	15 14.9	15 28.2	15 41.6	15 55.1	16 8.9	16 22.8	16 37.0	16 51.5	17 6.3	17 21.5	106	74
252	288	15 39.6	15 54.6	16 9.4	16 24.2	16 39.0	16 53.8	17 8.6	17 23.5	17 38.5	17 53.7	18 9.0	18 24.6	18 40.5	18 56.7	19 13.3	19 30.3	108	72
250	290	17 23.6	17 40.1	17 56.6	18 13.0	18 29.3	18 45.7	19 2.1	19 18.5	19 35.1	19 51.9	20 8.9	20 26.1	20 43.6	21 1.5	21 19.8	21 38.5	110	70
248	292	19 7.4	19 25.5	19 43.6	20 1.5	20 19.4	20 37.4	20 55.3	21 13.3	21 31.5	21 49.8	22 8.4	22 27.2	22 46.3	23 5.8	23 25.8	23 46.2	112	68
246	294	20 51.1	21 10.8	21 30.4	21 49.9	22 9.3	22 28.8	22 48.3	23 7.8	23 27.5	23 47.4	24 7.5	24 27.9	24 48.6	25 9.7	25 31.3	25 53.4	114	66
244	296	22 34.6	22 55.9	23 17.0	23 38.1	23 59.0	24 20.0	24 41.0	25 2.0	25 23.2	25 44.6	26 6.2	26 28.1	26 50.4	27 13.1	27 36.2	28 0.0	116	64
242	298	24 17.9	24 40.8	25 3.5	25 26.0	25 48.5	26 10.9	26 33.4	26 55.9	27 18.6	27 41.4	28 4.5	28 27.9	28 51.7	29 15.9	29 40.6	30 5.9	118	62
240	300	26 1.1	26 25.5	26 49.7	27 13.7	27 37.7	28 1.5	28 25.5	28 49.4	29 13.6	29 37.9	30 2.4	30 27.3	30 52.5	31 18.2	31 44.4	32 11.2	120	60
238	302	27 44.2	28 10.1	28 35.7	29 1.2	29 26.6	29 51.9	30 17.2	30 42.6	31 8.2	31 33.9	31 59.8	32 26.1	32 52.8	33 19.9	33 47.5	34 15.7	122	58
236	304	29 27.0	29 54.4	30 21.5	30 48.5	31 15.3	31 42.0	32 8.7	32 35.5	33 2.4	33 29.5	33 56.8	34 24.5	34 52.5	35 21.0	35 50.0	36 19.6	124	56
234	306	31 9.7	31 38.6	32 7.1	32 35.5	33 3.7	33 31.8	33 59.9	34 28.0	34 56.2	35 24.7	35 53.3	36 22.3	36 51.7	37 21.5	37 51.8	38 22.8	126	54
232	308	32 52.2	33 22.5	33 52.5	34 22.3	34 51.8	35 21.3	35 50.7	36 20.1	36 49.7	37 19.4	37 49.3	38 19.6	38 50.3	39 21.4	39 53.0	40 25.3	128	52
230	310	34 34.6	35 6.3	35 37.7	36 8.8	36 39.7	37 10.5	37 41.2	38 11.9	38 42.7	39 13.7	39 44.9	40 16.4	40 48.3	41 20.6	41 53.5	42 27.0	130	50
228	312	36 16.8	36 49.9	37 22.6	37 55.1	38 27.3	38 59.4	39 31.4	40 3.3	40 35.4	41 7.6	41 40.0	42 12.7	42 45.8	43 19.3	43 53.3	44 28.0	132	48
226	314	37 58.8	38 33.3	39 7.4	39 41.2	40 14.7	40 48.0	41 21.2	41 54.4	42 27.6	43 1.0	43 34.6	44 8.4	44 42.7	45 17.3	45 52.5	46 28.3	134	46
224	316	39 40.7	40 16.6	40 52.0	41 27.0	42 1.8	42 36.3	43 10.7	43 45.1	44 19.5	44 54.0	45 28.7	46 3.7	46 39.0	47 14.7	47 51.0	48 27.8	136	44
222	318	41 22.5	41 59.7	42 36.4	43 12.6	43 48.6	44 24.3	44 59.9	45 35.5	46 11.0	46 46.6	47 22.4	47 58.4	48 34.8	49 11.5	49 48.8	50 26.7	138	42
220	320	43 4.1	43 42.6	44 20.5	44 58.0	45 35.2	46 12.1	46 48.8	47 25.5	48 2.1	48 38.7	49 15.6	49 52.6	50 30.0	51 7.7	51 46.0	52 24.8	140	40
218	322	44 45.6	45 25.4	46 4.6	46 43.3	47 21.6	47 59.6	48 37.5	49 15.2	49 52.8	50 30.5	51 8.3	51 46.4	52 24.7	53 3.4	53 42.6	54 22.3	142	38
216	324	46 27.0	47 8.0	47 48.4	48 28.3	49 7.8	49 46.9	50 25.8	51 4.6	51 43.2	52 21.9	53 0.7	53 39.6	54 18.9	54 58.5	55 38.5	56 19.1	144	36
214	326	48 8.3	48 50.6	49 32.2	50 13.2	50 53.8	51 34.0	52 13.9	52 53.7	53 33.3	54 12.9	54 52.6	55 32.4	56 12.5	56 53.0	57 33.8	58 15.2	146	34
212	328	49 49.6	50 33.1	51 15.8	51 58.0	52 39.6	53 20.8	54 1.8	54 42.5	55 23.0	56 3.6	56 44.1	57 24.8	58 5.7	58 47.0	59 28.6	60 10.7	148	32
210	330	51 30.8	52 15.5	52 59.3	53 42.6	54 25.3	55 7.5	55 49.4	56 31.1	57 12.5	57 53.9	58 35.3	59 16.8	59 58.5	60 40.5	61 22.8	62 5.6	150	30
208	332	53 12.0	53 57.8	54 42.8	55 27.1	56 10.8	56 54.0	57 36.9	58 19.4	59 1.7	59 43.9	60 26.1	61 8.4	61 50.8	62 33.5	63 16.5	64 0.0	152	28
206	334	54 53.2	55 40.1	56 26.2	57 11.5	57 56.2	58 40.4	59 24.1	60 7.5	60 50.7	61 33.7	62 16.6	62 59.6	63 42.7	64 26.1	65 9.7	65 53.8	154	26
204	336	56 34.4	57 22.4	58 9.6	58 55.9	59 41.6	60 26.7	61 11.3	61 55.5	62 39.4	63 23.2	64 6.8	64 50.5	65 34.3	66 18.2	67 2.5	67 47.1	156	24
202	338	58 15.7	59 4.8	59 53.0	60 40.3	61 26.9	62 12.9	62 58.3	63 43.3	64 28.0	65 12.5	65 56.8	66 41.1	67 25.5	68 10.0	68 54.8	69 39.9	158	22
200	340	59 57.0	60 47.2	61 36.4	62 24.7	63 12.2	63 59.0	64 45.2	65 31.0	66 16.4	67 1.6	67 46.5	68 31.4	69 16.4	70 1.4	70 46.7	71 32.2	160	20
198	342	61 38.5	62 29.7	63 19.9	64 9.1	64 57.5	65 45.1	66 32.1	67 18.6	68 4.7	68 50.5	69 36.1	70 21.5	71 7.0	71 52.5	72 38.2	73 24.2	162	18
196	344	63 20.1	64 12.4	65 3.5	65 53.6	66 42.8	67 31.2	68 19.0	69 6.2	69 52.9	70 39.3	71 25.4	72 11.4	72 57.4	73 43.3	74 29.5	75 15.8	164	16
194	346	65 1.9	65 56.2	66 47.3	67 38.2	68 28.2	69 17.4	70 5.9	70 53.7	71 41.1	72 28.9	73 14.7	74 1.1	74 47.5	75 33.9	76 20.4	77 7.0	166	14

192	348	66 43.9	67 38.2	68 31.2	69 23.0	70 13.7	71 3.6	71 52.8	72 41.3	73 29.2	74 16.7	75 3.8	75 50.7	76 37.5	77 24.3	78 11.1	78 58.0	168	12
190	350	68 26.2	69 21.5	70 15.3	71 7.9	71 59.4	72 50.0	73 39.8	74 28.9	75 17.3	76 5.3	76 52.9	77 40.2	78 27.4	79 14.5	80 1.5	80 48.7	170	10
188	352	70 8.8	71 5.0	71 59.7	72 53.1	73 45.3	74 36.6	75 27.0	76 16.6	77 5.5	77 54.0	78 42.0	79 29.7	80 17.2	81 4.5	81 51.8	82 39.2	172	8
186	354	71 51.8	72 48.8	73 44.3	74 38.5	75 31.4	76 23.3	77 14.3	78 4'4	78 53.8	79 42.7	80 31.1	81 19.1	82 6.9	82 54.5	83 42.0	84 29.6	174	6
184	356	73 35.1	74 33.0	75 29.3	76 24.2	77 17.8	78 10.3	79 1.8	79 52.4	80 42.3	81 31.6	82 20.3	83 8.6	83 56.6	84 44.4	85 32.1	86 19.8	176	4
182	358	75 18.9	76 17.7	77 14.7	78 10.3	79 4.6	79 57.6	80 49.6	81 40.7	82 31.0	83 20.6	84 9.6	84 58.2	85 46.4	86 34.3	87 22.2	88 9.9	178	2
180	360	77 3.2	78 2.8	79 0.6	79 56.8	80 51.7	81 45.2	82 37.7	83 29.2	84 19.8	85 9.8	85 59.0	86 47.8	87 36.2	88 24.3	89 12.2	90 0.0	190	360
178	2	78 48.1	79 48.4	80 46.9	81 43.8	82 39.2	83 33.2	84 26.1	85 18.0	86 9.0	86 59.2	87 48.7	88 37.6	89 26.2	90 14.3	91 2.3	91 50.1	182	358
176	4	80 33.5	81 34.6	82 33.8	83 31.3	84 27.2	85 21.7	86 15.0	87 7.2	87 58.5	88 48.9	89 38.6	90 27.7	91 16.3	92 4.5	92 52.5	93 40.2	184	356
174	6	82 19.6	83 21.4	84 21.3	85 19.3	86 15.7	87 10.6	88 4.3	88 56.8	89 48.3	90 38.9	91 28.7	92 17.9	93 6.6	93 54.8	94 42.8	95 30.4	186	354
172	8	84 6.4	85 8.9	86 9.4	87 7.9	88 4.8	89 0.1	89 54.1	90 46.8	91 38.5	92 29.3	93 19.2	94 8.5	94 57.2	95 45.4	96 33.2	97 20.8	188	352
170	10	85 54.0	86 57.2	87 58.2	88 57.2	89 54.5	90 50.1	91 44.4	92 37.4	93 29.2	94 20.1	95 10.1	95 59.4	96 48.0	97 36.2	98 23.9	99 11.3	190	350
168	12	87 42.4	88 46.2	89 47.8	90 47.2	91 44.9	92 40.8	93 35.3	94 28.5	95 20.4	96 11.4	97 1.4	97 50.7	98 39.2	99 27.2	100 14.8	101 2.0	182	348
166	14	89 31.8	90 36.2	91 38.2	92 38.0	93 36.0	94 52.2	95 26.9	96 20.2	97 12.2	98 3.2	98 53.2	99 42.4	100 30.8	101 18.6	102 6.0	102 53.0	194	346
164	16	91 22.1	92 27.0	93 29.5	94 29.7	95 27.9	96 24.3	97 19.2	98 12.5	99 4.6	99 55.5	100 45.5	101 34.5	102 22.8	103 10.4	103 57.6	104 44.2	196	344
162	18	93 13.5	94 18.9	95 21.7	96 22.3	97 20.7	98 17.3	99 12.2	100 5.6	100 57.6	101 48.5	102 38.3	103 27.2	104 15.3	105 2.6	105 49.5	106 35.8	198	342
160	20	95 6.0	96 11.8	97 15.0	98 15.8	99 14.4	100 11.1	101 6.0	101 59.4	102 51.4	103 42.1	104 31.7	105 20.4	106 8.2	106 55.3	107 41.8	108 27.8	200	340
158	22	96 59.7	98 5.9	99 9.3	100 10.3	101 9.0	102 5.8	103 0.7	103 54.0	104 45.9	105 36.4	106 25.8	107 14.2	108 1.7	108 48.5	109 34.6	110 20.1	202	338
156	24	98 54.6	100 1.1	101 4.8	102 5.9	103 4.7	104 1.5	104 56.3	105 49.5	106 41.2	107 31.5	108 20.6	109 8.7	109 55.8	110 42.2	111 27.9	112 12.9	204	336
154	26	100 50.9	101 57.7	103 1.5	104 2.7	105 1.5	105 58.2	106 52.9	107 45.9	108 37.3	109 27.3	110 16.1	111 3.8	111 50.6	112 36.5	113 21.7	114 6.2	206	334
152	28	102 48.6	103 55.6	104 59.5	106 0.7	106 59.4	107 55.9	108 50.4	109 43.2	110 34.3	111 24.0	112 12.4	112 59.7	113 46.0	114 31.4	115 16.0	116 0.0	208	332
150	30	104 47.8	105 54.9	106 58.9	108 0.0	108 58.5	109 54.8	110 49.1	111 41.5	112 32.2	113 21.5	114 9.5	114 56.3	115 42.1	116 26.9	117 11.0	117 54.4	210	330
148	32	106 48.6	107 55.8	108 59.6	110 0.6	110 58.9	111 54.9	112 48.8	113 40.9	114 31.2	115 20.0	116 7.4	116 53.7	117 38.9	118 23.1	119 6.6	119 49.3	212	328
146	34	108 51.1	109 58.2	111 1.9	112 2.6	113 0.6	113 56.3	114 49.7	115 41.3	116 31.1	117 19.4	118 6.3	118 51.9	119 36.5	120 20.1	121 2.8	121 44.8	214	326
144	36	110 55.3	112 2.2	113 5.7	114 6.0	115 3.7	115 58.9	116 51.9	117 42.9	118 32.1	119 19.8	120 6.0	120 51.0	121 34.8	122 17.7	122 59.7	123 40.9	216	324
142	38	113 1.4	114 8.0	115 11.1	116 11.0	117 8.2	118 2.9	118 55.3	119 45.6	120 34.2	121 21.2	122 6.7	122 51.0	123 34.0	124 16.1	124 57.3	125 37.7	218	322
140	40	115 9.4	116 15.6	117 18.2	118 17.6	119 14.2	120 8.2	120 59.9	121 49.6	122 37.4	123 23.7	124 8.4	124 51.8	125 34.1	126 15.3	126 55.6	127 35.2	220	320
138	42	117 19.3	118 25.0	119 27.0	120 25.8	121 21.7	122 15.0	123 5.9	123 54.8	124 41.8	125 27.2	126 11.1	126 53.6	127 35.0	128 15.3	128 54.7	129 33.3	222	318
136	44	119 31.3	120 36.3	121 37.6	122 35.7	123 30.7	124 23.2	125 13.3	126 1.3	126 47.4	127 31.8	128 14.8	128 56.4	129 36.8	130 16.1	130 54.6	131 32.1	224	316
134	46	121 45.4	122 49.6	123 50.1	124 47.3	125 41.4	126 32.9	127 22.0	128 9.1	128 54.2	129 37.6	130 19.6	131 0.1	131 39.5	132 17.8	132 55.2	133 31.7	226	314
132	48	124 1.7	125 5.0	126 4.5	127 0.6	127 53.7	128 44.2	129 32.2	130 18.2	131 2.2	131 44.5	132 25.4	133 4.8	133 43.1	134 20.3	134 56.6	135 32.0	228	312
130	50	126 20.2	127 22.4	128 20.8	129 15.8	130 7.7	130 57.0	131 43.9	132 28.6	133 11.4	133 52.6	134 32.3	135 10.5	135 47.6	136 23.7	136 58.7	137 33.0	230	310
128	52	128 41.1	129 42.0	130 39.1	131 32.8	132 23.4	133 11.4	133 56.9	134 40.4	135 21.9	136 1.8	136 40.2	137 17.2	137 53.1	138 27.9	139 1.7	139 34.7	232	308
126	54	131 4.3	132 3.8	132 59.4	133 51.6	134 40.8	135 27.3	136 11.5	136 53.5	137 33.7	138 12.2	138 49.2	139 24.9	139 59.5	140 32.9	141 5.5	141 37.2	234	306
124	56	133 29.8	134 27.7	135 21.7	136 12.3	136 59.9	137 44.8	138 27.5	139 8.0	139 46.7	140 23.8	140 59.3	141 33.6	142 6.8	142 38.8	143 10.0	143 40.4	236	304
122	58	135 57.8	136 53.8	137 46.0	138 34.8	139 20.7	140 3.9	140 44.9	141 23.8	142 0.9	142 36.5	143 10.5	143 43.3	144 14.9	144 45.6	145 15.3	145 44.3	238	302
120	60	138 28.1	139 22.1	140 12.3	140 59.2	141 43.2	142 24.6	143 3.8	143 41.0	144 16.4	144 50.3	145 22.7	145 53.9	146 24.0	146 53.1	147 21.4	147 48.8	240	300
118	62	141 0.9	141 52.6	142 40.6	143 25.4	144 7.3	144 46.8	145 24.1	145 59.4	146 33.1	147 5.2	147 35.9	148 5.5	148 33.9	149 1.5	149 28.2	149 54.1	242	298
116	64	143 36.0	144 25.2	145 10.8	145 53.3	146 33.1	147 10.2	147 45.7	148 19.1	148 50.9	149 21.1	149 50.1	150 17.9	150 44.7	151 10.6	151 35.7	152 0.0	244	296
114	66	146 13.4	146 59.9	147 43.0	148 23.0	149 0.5	149 35.6	150 8.7	150 40.0	151 9.8	151 38.1	152 5.3	152 31.3	152 56.3	153 20.5	153 43.9	154 6.6	246	294
112	68	148 53.1	149 36.7	150 17.0	150 54.4	151 29.4	152 2.1	152 33.0	153 2.1	153 29.8	153 56.1	154 21.3	154 45.5	155 8.7	155 31.1	155 52.7	156 13.8	248	292
110	70	151 35.0	152 15.4	152 52.8	153 27.4	153 59.8	154 30.0	154 58.5	155 25.3	155 50.8	156 15.1	156 38.2	157 0.4	157 21.8	157 42.3	158 2.2	158 21.5	250	290
108	72	154 18.9	154 56.0	155 30.3	156 2.0	156 31.5	156 59.1	157 25.1	157 49.6	158 12.8	158 34.9	158 56.0	159 16.1	159 35.5	159 54.2	160 12.2	160 29.7	252	288
106	74	157 4.8	157 38.4	158 9.3	158 37.9	159 4.5	159 29.4	159 52.8	160 14.8	160 35.7	160 55.5	161 14.4	161 32.5	161 49.9	162 6.6	162 22.8	162 38.5	254	286
104	76	159 52.5	160 22.3	160 49.8	161 15.2	161 38.7	162 0.8	162 21.5	162 40.9	162 59.4	163 16.9	163 33.6	163 49.5	164 4.9	164 19.6	164 33.8	164 47.7	256	284
102	78	162 41.8	163 7.7	163 31.6	163 53.6	164 14.0	164 33.1	164 51.0	165 7.8	165 23.8	165 38.9	165 53.3	166 7.1	166 20.3	166 33.1	166 45.3	166 57.2	258	282
100	80	165 32.5	165 54.4	166 14.5	166 33.0	166 50.2	167 6.3	167 21.3	167 35.4	167 48.8	168 1.5	168 13.6	168 25.2	168 36.2	168 46.9	168 57.2	169 7.1	260	280
98	82	168 24.5	168 42.2	168 58.4	169 13.3	169 27.2	169 40.1	169 52.2	170 3.6	170 14.4	170 24.6	170 34.3	170 43.6	170 52.5	171 1.1	171 9.4	171 17.4	262	278
96	84	171 17.5	171 30.8	171 43.1	171 54.4	172 4.8	172 14.6	172 23.7	172 32.3	172 40.4	172 48.1	172 55.4	173 2.4	173 9.1	173 15.6	173 21.8	173 27.8	264	276
94	86	174 11.2	174 20.2	174 28.4	174 35.9	174 42.9	174 49.5	174 55.6	175 1.3	175 6.8	175 11.9	175 16.8	175 21.5	175 26.0	175 30.3	175 34.4	175 38.4	266	274
92	88	177 5.5	177 10.0	177 14.1	177 17.9	177 21.4	177 24.7	177 27.7	177 30.6	177 33.3	177 35.9	177 38.4	177 40.7	177 43.0	177 45.1	177 47.2	177 49.2	268	272
90	90	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 40	270	270

Supplement to Table F.—For those Stars which are near, both to the Ecliptic, and the Colure of the Solstices.

Subtract from 180°	Add to 180°	Declination of the Star. 15°	16°	17°	18°	19°	20°	21°	22°	23°	23° 30′	24°	25°	Add to 180°	Subtract from 180°
110	70	28° 43′.6	33° 0′.2	37° 50′.1	43° 15′.6	49° 16′.9	55° 51′.2	62° 51′.9	70° 8′.6	77° 27′.9	81° 4′.2	84° 36′.1	91° 21′.5	250	290
109	71	27 45.3	31 59.3	36 48.7	42 16.5	48 24.1	55 9.5	62 26.2	70 3.0	77 44.9	81 32.7	85 15.8	92 22.0	251	289
108	72	26 43.9	30 54.5	35 42.4	41 11.8	47 25.2	54 21.7	61 55.3	69 53.9	78 0.7	82 1.2	85 56.9	93 26.0	252	288
107	73	25 39.2	29 45.6	34 31.0	40 1.1	46 19.6	53 27.0	61 18.3	69 40.7	78 15.1	82 29.9	86 39.4	94 34.0	253	287
106	74	24 31.3	28 32.4	33 14.4	38 43.9	45 6.5	52 24.6	60 34.3	69 22.6	78 27.9	82 58.6	87 23.8	95 46.7	254	286
105	75	23 20.1	27 15.0	31 52.2	37 19.8	43 45.3	51 13.5	59 42.3	68 58.8	78 38.7	83 27.5	88 10.4	97 5.2	255	285
104	76	22 5.7	25 53.2	30 24.2	35 48.3	42 15.2	49 52.5	58 40.8	68 28.2	78 47.1	83 56.6	88 59.7	98 30.4	256	284
103	77	20 48.0	24 26.9	28 50.2	34 8.9	40 35.2	48 20.2	57 28.2	67 49.2	78 52.6	84 25.9	89 52.3	100 3.8	257	283
102	78	19 27.1	22 56.2	27 10.1	32 21.3	38 44.6	46 35.2	56 2.6	67 0.2	78 54.4	84 55.5	90 48.9	101 47.2	258	282
101	79	18 3.0	21 21.0	25 23.8	30 25.1	36 42.5	44 35.9	54 21.5	65 58.6	78 51.6	85 25.4	91 50.8	103 43.1	259	281
100	80	16 35.8	19 41.5	23 31.2	28 19.9	34 27.9	42 20.2	52 21.9	64 41.2	78 42.8	85 55.8	92 59.4	105 54.5	260	280
99	81	15 5.7	17 57.7	21 32.3	26 5.5	32 0.0	39 46.3	50 0.1	63 3.8	78 26.0	86 26.8	94 16.9	108 25.9	261	279
98	82	13 32.8	16 9.8	19 27.4	23 41.9	29 18.1	36 52.0	47 11.8	61 0.4	77 58.4	86 58.7	95 46.4	111 23.2	262	278
97	83	11 57.4	14 18.0	17 16.5	21 9.1	26 21.8	33 35.4	43 51.9	58 22.8	77 15.5	87 31.8	97 33.4	114 54.5	263	277
96	84	10 19.6	12 22.7	15 0.1	18 27.5	23 11.0	29 54.8	39 54.4	54 59.3	76 9.8	88 6.8	99 45.6	119 11.7	264	276
95	85	8 39.7	10 24.1	12 38.6	15 37.5	19 45.9	25 49.1	35 13.3	50 33.7	74 28.5	88 44.7	102 37.2	124 31.1	265	275
94	86	6 58.0	8 22.8	10 12.6	12 40.0	16 7.5	21 18.2	29 43.2	44 43.5	71 47.1	89 27.8	106 34.9	131 15.6	266	274
93	87	5 14.8	6 19.2	7 42.9	9 36.1	12 17.2	16 23.5	23 21.1	37 0.0	67 13.3	90 21.3	112 33.9	139 54.6	267	273
92	88	3 30.5	4 13.8	5 10.3	6 27.0	8 17.2	11 8.5	16 9.0	26 54.7	58 37.7	91 40.6	122 44.8	150 57.9	268	272
91	89	1 45.4	2 7.2	2 35.7	3 14.4	4 10.4	5 38.3	8 17.3	14 19.6	39 55.0	94 43.2	142 38.8	164 35.0	269	271
90	90	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	180 0	180 0	180 0	270	270

Subtract from 180°	Add to 180°	Declination of the Star. 25°	26°	27°	28°	29°	30°	31°	32°	33°	34°	35°	36°	Add to 180°	Subtract from 180°
110	70	91° 21′.5	97° 35′.8	103° 15′.0	108° 18′.2	112° 47′.1	116° 44′.6	120° 14′.0	123° 18′.8	126° 2′.4	128° 27′.7	130° 37′.2	132° 33′.0	250	290
109	71	92 22.0	98 53.8	104 46.6	109 59.9	114 35.6	118 37.5	122 9.3	125 15.3	127 59.0	130 23.7	132 32.1	134 26.6	251	289
108	72	93 26.0	100 16.8	106 24.1	111 47.7	116 30.2	120 36.1	124 10.0	127 16.6	129 59.9	132 23.6	134 30.6	136 23.4	252	288
107	73	94 34.0	101 45.5	108 8.3	113 42.3	118 31.5	122 41.0	126 16.4	129 23.0	132 5.5	134 27.6	136 32.7	138 23.4	253	287
106	74	95 46.7	103 20.9	110 0.0	115 44.7	120 40.1	124 52.6	128 28.9	131 34.9	134 15.9	136 35.9	138 38.6	140 26.8	254	286
105	75	97 5.2	105 4.1	112 0.3	117 55.7	122 56.8	127 11.6	130 47.8	133 52.5	136 31.2	138 48.5	140 48.3	142 33.5	255	285
104	76	98 30.4	106 56.5	114 10.6	120 16.3	125 22.3	129 38.3	133 13.6	136 16.0	138 51.6	141 5.6	143 1.9	144 43.7	256	284
103	77	100 3.8	108 59.6	116 32.3	122 47.8	127 57.4	132 13.5	135 46.6	138 45.6	141 17.3	143 27.2	145 19.4	146 57.2	257	283
102	78	101 47.2	111 15.6	119 7.1	125 31.2	130 43.1	134 57.5	138 27.1	141 21.5	143 48.3	145 53.3	147 40.7	149 14.0	258	282
101	79	103 43.1	113 46.9	121 57.0	128 28.1	133 40.1	137 51.0	141 15.3	144 3.9	146 24.7	148 23.9	150 5.9	151 34.1	259	281
100	80	105 54.5	116 36.7	125 4.4	131 39.9	136 49.3	140 54.4	144 11.6	146 52.8	149 6.5	150 59.0	152 34.8	153 57.4	260	280
99	81	108 25.9	119 48.9	128 31.7	135 8.0	140 11.5	144 8.0	147 15.9	149 48.2	151 53.6	153 38.5	155 7.4	156 23.7	261	279
98	82	111 23.2	123 28.2	132 21.9	138 54.0	143 47.2	147 32.0	150 28.4	152 49.9	154 45.7	156 22.1	157 43.4	158 53.0	262	278
97	83	114 54.5	127 40.4	136 37.9	142 59.0	147 37.0	151 6.4	153 48.7	155 57.9	157 42.8	159 9.6	160 22.7	161 24.9	263	277
96	84	119 11.7	132 32.3	141 22.6	147 23.9	151 40.8	154 51.0	157 16.7	159 11.6	160 44.4	162 0.8	163 4.9	163 59.3	264	276
95	85	124 31.1	138 11.5	146 38.2	152 9.1	155 58.3	158 45.2	160 51.7	162 30.7	163 50.1	164 55.3	165 49.8	166 35.9	265	275
94	86	131 15.6	144 45.0	152 26.0	157 14.1	160 28.7	162 48.3	164 33.1	165 54.5	166 59.5	167 52.7	168 36.9	169 14.4	266	274
93	87	139 54.6	152 18.0	158 45.2	162 37.2	165 10.5	166 59.0	168 19.8	169 22.2	170 11.9	170 52.4	171 26.0	171 54.4	267	273
92	88	150 57.9	160 49.7	165 32.2	168 15.5	170 1.5	171 15.8	172 10.7	172 53.0	173 26.6	173 53.9	174 16.5	174 35.6	268	272
91	89	164 35.0	170 10.6	172 40.3	174 4.8	174 59.1	175 36.8	176 4.6	176 26.0	176 42.9	176 56.6	177 8.0	177 17.6	269	271
90	90	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	180 0	270	270

Table G.—Semiannular Solar Equation of Stars in North Polar Distance.

Argument. 2 Sun's Long.—Æ of Star.

Signs.	O. + VI. −	I. + VII. −	II. + VIII. −	Signs.
D	"	"	"	D
0	0.0000	0.2172	0.3763	30
1	0.0076	0.2238	0.3800	29
2	0.0152	0.2302	0.3836	28
3	0.0227	0.2366	0.3871	27
4	0.0303	0.2430	0.3905	26
5	0.0379	0.2492	0.3938	25
6	0.0454	0.2554	0.3969	24
7	0.0530	0.2615	0.4000	23
8	0.0605	0.2675	0.4029	22
9	0.0680	0.2734	0.4056	21
10	0.0754	0.2793	0.4083	20
11	0.0829	0.2851	0.4108	19
12	0.0903	0.2907	0.4132	18
13	0.0977	0.2963	0.4155	17
14	0.1051	0.3018	0.4177	16
15	0.1125	0.3072	0.4197	15
16	0.1198	0.3125	0.4216	14
17	0.1270	0.3178	0.4234	13
18	0.1343	0.3229	0.4250	12
19	0.1415	0.3279	0.4265	11
20	0.1486	0.3328	0.4279	10
21	0.1557	0.3377	0.4292	9
22	0.1628	0.3424	0.4303	8
23	0.1698	0.3470	0.4313	7
24	0.1767	0.3515	0.4321	6
25	0.1836	0.3559	0.4328	5
26	0.1905	0.3602	0.4334	4
27	0.1973	0.3644	0.4339	3
28	0.2040	0.3685	0.4342	2
29	0.2106	0.3724	0.4344	1
30	0.2172	0 3763	0.4345	0
Signs.	V. + XI. −	IV. + X. −	III. + IX. −	Signs.

Table H.—For the Maximum of Aberration of a Star in North Polar Distance.

Enter with the Right Ascension on the left side if the Declination is North, on the right side if South.

Declination of the Star.

Right Ascen.		0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	23°	24°	26°	28°	30°	32°	34°	36°	38°	40°	42°	44°	Right Ascen.	
270	270	8.065	8.708	9.341	9.963	10.572	11.169	11.752	12.319	12.874	13.413	13.934	14.438	14.684	14.925	15.394	15.844	16.275	16.685	17.076	17.446	17.795	18.121	18.426	18.708	90	90
268	272	8.065	8.707	9.340	9.962	10.571	11.168	11.750	12.318	12.872	13.411	13.932	14.436	14.682	14.923	15.392	15.842	16.273	16.683	17.074	17.444	17.793	18.119	18.424	18.706	92	88
266	274	8.065	8.706	9.338	9.959	10.568	11.164	11.746	12.314	12.867	13.406	13.927	14.431	14.677	14.918	15.387	15.837	16.267	16.678	17.069	17.439	17.788	18.114	18.420	18.702	94	86
264	276	8.065	8.705	9.335	9.954	10.562	11.157	11.739	12.306	12.859	13.396	13.918	14.422	14.668	14.909	15.378	15.828	16.258	16.669	17.060	17.430	17.780	18.106	18.413	18.695	96	84
262	278	8.065	8.702	9.330	9.948	10.554	11.148	11.729	12.295	12.847	13.384	13.906	14.410	14.654	14.896	15.365	15.815	16.245	16.657	17.048	17.418	17.768	18.095	18.402	18.686	98	82
260	280	8.065	8.699	9.324	9.940	10.544	11.136	11.716	12.281	12.832	13.369	13.889	14.393	14.638	14.880	15.348	15.798	16.229	16.641	17.032	17.403	17.753	18.081	18.388	18.673	100	80
258	282	8.065	8.695	9.317	9.930	10.532	11.122	11.700	12.264	12.814	13.350	13.869	14.373	14.618	14.859	15.328	15.777	16.209	16.621	17.013	17.384	17.735	18.064	18.372	18.657	102	78
256	284	8.065	8.690	9.309	9.918	10.518	11.106	11.681	12.244	12.793	13.328	13.846	14.349	14.594	14.835	15.304	15.753	16.185	16.597	16.990	17.362	17.713	18.043	18.352	18.638	104	76
254	286	8.065	8.685	9.299	9.904	10.501	11.086	11.659	12.221	12.768	13.302	13.820	14.322	14.566	14.807	15.276	15.726	16.157	16.570	16.963	17.336	17.688	18.019	18.329	18.616	106	74
252	288	8.065	8.679	9.288	9.889	10.482	11.064	11.635	12.194	12.740	13.272	13.790	14.291	14.535	14.776	15.244	15.695	16.126	16.539	16.933	17.307	17.660	17.992	18.303	18.592	108	72
250	290	8.065	8.673	9.276	9.872	10.461	11.040	11.608	12.164	12.709	13.239	13.756	14.256	14.500	14.741	15.208	15.660	16.091	16.504	16.899	17.274	17.628	17.962	18.274	18.565	110	70
248	292	8.065	8.665	9.262	9.853	10.438	11.013	11.578	12.132	12.674	13.203	13.718	14.218	14.462	14.702	15.169	15.621	16.053	16.466	16.862	17.238	17.593	17.928	18.241	18.534	112	68
246	294	8.065	8.657	9.247	9.832	10.413	10.983	11.545	12.097	12.636	13.164	13.677	14.177	14.420	14.660	15.127	15.578	16.010	16.424	16.821	17.198	17.554	17.890	18.205	18.500	114	66
244	296	8.065	8.649	9.231	9.810	10.385	10.951	11.509	12.058	12.595	13.121	13.632	14.131	14.374	14.614	15.081	15.531	15.964	16.379	16.776	17.154	17.511	17.849	18.166	18.463	116	64
242	298	8.065	8.639	9.214	9.786	10.355	10.917	11.471	12.016	12.551	13.074	13.584	14.081	14.324	14.564	15.030	15.480	15.914	16.330	16.727	17.106	17.465	17.805	18.123	18.422	118	62
240	300	8.065	8.629	9.195	9.760	10.323	10.880	11.430	11.971	12.504	13.024	13.532	14.028	14.270	14.510	14.976	15.426	15.860	16.277	16.675	17.055	17.415	17.757	18.077	18.379	120	60
238	302	8.065	8.618	9.175	9.732	10.289	10.840	11.386	11.923	12.453	12.971	13.477	13.971	14.212	14.452	14.918	15.368	15.802	16.220	16.619	17.000	17.362	17.705	18.028	18.332	122	58
236	304	8.065	8.606	9.154	9.703	10.253	10.798	11.339	11.872	12.398	12.914	13.418	13.911	14.151	14.390	14.856	15.306	15.740	16.159	16.559	16.941	17.306	17.650	17.976	18.281	124	56
234	306	8.065	8.594	9.132	9.672	10.214	10.753	11.289	11.818	12.340	12.853	13.355	13.847	14.086	14.324	14.790	15.240	15.674	16.093	16.495	16.878	17.245	17.592	17.920	18.227	126	54
232	308	8.065	8.582	9.109	9.639	10.173	10.706	11.236	11.761	12.279	12.789	13.289	13.778	14.018	14.255	14.720	15.170	15.604	16.024	16.427	16.812	17.180	17.529	17.860	18.170	128	52
230	310	8.065	8.569	9.084	9.605	10.130	10.656	11.180	11.700	12.215	12.721	13.219	13.706	13.946	14.182	14.646	15.096	15.530	15.951	16.355	16.742	17.111	17.463	17.796	18.109	130	50
228	312	8.065	8.555	9.058	9.569	10.085	10.604	11.122	11.637	12.147	12.650	13.145	13.630	13.869	14.105	14.568	15.018	15.453	15.874	16.279	16.668	17.039	17.393	17.728	18.044	132	48
226	314	8.065	8.540	9.031	9.532	10.039	10.550	11.061	11.571	12.076	12.576	13.068	13.551	13.789	14.024	14.486	14.936	15.371	15.793	16.199	16.590	16.962	17.319	17.656	17.975	134	46
224	316	8.065	8.525	9.003	9.493	9.991	10.493	10.997	11.502	12.002	12.498	12.987	13.468	13.705	13.939	14.400	14.850	15.285	15.708	16.115	16.507	16.881	17.241	17.580	17.902	136	44
222	318	8.065	8.510	8.974	9.452	9.940	10.434	10.931	11.429	11.925	12.416	12.902	13.381	13.617	13.850	14.310	14.760	15.195	15.618	16.027	16.420	16.797	17.158	17.501	17.826	138	42
220	320	8.065	8.494	8.944	9.410	9.887	10.373	10.862	11.353	11.844	12.331	12.814	13.290	13.525	13.757	14.216	14.665	15.101	15.524	15.934	16.328	16.708	17.071	17.418	17.746	140	40
218	322	8.065	8.478	8.913	9.366	9.832	10.309	10.790	11.275	11.760	12.243	12.722	13.195	13.429	13.660	14.118	14.567	15.003	15.426	15.837	16.233	16.615	16.981	17.330	17.662	142	38
216	324	8.065	8.461	8.881	9.321	9.776	10.242	10.715	11.193	11.673	12.151	12.626	13.097	13.330	13.560	14.016	14.465	14.900	15.324	15.736	16.134	16.518	16.886	17.238	17.573	144	36
214	326	8.065	8.443	8.848	9.274	9.718	10.173	10.638	11.109	11.582	12.055	12.526	12.995	13.226	13.456	13.910	14.358	14.793	15.218	15.631	16.031	16.417	16.787	17.142	17.480	146	34
212	328	8.065	8.425	8.814	9.226	9.658	10.102	10.558	11.021	11.488	11.956	12.423	12.889	13.118	13.347	13.800	14.246	14.682	15.107	15.521	15.923	16.311	16.683	17.041	17.383	148	32
210	330	8.065	8.407	8.779	9.177	9.596	10.029	10.476	10.930	11.391	11.853	12.316	12.779	13.006	13.234	13.686	14.130	14.566	14.992	15.407	15.810	16.200	16.575	16.936	17.281	150	30
208	332	8.065	8.388	8.743	9.126	9.532	9.954	10.391	10.836	11.290	11.747	12.206	12.664	12.890	13.117	13.567	14.010	14.446	14.873	15.288	15.693	16.084	16.463	16.827	17.175	152	28
[illegible]	[illegible]	8.065	8.369	8.706	9.074	9.466	9.877	10.303	10.740	11.186	11.637	12.091	12.544	12.771	12.996	13.444	13.886	14.321	14.749	15.165	15.571	15.965	16.347	16.713	17.065	154	26
[illegible]	[illegible]	8.065	8.349	8.669	9.021	9.398	9.797	10.213	10.641	11.079	11.524	11.973	12.421	12.647	12.871	13.317	13.758	14.192	14.621	15.038	15.445	15.841	16.226	16.595	16.951	156	24
[illegible]	[illegible]	8.065	8.329	8.631	8.966	9.328	9.715	10.119	10.539	10.969	11.407	11.851	12.296	12.519	12.742	13.186	13.625	14.059	14.488	14.906	15.315	15.713	16.100	16.473	16.832	158	22
[illegible]	[illegible]	[illegible]	8.308	8.592	8.910	9.257	9.631	10.023	10.434	10.856	11.287	11.726	12.166	12.388	12.609	13.050	13.488	13.922	14.350	14.769	15.180	15.580	15.969	16.346	16.708	160	20
[illegible]	[illegible]	[illegible]	8.287	8.552	8.853	9.185	9.545	9.925	10.326	10.740	11.164	11.596	12.032	12.252	12.472	12.910	13.347	13.780	14.208	14.628	15.040	15.442	15.834	16.214	16.580	162	18
[illegible]	[illegible]	8.065	8.267	8.511	8.795	9.111	9.457	9.825	10.215	10.621	11.037	11.463	11.895	12.113	12.330	12.766	13.201	13.633	14.061	14.482	14.896	15.300	15.695	16.077	16.447	164	16
[illegible]	346	8.065	8.246	8.470	8.736	9.036	9.367	9.723	10.101	10.498	10.906	11.327	11.753	11.969	12.184	12.618	13.051	13.482	13.910	14.332	14.747	15.153	15.550	15.936	16.310	166	14

192	348	8.065	8.224	8.429	8.675	8.959	9.275	9.618	9.985	10.372	10.772	11.186	11.607	11.821	12.035	12.466	12.897	13.327	13.755	14.177	14.594	15.002	15.401	15.790	16.168	168	12
190	350	8.065	8.202	8.387	8.614	8.881	9.181	9.511	9.866	10.243	10.635	11.042	11.457	11.669	11.881	12.309	12.738	13.167	13.595	14.018	14.436	14.846	15.248	15.640	16.022	170	10
188	352	8.065	8.180	8.344	8.552	8.801	9.085	9.402	9.744	10.111	10.495	10.894	11.304	11.513	11.723	12.148	12.575	13.003	13.430	13.854	14.273	14.685	15.090	15.485	15.871	172	8
186	354	8.065	8.158	8.300	8.489	8.720	8.988	9.290	9.620	9.976	10.351	10.743	11.147	11.354	11.561	11.983	12.408	12.834	13.261	13.685	14.105	14.520	14.927	15.325	15.715	174	6
184	356	8.065	8.135	8.256	8.425	8.638	8.889	9.176	6.494	9.838	10.204	10.588	10.986	11.190	11.396	11.814	12.236	12.661	13.087	13.512	13.933	14.350	14.759	15.161	15.554	176	4
182	358	8.065	8.113	8.212	8.360	8.554	8.788	9.060	9.365	9.697	10.053	10.429	10.822	11.023	11.226	11.640	12.060	12.484	12.909	13.334	13.757	14.175	14.587	14.993	15.389	178	2
180	360	8.065	8.091	8.168	8.295	8.469	8.686	8.942	9.233	9.553	9.899	10.267	10.653	10.852	11.052	11.462	11.880	12.303	12.727	13.152	13.576	13.996	14.411	14.820	15.219	180	360
178	2	8.065	8.069	8.123	8.229	8.383	8.583	8.822	9.099	9.406	9.742	10.102	10.480	10.677	10.874	11.280	11.696	12.117	12.540	12.966	13.390	13.812	14.230	14.642	15.045	182	358
176	4	8.065	8.046	8.078	8.163	8.297	8.478	8.700	8.962	9.257	9.582	9.933	10.303	10.498	10.692	11.094	11.507	11.926	12.349	12.775	13.200	13.624	14.044	14.459	14.867	184	356
174	6	8.065	8.023	8.033	8.096	8.209	8.371	8.577	8.823	9.105	9.419	9.760	10.123	10.314	10.506	10.904	11.314	11.731	12.154	12.580	13.006	13.431	13.854	14.273	14.685	186	354
172	8	8.065	8.001	7.988	8.029	8.121	8.263	8.452	8.682	8.951	9.253	9.584	9.940	10.127	10.317	10.710	11.117	11.532	11.954	12.380	12.808	13.235	13.660	14.082	14.498	188	352
170	10	8.065	7.978	7.943	7.961	8.032	8.154	8.325	8.539	8.794	9.084	9.405	9.753	9.936	10.123	10.512	10.915	11.328	11.750	12.176	12.605	13.035	13.462	13.886	14.307	190	350
168	12	8.065	7.955	7.897	7.893	7.943	8.044	8.196	8.394	8.634	8.912	9.223	9.563	9.741	9.926	10.310	10.709	11.120	11.542	11.968	12.398	12.830	13.260	13.687	14.112	192	348
166	14	8.065	7.933	7.852	7.825	7.853	7.933	8.066	8.247	8.472	8.737	9.037	9.369	9.543	9.725	10.104	10.499	10.908	11.329	11.756	12.187	12.621	13.054	13.485	13.913	194	346
164	16	8.065	7.911	7.807	7.757	7.762	7.821	7.934	8.098	8.307	8.559	8.848	9.171	9.341	9.521	9.893	10.285	10.692	11.112	11.539	11.972	12.407	12.844	13.279	13.710	196	344
162	18	8.065	7.888	7.762	7.689	7.672	7.709	7.801	7.947	8.140	8.378	8.656	8.969	9.136	9.313	9.679	10.068	10.473	10.891	11.318	11.753	12.190	12.630	13.068	13.504	198	342
160	20	8.065	7.866	7.717	7.621	7.580	7.596	7.667	7.794	7.971	8.195	8.461	8.765	8.928	9.100	9.462	9.847	10.250	10.667	11.094	11.530	11.970	12.412	12.854	13.294	200	340
158	22	8.065	7.844	7.673	7.553	7.488	7.482	7.532	7.639	7.800	8.009	8.263	8.557	8.717	8.884	9.241	9.622	10.022	10.439	10.866	11.303	11.746	12.191	12.637	13.081	202	338
156	24	8.065	7.822	7.629	7.485	7.397	7.368	7.396	7.483	7.626	7.821	8.062	8.345	8.501	8.665	9.016	9.393	9.791	10.207	10.635	11.073	11.519	11.967	12.417	12.866	204	336
154	26	8.065	7.801	7.585	7.417	7.306	7.254	7.260	7.326	7.450	7.630	7.858	8.130	8.282	8.442	8.787	9.160	9.557	9.971	10.400	10.840	11.288	11.740	12.194	12.647	206	334
152	28	8.065	7.780	7.541	7.350	7.215	7.139	7.123	7.168	7.273	7.436	7.651	7.913	8.059	8.216	8.555	8.924	9.319	9.733	10.162	10.604	11.054	11.510	11.968	12.426	208	332
150	30	8.065	7.760	7.497	7.284	7.124	7.024	6.985	7.008	7.094	7.240	7.441	7.692	7.834	7.987	8.319	8.685	9.077	9.491	9.921	10.365	10.818	11.277	11.740	12.202	210	330
148	32	8.065	7.740	7.455	7.219	7.034	6.909	6.847	6.848	6.914	7.042	7.228	7.468	7.606	7.754	8.081	8.442	8.832	9.246	9.677	10.123	10.579	11.042	11.509	11.977	212	328
146	34	8.065	7.720	7.413	7.154	6.945	6.795	6.708	6.687	6.732	6.842	7.013	7.241	7.374	7.518	7.839	8.196	8.584	8.998	9.430	9.879	10.338	10.805	11.277	11.750	214	326
144	36	8.065	7.700	7.372	7.089	6.857	6.682	6.570	6.525	6.549	6.640	6.796	7.011	7.139	7.279	7.594	7.947	8.333	8.747	9.181	9.632	10.095	10.567	11.043	11.522	216	324
142	38	8.065	7.680	7.332	7.026	6.769	6.570	6.432	6.363	6.364	6.436	6.576	6.779	6.901	7.037	7.346	7.695	8.079	8.493	8.930	9.384	9.850	10.327	10.808	11.293	218	322
140	40	8.065	7.662	7.293	6.964	6.682	6.458	6.295	6.201	6.179	6.231	6.354	6.543	6.660	6.792	7.094	7.440	7.823	8.237	8.677	9.134	9.604	10.086	10.573	11.063	220	320
138	42	8.065	7.644	7.254	6.903	6.597	6.347	6.158	6.039	5.993	6.024	6.130	6.305	6.417	6.544	6.839	7.182	7.565	7.980	8.422	8.882	9.358	9.844	10.337	10.833	222	318
136	44	8.065	7.627	7.217	6.843	6.514	6.238	6.022	5.877	5.806	5.816	5.904	6.064	6.171	6.293	6.582	6.922	7.304	7.721	8.165	8.629	9.110	9.602	10.101	10.604	224	316
134	46	8.065	7.610	7.180	6.785	6.432	6.129	5.888	5.716	5.620	5.607	5.675	5.821	5.923	6.039	6.323	6.660	7.042	7.460	7.907	8.376	8.862	9.361	9.866	10.376	226	314
132	48	8.065	7.593	7.145	6.729	6.351	6.022	5.755	5.556	5.434	5.397	5.445	5.576	5.672	5.783	6.061	6.395	6.778	7.198	7.649	8.123	8.615	9.120	9.632	10.149	228	312
130	50	8.065	7.576	7.111	6.674	6.273	5.919	5.624	5.397	5.248	5.186	5.214	5.328	5.418	5.525	5.796	6.128	6.512	6.935	7.391	7.871	8.369	8.880	9.400	9.924	230	310
128	52	8.065	7.561	7.078	6.621	6.197	5.818	5.495	5.239	5.062	4.974	4.981	5.078	5.162	5.265	5.529	5.859	6.245	6.672	7.133	7.620	8.124	8.643	9.170	9.702	232	308
126	54	8.065	7.547	7.046	6.569	6.123	5.719	5.369	5.083	4.878	4.763	4.746	4.827	4.904	5.002	5.260	5.589	5.976	6.409	6.876	7.370	7.882	8.409	8.943	9.483	234	306
124	56	8.065	7.533	7.015	6.519	6.051	5.623	5.245	4.930	4.695	4.552	4.510	4.573	4.643	4.737	4.989	5.318	5.708	6.146	6.620	7.122	7.643	8.178	8.720	9.268	236	304
122	58	8.065	7.520	6.986	6.471	5.982	5.530	5.124	4.780	4.514	4.342	4.274	4.318	4.381	4.470	4.717	5.046	5.440	5.885	6.367	6.877	7.407	7.950	8.502	9.058	238	302
120	60	8.065	7.507	6.959	6.426	5.916	5.440	5.007	4.633	4.335	4.132	4.038	4.061	4.117	4.201	4.444	4.774	5.174	5.626	6.117	6.636	7.175	7.728	8.289	8.854	240	300
118	62	8.065	7.494	6.933	6.383	5.854	5.354	4.894	4.490	4.159	3.924	3.801	3.803	3.851	3.930	4.169	4.501	4.909	5.369	5.870	6.400	6.949	7.512	8.082	8.656	242	298
116	64	8.065	7.483	6.908	6.343	5.795	5.273	4.786	4.352	3.988	3.718	3.565	3.544	3.584	3.657	3.893	4.230	4.645	5.116	5.629	6.170	6.730	7.303	7.883	8.466	244	296
114	66	8.065	7.473	6.885	6.305	5.739	5.195	4.684	4.219	3.822	3.516	3.329	3.283	3.315	3.383	3.617	3.960	4.384	4.868	5.394	5.947	6.579	7.102	7.692	8.284	246	294
112	68	8.065	7.464	6.863	6.270	5.687	5.123	4.587	4.092	3.661	3.318	3.095	3.021	3.045	3.107	3.340	3.692	4.129	4.627	5.166	5.732	6.316	6.910	7.510	8.111	248	292
110	70	8.065	7.455	6.844	6.237	5.639	5.055	4.496	3.972	3.506	3.124	2.863	2.758	2.773	2.830	3.064	3.426	3.879	4.393	4.947	5.527	6.123	6.729	7.339	7.949	250	290
108	72	8.065	7.447	6.827	6.207	5.595	4.994	4.411	3.859	3.359	2.937	2.634	2.496	2.501	2.551	2.789	3.165	3.636	4.168	4.739	5.334	5.942	6.560	7.179	7.798	252	288
106	74	8.065	7.440	6.811	6.180	5.555	4.937	4.334	3.755	3.221	2.758	2.410	2.235	2.226	2.273	2.516	2.910	3.403	3.955	4.544	5.153	5.774	6.403	7.032	7.660	254	286
104	76	8.065	7.433	6.797	6.157	5.519	4.886	4.264	3.661	3.094	2.589	2.192	1.974	1.951	1.993	2.246	2.664	3.182	3.756	4.363	4.988	5.621	6.260	6.899	7.535	256	284
102	78	8.065	7.427	6.784	6.137	5.488	4.842	4.202	3.577	2.979	2.432	1.983	1.714	1.676	1.712	1.979	2.429	2.976	3.574	4.199	4.839	5.485	6.133	6.781	7.424	258	282
100	80	8.065	7.422	6.773	6.119	5.461	4.804	4.149	3.504	2.877	2.290	1.786	1.458	1.401	1.432	1.724	2.210	2.789	3.411	4.055	4.709	5.366	6.023	6.679	7.329	260	280
98	82	8.065	7.418	6.764	6.105	5.439	4.773	4.105	3.442	2.791	2.167	1.607	1.208	1.126	1.152	1.481	2.013	2.625	3.272	3.932	4.599	5.266	5.932	6.594	7.250	262	278
96	84	8.065	7.415	6.758	6.094	5.422	4.748	4.071	3.394	2.722	2.066	1.452	0.967	0.851	0.874	1.259	1.845	2.491	3.159	3.834	4.512	5.187	5.860	6.527	7.187	264	276
94	86	8.065	7.413	6.753	6.086	5.410	4.730	4.046	3.359	2.672	1.991	1.329	0.752	0.581	0.599	1.073	1.714	2.390	3.075	3.762	4.448	5.130	5.807	6.479	7.142	266	274
92	88	8.065	7.412	6.750	6.081	5.403	4.719	4.031	3.338	2.641	1.944	1.250	0.585	0.323	0.341	0.944	1.631	2.327	3.024	3.719	4.409	5.095	5.775	6.450	7.114	268	272
90	90	8.065	7.412	6.749	6.079	5.401	4.716	4.026	3.330	2.631	1.929	1.224	0.517	0.164	0.189	0.896	1.602	2.306	3.007	3.704	4.396	5.084	5.765	6.441	7.105	270	270

Table H.—For the Maximum of Aberration of a Star in North Polar Distance.

Enter with the Right Ascension on the left side if the Declination is North, on the right side if South.

Right Ascen.		44°	46°	48°	50°	52°	54°	56°	58°	60°	62°	64°	66°	68°	70°	72°	74°	76°	78°	80°	82°	84°	86°	88°	90°	Right Ascen.	
		Declination of the Star.																									
270	270	18.708	18.968	19.204	19.417	19.607	19.772	19.913	20.031	20.123	20.192	20.235	20.254	20.248	20.218	20.163	20.083	19.979	19.851	19.698	19.522	19.321	19.097	18.850	18.580	90	90
268	272	18.706	18.966	19.203	19.416	19.606	19.771	19.912	20.030	20.123	20.191	20.235	20.254	20.249	20.219	20.164	20.084	19.980	19.852	19.699	19.523	19.323	19.099	18.852	18.582	92	88
266	274	18.702	18.963	19.200	19.413	19.603	19.769	19.910	20.028	20.121	20.190	20.234	20.254	20.249	20.220	20.165	20.086	19.983	19.855	19.703	19.527	19.328	19.104	18.858	18.589	94	86
264	276	18.695	18.956	19.194	19.407	19.598	19.765	19.906	20.025	20.119	20.188	20.233	20.254	20.249	20.221	20.167	20.089	19.987	19.860	19.709	19.534	19.335	19.113	18.868	18.599	96	84
262	278	18.686	18.947	19.185	19.399	19.591	19.758	19.901	20.021	20.115	20.186	20.232	20.253	20.250	20.223	20.170	20.093	19.992	19.866	19.716	19.543	19.346	19.125	18.881	18.614	98	82
260	280	18.673	18.935	19.174	19.389	19.582	19.749	19.894	20.015	20.110	20.183	20.230	20.253	20.251	20.225	20.174	20.098	19.999	19.875	19.726	19.555	19.360	19.140	18.898	18.633	100	80
258	282	18.657	18.920	19.160	19.377	19.570	19.739	19.885	20.007	20.105	20.179	20.228	20.252	20.252	20.228	20.178	20.105	20.007	19.885	19.739	19.569	19.376	19.158	18.918	18.656	102	78
256	284	18.638	18.902	19.144	19.362	19.556	19.728	19.875	19.998	20.099	20.174	20.225	20.251	20.253	20.231	20.183	20.112	20.017	19.897	19.753	19.586	19.395	19.180	18.942	18.682	104	76
254	286	18.616	18.882	19.125	19.345	19.541	19.714	19.864	19.988	20.091	20.168	20.221	20.249	20.254	20.234	20.189	20.120	20.028	19.910	19.769	19.605	19.417	19.205	18.969	18.713	106	74
252	288	18.592	18.859	19.103	19.325	19.523	19.698	19.850	19.977	20.081	20.160	20.216	20.247	20.254	20.237	20.195	20.129	20.039	19.925	19.787	19.626	19.441	19.232	19.000	18.747	108	72
250	290	18.565	18.833	19.079	19.303	19.503	19.680	19.834	19.964	20.070	20.152	20.211	20.245	20.255	20.240	20.201	20.139	20.052	19.941	19.807	19.649	19.467	19.262	19.034	18.784	110	70
248	292	18.534	18.804	19.052	19.278	19.481	19.660	19.816	19.949	20.058	20.143	20.205	20.242	20.255	20.244	20.208	20.150	20.066	19.958	19.828	19.673	19.495	19.294	19.071	18.824	112	68
246	294	18.500	18.772	19.022	19.250	19.456	19.638	19.797	19.933	20.045	20.133	20.197	20.238	20.255	20.247	20.215	20.160	20.081	19.977	19.850	19.700	19.526	19.329	19.110	18.867	114	66
244	296	18.463	18.737	18.990	19.220	19.428	19.614	19.776	19.914	20.030	20.121	20.189	20.233	20.254	20.250	20.222	20.171	20.095	19.996	19.874	19.728	19.558	19.366	19.151	18.913	116	64
242	298	18.422	18.699	18.954	19.187	19.398	19.587	19.752	19.894	20.013	20.108	20.180	20.228	20.252	20.252	20.229	20.182	20.111	20.016	19.898	19.757	19.592	19.405	19.194	18.962	118	62
240	300	18.379	18.658	18.916	19.152	19.366	19.557	19.726	19.871	19.994	20.093	20.169	20.222	20.249	20.254	20.235	20.192	20.126	20.036	19.923	19.787	19.627	19.445	19.240	19.013	120	60
238	302	18.332	18.613	18.875	19.114	19.330	19.525	19.697	19.846	19.973	20.076	20.156	20.214	20.246	20.255	20.241	20.202	20.141	20.057	19.949	19.818	19.663	19.486	19.287	19.065	122	58
236	304	18.281	18.565	18.830	19.072	19.292	19.490	19.666	19.819	19.950	20.057	20.142	20.204	20.241	20.255	20.246	20.212	20.157	20.077	19.975	19.849	19.700	19.529	19.335	19.119	124	56
234	306	18.227	18.514	18.782	19.027	19.251	19.453	19.633	19.790	19.925	20.037	20.126	20.192	20.235	20.254	20.250	20.222	20.172	20.098	20.001	19.881	19.738	19.573	19.385	19.175	126	54
232	308	18.170	18.460	18.730	18.979	19.207	19.413	19.597	19.758	19.898	20.015	20.108	20.179	20.227	20.251	20.253	20.231	20.186	20.118	20.027	19.913	19.776	19.617	19.436	19.232	128	52
230	310	18.109	18.402	18.675	18.928	19.160	19.370	19.558	19.724	19.868	19.990	20.088	20.164	20.217	20.248	20.255	20.239	20.199	20.137	20.053	19.945	19.814	19.661	19.487	19.290	130	50
228	312	18.044	18.341	18.617	18.874	19.109	19.324	19.516	19.687	19.836	19.963	20.066	20.148	20.206	20.242	20.255	20.245	20.211	20.156	20.078	19.976	19.852	19.706	19.538	19.348	132	48
226	314	17.975	18.275	18.556	18.816	19.055	19.274	19.471	19.647	19.801	19.933	20.042	20.129	20.193	20.235	20.254	20.250	20.223	20.174	20.102	20.007	19.890	19.751	19.589	19.406	134	46
224	316	17.902	18.206	18.490	18.754	18.998	19.221	19.423	19.604	19.763	19.900	20.015	20.108	20.178	20.226	20.251	20.253	20.233	20.190	20.125	20.037	19.927	19.795	19.640	19.465	136	44
222	318	17.826	18.133	18.421	18.689	18.937	19.165	19.372	19.558	19.722	19.864	19.985	20.084	20.161	20.215	20.246	20.255	20.241	20.205	20.147	20.066	19.963	19.838	19.691	19.523	138	42
220	320	17.746	18.056	18.348	18.620	18.873	19.105	19.317	19.506	19.678	19.825	19.952	20.058	20.141	20.201	20.239	20.255	20.248	20.219	20.168	20.094	19.998	19.881	19.741	19.580	140	40
218	322	17.662	17.975	18.271	18.548	18.805	19.042	19.259	19.455	19.631	19.784	19.917	20.029	20.118	20.185	20.230	20.252	20.252	20.231	20.187	20.120	20.032	19.922	19.790	19.637	142	38
216	324	17.573	17.890	18.190	18.472	18.733	18.975	19.197	19.399	19.580	19.740	19.879	19.997	20.092	20.166	20.218	20.247	20.255	20.240	20.204	20.145	20.064	19.962	19.838	19.692	144	36
214	326	17.480	17.801	18.105	18.391	18.657	18.904	19.132	19.339	19.526	19.692	19.837	19.962	20.063	20.144	20.203	20.240	20.255	20.248	20.219	20.168	20.095	20.000	19.884	19.746	146	34
212	328	17.383	17.708	18.016	18.306	18.578	18.830	19.063	19.275	19.469	19.641	19.793	19.923	20.032	20.119	20.185	20.230	20.252	20.253	20.231	20.188	20.123	20.036	19.928	19.799	148	32
210	330	17.281	17.610	17.922	18.217	18.494	18.752	18.990	19.208	19.408	19.585	19.744	19.881	19.997	20.091	20.165	20.217	20.246	20.255	20.241	20.206	20.149	20.070	19.971	19.850	150	30
208	332	17.175	17.509	17.825	18.124	18.406	18.669	18.913	19.137	19.343	19.527	19.692	19.836	19.959	20.060	20.142	20.201	20.238	20.254	20.249	20.222	20.173	20.102	20.011	19.898	152	28
206	334	17.065	17.403	17.723	18.027	18.314	18.582	18.832	19.062	19.274	19.465	19.636	19.787	19.918	20.026	20.115	20.182	20.227	20.251	20.253	20.235	20.194	20.132	20.049	19.944	154	26
204	336	16.951	17.292	17.617	17.925	18.217	18.491	18.746	18.983	19.201	19.399	19.577	19.735	19.873	19.989	20.085	20.159	20.212	20.244	20.255	20.244	20.212	20.158	20.084	19.988	156	24
202	338	16.832	17.177	17.506	17.819	18.116	18.396	18.657	18.900	19.124	19.328	19.514	19.679	19.824	19.948	20.051	20.133	20.194	20.235	20.253	20.251	20.227	20.182	20.116	20.028	158	22
200	340	16.708	17.058	17.392	17.709	18.010	18.296	18.563	18.813	19.043	19.254	19.446	19.619	19.771	19.903	20.014	20.104	20.173	20.222	20.249	20.255	20.239	20.203	20.145	20.066	160	20
198	342	16.580	16.933	17.272	17.595	17.900	18.192	18.465	18.721	18.958	19.176	19.375	19.555	19.714	19.854	19.973	20.071	20.148	20.205	20.241	20.255	20.248	20.220	20.171	20.101	162	18
196	344	16.447	16.804	17.148	17.476	17.786	18.084	18.363	18.625	18.869	19.093	19.300	19.486	19.654	19.801	19.928	20.034	20.119	20.184	20.229	20.251	20.253	20.234	20.194	20.132	164	16
194	346	16.310	16.671	17.019	17.352	17.669	17.971	18.257	18.525	18.775	19.007	19.220	19.414	19.590	19.744	19.879	19.993	20.087	20.160	20.213	20.244	20.255	20.245	20.213	20.160	164	14

192	348	16.168	16.533	16.885	17.223	17.547	17.854	18.146	18.420	18.677	18.916	19.137	19.338	19.521	19.683	19.826	19.949	20.051	20.132	20.193	20.234	20.253	20.252	20.229	20.185	168	12
190	350	16.022	16.391	16.748	17.091	17.420	17.733	18.030	18.311	18.575	18.821	19.049	19.258	19.448	19.618	19.769	19.900	20.011	20.101	20.170	20.220	20.248	20.255	20.241	20.207	170	10
188	352	15.871	16.245	16.605	16.953	17.288	17.607	17.910	18.198	18.468	18.722	18.957	19.173	19.371	19.549	19.708	19.848	19.967	20.065	20.143	20.201	20.238	20.254	20.249	20.224	172	8
186	354	15.715	16.092	16.458	16.812	17.151	17.476	17.786	18.080	18.357	18.618	18.860	19.084	19.290	19.476	19.643	19.791	19.919	20.026	20.113	20.179	20.225	20.250	20.254	20.237	174	6
184	356	15.554	15.936	16.307	16.665	17.010	17.341	17.657	17.958	18.242	18.509	18.759	18.991	19.205	19.399	19.574	19.730	19.866	19.983	20.078	20.153	20.208	20.242	20.255	20.247	176	4
182	358	15.389	15.775	16.151	16.514	16.865	17.202	17.524	17.832	18.123	18.397	18.654	18.894	19.116	19.318	19.502	19.666	19.810	19.935	20.039	20.123	20.187	20.230	20.252	20.253	178	2
180	360	15.219	15.610	15.990	16.359	16.715	17.058	17.387	17.701	17.999	18.280	18.545	18.793	19.022	19.232	19.425	19.597	19.750	19.883	19.996	20.089	20.162	20.213	20.245	20.255	180	360
178	2	15.045	15.441	15.825	16.200	16.561	16.910	17.246	17.566	17.871	18.159	18.432	18.687	18.924	19.143	19.344	19.524	19.686	19.828	19.950	20.051	20.132	20.194	20.234	20.253	182	358
176	4	14.867	15.267	15.656	16.036	16.403	16.758	17.100	17.427	17.739	18.035	18.315	18.577	18.822	19.050	19.258	19.447	19.618	19.768	19.900	20.010	20.100	20.170	20.219	20.247	184	356
174	6	14.685	15.088	15.483	15.868	16.241	16.602	16.950	17.284	17.602	17.907	18.194	18.464	18.717	18.952	19.169	19.367	19.546	19.705	19.845	19.964	20.064	20.142	20.200	20.237	186	354
172	8	14.498	14.906	15.306	15.695	16.074	16.442	16.796	17.137	17.462	17.774	18.069	18.347	18.608	18.851	19.076	19.283	19.470	19.638	19.786	19.915	20.023	20.110	20.178	20.224	188	352
170	10	14.307	14.719	15.124	15.519	15.904	16.278	16.638	16.986	17.319	17.637	17.940	18.226	18.495	18.746	18.980	19.195	19.391	19.567	19.724	19.862	19.979	20.075	20.152	20.207	190	350
168	12	14.112	14.529	14.939	15.339	15.730	16.110	16.477	16.831	17.172	17.497	17.807	18.101	18.378	18.638	18.880	19.103	19.308	19.493	19.659	19.805	19.931	20.037	20.122	20.185	192	348
166	14	13.913	14.335	14.750	15.156	15.552	15.938	16.312	16.673	17.021	17.354	17.671	17.973	18.258	18.526	18.776	19.008	19.221	19.416	19.596	19.745	19.879	19.994	20.088	20.160	194	346
164	16	13.710	14.136	14.557	14.969	15.371	15.763	16.144	16.512	16.867	17.207	17.531	17.841	18.135	18.411	18.670	18.910	19.132	19.335	19.518	19.681	19.825	19.948	20.051	20.132	196	344
162	18	13.504	13.935	14.360	14.778	15.186	15.585	15.972	16.348	16.709	17.057	17.390	17.707	18.009	18.293	18.560	18.809	19.039	19.250	19.442	19.614	19.767	19.899	20.011	20.101	198	342
160	20	13.294	13.731	14.160	14.584	14.998	15.404	15.798	16.180	16.549	16.904	17.245	17.570	17.880	18.172	18.447	18.704	18.943	19.163	19.364	19.545	19.706	19.846	19.967	20.066	200	340
158	22	13.081	13.522	13.957	14.387	14.808	15.220	15.620	16.009	16.386	16.749	17.097	17.430	17.748	18.048	18.332	18.597	18.844	19.073	19.283	19.472	19.642	19.791	19.920	20.028	202	338
156	24	12.866	13.310	13.752	14.188	14.615	15.033	15.440	15.836	16.221	16.591	16.947	17.288	17.613	17.922	18.214	18.488	18.743	18.980	19.199	19.397	19.575	19.733	19.871	19.988	204	336
154	26	12.647	13.097	13.545	13.986	14.420	14.844	15.258	15.661	16.053	16.431	16.795	17.144	17.477	17.794	18.094	18.376	18.640	18.886	19.112	19.319	19.506	19.672	19.819	19.944	206	334
152	28	12.426	12.882	13.335	13.782	14.222	14.653	15.074	15.484	15.883	16.269	16.640	16.997	17.338	17.664	17.972	18.262	18.535	18.789	19.023	19.239	19.435	19.609	19.764	19.898	208	332
150	30	12.202	12.664	13.122	13.576	14.022	14.460	14.889	15.306	15.712	16.106	16.484	16.849	17.198	17.532	17.848	18.147	18.428	18.690	18.933	19.157	19.361	19.544	19.707	19.850	210	330
148	32	11.977	12.444	12.908	13.368	13.821	14.266	14.702	15.127	15.540	15.941	16.327	16.700	17.057	17.399	17.723	18.030	18.319	18.590	18.841	19.073	19.285	19.477	19.648	19.799	212	328
146	34	11.750	12.223	12.694	13.159	13.619	14.071	14.514	14.946	15.367	15.775	16.170	16.550	16.915	17.264	17.597	17.912	18.209	18.488	18.748	18.988	19.208	19.408	19.588	19.746	214	326
144	36	11.522	12.000	12.478	12.950	13.416	13.875	14.325	14.765	15.193	15.609	16.011	16.399	16.772	17.129	17.470	17.793	18.098	18.385	18.653	18.901	19.130	19.338	19.526	19.692	216	324
142	38	11.293	11.777	12.262	12.740	13.213	13.679	14.136	14.584	15.019	15.443	15.852	16.248	16.629	16.994	17.342	17.674	17.987	18.282	18.558	18.814	19.051	19.267	19.462	19.637	218	322
140	40	11.063	11.554	12.045	12.530	13.010	13.483	13.947	14.403	14.845	15.277	15.694	16.098	16.486	16.859	17.215	17.555	17.876	18.178	18.462	18.727	18.971	19.195	19.398	19.580	220	320
138	42	10.833	11.332	11.828	12.320	12.808	13.288	13.759	14.222	14.672	15.112	15.537	15.948	16.344	16.725	17.088	17.436	17.764	18.075	18.366	18.639	18.890	19.122	19.333	19.523	222	318
136	44	10.604	11.109	11.612	12.112	12.606	13.094	13.573	14.042	14.500	14.947	15.380	15.799	16.203	16.591	16.962	17.317	17.653	17.971	18.271	18.551	18.810	19.049	19.268	19.465	224	316
134	46	10.376	10.887	11.398	11.905	12.406	12.901	13.388	13.864	14.330	14.784	15.225	15.652	16.063	16.458	16.837	17.199	17.543	17.869	18.176	18.463	18.730	18.976	19.220	19.406	226	314
132	48	10.149	10.667	11.185	11.700	12.208	12.711	13.205	13.689	14.162	14.623	15.072	15.506	15.925	16.327	16.713	17.083	17.434	17.767	18.082	18.376	18.650	18.903	19.136	19.348	228	312
130	50	9.924	10.450	10.975	11.497	12.013	12.524	13.025	13.517	13.997	14.465	14.921	15.362	15.789	16.198	16.592	16.969	17.327	17.667	17.988	18.290	18.571	18.831	19.071	19.290	230	310
128	52	9.702	10.235	10.768	11.298	11.822	12.340	12.848	13.348	13.835	14.310	14.773	15.221	15.655	16.072	16.473	16.857	17.222	17.569	17.896	18.205	18.493	18.760	19.007	19.232	232	308
126	54	9.483	10.024	10.565	11.102	11.634	12.159	12.675	13.182	13.677	14.159	14.629	15.084	15.524	15.949	16.356	16.747	17.119	17.473	17.807	18.122	18.417	18.691	18.944	19.175	234	306
124	56	9.268	9.817	10.366	10.911	11.450	11.983	12.506	13.020	13.522	14.012	14.489	14.951	15.398	15.829	16.243	16.640	17.019	17.379	17.720	18.041	18.343	18.623	18.882	19.119	236	304
122	58	9.058	9.615	10.172	10.725	11.272	11.812	12.343	12.863	13.372	13.869	14.353	14.822	15.275	15.713	16.134	16.537	16.922	17.288	17.636	17.963	18.270	18.557	18.822	19.065	238	302
120	60	8.854	9.419	9.984	10.545	11.100	11.647	12.185	12.712	13.228	13.732	14.222	14.697	15.157	15.601	16.028	16.438	16.829	17.201	17.554	17.888	18.200	18.493	18.763	19.013	240	300
118	62	8.656	9.230	9.803	10.372	10.934	11.488	12.033	12.567	13.090	13.600	14.096	14.578	15.044	15.494	15.927	16.342	16.739	17.117	17.476	17.815	18.133	18.431	18.707	18.962	242	298
116	64	8.466	9.049	9.629	10.206	10.775	11.336	11.888	12.429	12.958	13.474	13.976	14.464	14.936	15.392	15.830	16.251	16.654	17.037	17.402	17.746	18.069	18.373	18.654	18.913	244	296
114	66	8.284	8.876	9.464	10.048	10.624	11.192	11.751	12.298	12.833	13.355	13.863	14.356	14.834	15.296	15.739	16.166	16.574	16.962	17.332	17.681	18.009	18.317	18.603	18.867	246	294
112	68	8.111	8.712	9.308	9.899	10.482	11.057	11.622	12.175	12.716	13.243	13.757	14.255	14.739	15.205	15.654	16.086	16.498	16.892	17.266	17.620	17.953	18.265	18.555	18.824	248	292
110	70	7.949	8.558	9.162	9.760	10.349	10.931	11.502	12.060	12.607	13.139	13.659	14.162	14.650	15.121	15.575	16.011	16.428	16.826	17.204	17.563	17.900	18.216	18.511	18.784	250	290
108	72	7.798	8.415	9.026	9.631	10.226	10.814	11.391	11.954	12.507	13.044	13.568	14.076	14.569	15.044	15.502	15.942	16.364	16.766	17.148	17.511	17.851	18.172	18.470	18.747	252	288
106	74	7.660	8.284	8.902	9.513	10.114	10.708	11.290	11.858	12.415	12.958	13.486	13.998	14.495	14.974	15.436	15.880	16.305	16.711	17.097	17.463	17.807	18.131	18.433	18.713	254	286
104	76	7.535	8.166	8.790	9.408	10.014	10.613	11.200	11.772	12.333	12.880	13.412	13.928	14.429	14.911	15.377	15.824	16.253	16.662	17.051	17.420	17.768	18.094	18.400	18.682	256	284
102	78	7.424	8.062	8.692	9.315	9.926	10.529	11.120	11.697	12.262	12.812	13.347	13.867	14.371	14.856	15.325	15.775	16.207	16.619	17.011	17.382	17.733	18.063	18.370	18.656	258	282
100	80	7.329	7.972	8.608	9.235	9.851	10.457	11.051	11.632	12.201	12.754	13.292	13.814	14.321	14.809	15.281	15.734	16.168	16.582	16.977	17.350	17.703	18.035	18.345	18.633	260	280
98	82	7.250	7.897	8.538	9.169	9.789	10.398	10.995	11.579	12.150	12.705	13.246	13.771	14.280	14.771	15.244	15.699	16.135	16.552	16.949	17.324	17.679	18.012	18.325	18.614	262	278
96	84	7.187	7.839	8.483	9.117	9.740	10.352	10.951	11.537	12.110	12.667	13.210	13.737	14.248	14.741	15.215	15.672	16.110	16.528	16.926	17.304	17.660	17.994	18.309	18.599	264	276
94	86	7.142	7.797	8.443	9.079	9.705	10.319	10.919	11.507	12.081	12.640	13.184	13.713	14.224	14.719	15.195	15.653	16.092	16.511	16.910	17.289	17.646	17.982	18.297	18.589	266	274
92	88	7.114	7.772	8.419	9.057	9.683	10.298	10.899	11.489	12.064	12.624	13.169	13.698	14.210	14.706	15.183	15.641	16.081	16.501	16.900	17.280	17.638	17.975	18.290	18.582	268	272
90	90	7.105	7.763	8.411	9.049	9.676	10.291	10.892	11.483	12.058	12.619	13.164	13.693	14.206	14.701	15.179	15.637	16.077	16.497	16.897	17.277	17.635	17.972	18.287	18.580	270	270

Table I.—For the Aberration of a Star, in North Polar Distance, or Right Ascension.

The Argument at top, is the Maximum of Aberration, to each half-second.

The Argument at the sides, is the difference of Sun's Longitude, and the Star's annual number.

The Aberration is *minus*, if the difference is less than 3 signs; and *plus*, if the difference is more than 3 signs.

Diff. Long.	Maximum of Aberration.																								Diff. Long.
S. D.	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	10.50	11.00	11.50	12.00	12.50	S. D.
O 0	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	10.50	11.00	11.50	12.00	12.50	VI. 0
1	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50	10.00	10.50	11.00	11.50	12.00	12.50	29
2	0.50	1·00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.49	8.99	9.49	9.99	10.49	10.99	11.49	11.99	12.49	28
3	0.50	1.00	1.50	2.00	2.50	3.00	3.50	3.99	4.49	4.99	5.49	5.99	6.49	6.99	7.49	7.99	8.49	8.99	9.49	9.99	10.49	10.98	11.48	11.98	12.48	27
4	0.50	1.00	1.50	2.00	2.49	2.99	3.49	3.99	4.49	4.99	5.49	5.99	6.48	6.98	7.48	7.98	8.48	8.98	9.48	9.98	10.47	10.97	11.47	11.97	12.47	26
5	0.50	1.00	1.49	1.99	2.49	2.99	3.49	3.98	4.48	4.98	5.48	5.98	6.47	6.97	7.47	7.97	8.47	8.97	9.46	9.96	10.46	10.96	11.46	11.95	12.45	25
6	0.50	0.99	1.49	1.99	2.49	2.98	3.48	3.98	4.48	4.97	5.47	5.97	6.46	6.96	7.46	7.96	8.45	8.95	9.45	9.95	10.44	10.94	11.44	11.93	12.43	24
7	0.50	0.99	1.49	1.99	2.48	2.98	3.47	3.97	4.47	4.96	5.46	5.96	6.45	6.95	7.44	7.94	8.44	8.93	9.43	9.93	10.42	10.92	11.41	11.91	12.41	23
8	0.50	0.99	1.49	1.98	2.48	2.97	3.47	3.96	4.46	4.95	5.45	5.94	6.44	6.93	7.43	7.92	8.42	8.91	9.41	9.90	10.40	10.89	11.39	11.88	12.38	22
9	0.49	0.99	1.48	1.98	2.47	2.96	3.46	3.95	4·45	4.94	5.43	5.93	6.42	6.91	7.41	7.90	8.40	8.89	9.38	9.88	10.37	10.86	11.36	11.85	12.35	21
10	0.49	0.98	1.48	1.97	2.46	2.95	3.45	3.94	4.43	4.92	5.42	5.91	6.40	6.89	7.39	7.88	8.37	8.86	9.36	9.85	10.34	10.83	11.33	11.82	12.31	20
11	0.49	0.98	1.47	1.96	2.45	2.94	3.44	3.93	4.42	4.91	5.40	5.89	6.38	6.87	7.36	7.85	8.34	8.83	9.33	9.82	10.31	10.80	11.29	11.78	12.27	19
12	0.49	0.98	1.47	1.96	2.45	2.93	3.42	3.91	4.40	4.89	5.38	5.87	6.36	6.85	7.34	7.83	8.31	8.80	9.29	9.78	10.27	10.76	11.25	11.74	12.23	18
13	0·49	0.97	1·46	1.95	2.44	2.92	3.41	3.90	4.38	4.87	5.36	5.85	6.33	6.82	7.31	7.79	8.28	8.77	9.26	9.74	10.23	10.72	11.21	11.69	12.18	17
14	0.49	0.97	1.46	1.94	2.43	2.91	3.40	3.88	4.37	4.85	5.34	5.82	6.31	6.79	7.28	7.76	8.25	8.73	9.22	9.70	10.19	10.67	11.16	11.64	12.13	16
15	0.48	0.97	1.45	1.93	2.41	2.90	3.38	3.86	4.35	4.83	5.31	5.80	6.28	6.76	7.24	7.73	8.21	8.69	9.18	9.66	10.14	10.63	11.11	11.59	12.07	15
16	0.48	0·96	1.44	1.92	2.40	2.88	3.36	3.85	4.33	4.81	5.29	5.77	6.25	6.73	7.21	7.69	8.17	8.65	9.13	9.61	10.09	10.57	11.05	11.54	12.02	14
17	0.48	0·96	1.43	1.91	2.39	2.87	3.35	3.83	4.30	4.78	5.26	5.74	6.22	6.69	7.17	7.65	8.13	8.61	9.08	9.56	10.04	10.52	11.00	11.48	11.95	13
18	0.48	0.95	1.43	1.90	2.38	2.85	3.33	3.80	4.28	4.76	5.23	5.71	6.18	6.66	7.13	7.61	8.08	8.56	9.04	9.51	9.99	10.46	10.94	11.41	11.89	12
19	0.47	0·95	1.42	1.89	2.36	2.84	3.31	3.78	4.25	4.73	5.20	5.67	6.15	6.62	7.09	7.56	8.04	8.51	8.98	9.46	9.93	10.40	10.87	11.35	11.82	11
20	0.47	0.94	1.41	1.88	2.35	2.82	3.29	3.76	4.23	4.70	5.17	5.64	6.11	6.58	7.05	7.52	7.99	8.46	8.93	9.40	9.87	10.34	10.81	11.28	11.75	10
21	0.47	0.93	1.40	1.87	2.33	2.80	3.27	3.73	4.20	4.67	5.13	5.60	6.07	6.54	7.00	7.47	7.94	8.40	8.87	9.34	9.80	10.27	10.74	11.20	11.67	9
22	0.46	0·93	1.39	1.85	2.32	2.78	3.25	3.71	4.17	4.64	5.10	5.56	6.03	6.49	6.95	7.42	7.88	8.34	8.81	9.27	9.74	10.20	10.66	11.13	11.59	8
23	0.46	0.92	1.38	1.84	2.30	2.76	3.22	3.68	4.14	4.60	5.06	5.52	5.98	6.44	6.90	7.36	7.82	8.28	8.74	9.21	9.67	10.13	10.59	11.05	11.51	7
24	0.46	0.91	1.37	1.83	2.28	2.74	3.20	3.65	4.11	4.57	5.02	5.48	5.94	6.39	6.85	7.31	7.77	8.22	8.68	9.14	9.59	10.05	10.51	10.96	11.42	6
25	0.45	0.91	1.36	1.81	2.27	2.72	3.17	3.63	4.08	4.53	4.98	5.44	5.89	6.34	6.80	7.25	7.70	8.16	8.61	9.06	9.52	9.97	10.42	10.88	11.33	5
26	0.45	0.90	1.35	1.80	2.25	2.70	3.15	3.60	4.04	4.49	4.94	5.39	5.84	6.29	6.74	7.19	7.64	8.09	8.54	8.99	9.44	9.89	10.34	10.79	11.23	4
27	0.45	0.89	1.34	1.78	2.23	2.67	3.12	3.56	4.01	4.46	4.90	5.35	5.79	6.24	6.68	7.13	7.57	8.02	8.46	8.91	9.36	9.80	10.25	10.69	11.14	3
28	0.44	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.97	4.41	4.86	5.30	5.74	6.18	6.62	7.06	7.50	7.95	8.38	8.83	9.27	9.71	10.15	10.60	11.04	2
29	0.44	0.87	1.31	1.75	2.19	2.62	3.06	3.50	3.94	4.37	4.81	5.25	5.69	6.12	6.56	7.00	7.43	7.87	8.31	8.75	9.18	9.62	10.06	10.50	10.93	1
I. 0	0.43	0.87	1.30	1.73	2.17	2.60	3.03	3.46	3.90	4.33	4.76	5.20	5.63	6.06	6.50	6.93	7.36	7.79	8.23	8.66	9.09	9.53	9.96	10.39	10.83	V. 0
1	0.43	0.86	1.29	1.71	2.14	2.57	3.00	3.43	3.86	4.29	4.71	5.14	5.57	6.00	6.43	6.86	7.29	7.71	8.14	8.57	9.00	9.43	9.86	10.29	10.71	29
2	0.42	0.85	1.27	1.70	2.12	2.54	2.97	3.39	3.82	4.24	4.66	5.09	5.51	5.94	6.36	6.78	7.21	7.63	8.06	8.48	8.90	9.33	9.75	10.18	10.60	28
3	0.42	0.84	1.26	1.68	2.10	2.52	2.94	3.35	3.77	4.19	4.61	5.03	5.45	5.87	6.29	6.71	7.13	7.55	7.97	8.39	8.80	9.23	9.65	10.06	10.48	27
4	0.41	0.83	1.24	1.66	2.07	2.49	2.90	3.32	3.73	4.15	4.56	4.97	5.39	5.80	6.22	6.63	7.05	7.46	7.88	8.29	8.70	9.12	9.53	9.95	10.36	26
5	0.41	0.82	1.23	1.64	2.05	2.46	2.87	3.28	3.69	4.10	4.51	4.91	5.32	5.73	6.14	6.55	6.96	7.37	7.78	8.19	8.60	9.01	9.42	9.83	16.24	25
6	0.40	0.81	1.21	1.62	2.02	2.43	2.83	3.24	3.64	4.05	4.45	4.85	5.26	5.66	6.07	6.47	6.88	7.28	7.69	8.09	8.49	8.90	9.30	9.71	10.11	24
7	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.19	3.59	3.99	4.39	4.79	5.19	5.59	5.99	6.39	6.79	7.19	7.59	7.99	8.39	8.79	9.18	9.58	9.98	23
8	0.39	0.79	1.18	1.58	1.97	2.36	2.76	3.15	3.55	3.94	4.33	4.73	5.12	5.52	5.91	6.30	6.70	7.09	7.49	7.88	8.27	8.67	9.06	9.46	9.85	22

9	0.39	0.78	1.17	1.55	1.94	2.33	2.72	3.11	3.50	3.89	4.27	4.66	5.05	5.44	5.83	6.22	6.61	6.99	7.38	7.77	8.16	8.55	8.94	9.33	9.71	21
10	0.38	0.77	1.15	1.53	1.92	2.30	2.68	3.06	3.45	3.83	4.21	4.60	4.98	5.36	5.75	6.13	6.51	6.89	7.28	7.66	8.04	8.43	8.81	9.19	9.58	20
11	0.38	0.75	1.13	1.51	1.89	2.26	2.64	3.02	3.40	3.77	4.15	4.53	4.91	5.28	5.66	6.04	6.42	6.79	7.17	7.55	7.92	8.30	8.68	9.06	9.43	19
12	0.37	0.74	1.11	1.49	1.87	2.23	2.60	2.97	3.34	3.72	4.09	4.46	4.83	5.20	5.57	5.95	6.32	6.69	7.06	7.43	7.80	8.17	8.55	8.92	9.29	18
13	0.37	0.73	1.10	1.46	1.83	2.19	2.56	2.93	3.29	3.66	4.02	4.39	4.75	5.12	5.49	5.85	6.22	6.58	6.95	7.31	7.68	8.04	8.41	8.78	9.14	17
14	0.36	0.72	1.08	1.44	1.80	2.16	2.52	2.88	3.24	3.60	3.96	4.32	4.68	5.04	5.40	5.75	6.11	6.47	6.83	7.19	7.55	7.91	8.27	8.63	8.99	16
15	0.35	0.71	1.06	1.41	1.77	2.12	2.47	2.83	3.18	3.54	3.89	4.24	4.60	4.95	5.30	5.66	6.01	6.36	6.72	7.07	7.42	7.78	8.13	8.49	8.84	15
16	0.35	0.69	1.04	1.39	1.74	2.08	2.43	2.78	3.13	3.47	3.82	4.17	4.52	4.86	5.21	5.56	5.90	6.25	6.60	6.95	7.29	7.64	7.99	8.34	8.68	14
17	0.34	0.68	1.02	1.36	1.71	2.05	2.39	2.73	3.07	3.41	3.75	4.09	4.43	4.77	5.12	5.46	5.80	6.14	6.48	6.82	7.16	7.50	7.84	8.18	8.53	13
18	0.33	0.67	1.00	1.34	1.67	2.01	2.34	2.68	3.01	3.35	3.68	4.01	4.35	4.68	5.02	5.35	5.69	6.02	6.36	6.69	7.03	7.36	7.69	8.03	8.36	12
19	0.33	0.66	0.98	1.31	1.64	1.97	2.30	2.62	2.95	3.28	3.61	3.94	4.26	4.59	4.92	5.25	5.58	5.90	6.23	6.56	6.89	7.22	7.54	7.87	8.20	11
20	0.32	0.64	0.96	1.29	1.61	1.93	2.25	2.57	2.89	3.21	3.54	3.86	4.18	4.50	4.82	5.14	5.46	5.79	6.11	6.43	6.75	7.07	7.39	7.71	8.03	10
21	0.31	0.63	0.94	1.26	1.57	1.89	2.20	2.52	2.83	3.15	3.46	3.78	4.09	4.41	4.72	5.03	5.35	5.66	5.98	6.29	6.61	6.92	7.24	7.55	7.87	9
22	0.31	0.62	0.92	1.23	1.54	1.85	2.16	2.46	2.77	3.08	3.39	3.70	4.00	4.31	4.62	4.93	5.23	5.54	5.85	6.16	6.46	6.77	7.08	7.39	7.70	8
23	0.30	0.60	0.90	1.20	1.50	1.81	2.11	2.41	2.71	3.01	3.31	3.61	3.91	4.21	4.51	4.81	5.12	5.42	5.72	6.02	6.32	6.62	6.92	7.22	7.52	7
24	0.29	0.59	0.88	1.18	1.47	1.76	2.06	2.35	2.65	2.94	3.23	3.53	3.82	4.11	4.41	4.70	5.00	5.29	5.58	5.88	6.17	6.47	6.76	7.05	7.35	6
25	0.29	0.57	0.86	1.15	1.43	1.72	2.01	2.29	2.58	2.87	3.15	3.44	3.73	4.02	4.30	4.59	4.88	5.16	5.44	5.74	6.02	6.31	6.60	6.88	7.17	5
26	0.28	0.56	0.84	1.12	1.40	1.68	1.96	2.24	2.52	2.80	3.08	3.36	3.63	3.91	4.19	4.47	4.75	5.03	5.31	5.59	5.87	6.15	6.43	6.71	6.99	4
27	0.27	0.54	0.82	1.09	1.36	1.63	1.91	2.18	2.45	2.72	3.00	3.27	3.54	3.81	4.08	4.36	4.63	4.90	5.17	5.45	5.72	5.99	6.26	6.54	6.81	3
28	0.26	0.53	0.79	1.06	1.32	1.59	1.85	2.12	2.38	2.65	2.91	3.18	3.44	3.71	3.97	4.24	4.50	4.77	5.03	5.30	5.56	5.83	6.09	6.36	6.62	2
29	0.26	0.52	0.77	1.03	1.29	1.55	1.80	2.06	2.32	2.58	2.83	3.09	3.35	3.61	3.86	4.12	4.38	4.64	4.89	5.15	5.41	5.67	5.92	6.18	6.44	1
II. 0	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25	IV. 0
1	0.24	0.48	0.73	0.97	1.21	1.45	1.70	1.94	2.18	2.42	2.67	2.91	3.15	3.40	3.64	3.88	4.12	4.36	4.61	4.85	5.09	5.33	5.58	5.82	6.06	29
2	0.23	0.47	0.70	0.94	1.17	1.41	1.64	1.88	2.11	2.35	2.58	2.82	3.05	3.29	3.52	3.76	3.99	4.23	4.46	4.69	4.93	5.16	5.40	5.63	5.87	28
3	0.23	0.45	0.68	0.91	1.13	1.36	1.59	1.82	2.04	2.27	2.50	2.72	2.95	3.18	3.40	3.63	3.86	4.09	4.31	4.54	4.77	4.99	5.22	5.45	5.67	27
4	0.22	0.44	0.66	0.88	1.10	1.32	1.53	1.75	1.97	2.19	2.41	2.63	2.85	3.07	3.29	3.51	3.73	3.95	4.16	4.38	4.60	4.82	5.04	5.26	5.48	26
5	0.21	0.42	0.63	0.85	1.06	1.27	1.48	1.69	1.90	2.11	2.32	2.54	2.75	2.96	3.17	3.38	3.59	3.80	4.01	4.23	4.44	4.65	4.86	5.07	5.28	25
6	0.20	0.41	0.61	0.81	1.02	1.22	1.42	1.63	1.83	2.03	2.24	2.44	2.64	2.85	3.05	3.25	3.46	3.66	3.86	4.07	4.27	4.47	4.68	4.88	5.08	24
7	0.20	0.39	0.59	0.78	0.98	1.17	1.37	1.56	1.76	1.95	2.15	2.34	2.54	2.74	2.93	3.13	3.32	3.52	3.71	3.91	4.10	4.30	4.49	4.69	4.88	23
8	0.19	0.37	0.56	0.75	0.94	1.12	1.31	1.50	1.69	1.87	2.06	2.25	2.43	2.62	2.81	3.00	3.18	3.37	3.56	3.75	3.93	4.12	4.31	4.50	4.68	22
9	0.18	0.36	0.54	0.72	0.90	1.08	1.25	1.43	1.61	1.79	1.97	2.15	2.33	2.51	2.69	2.87	3.05	3.23	3.40	3.58	3.76	3.94	4.12	4.30	4.48	21
10	0.17	0.34	0.51	0.68	0.86	1.03	1.20	1.37	1.54	1.71	1.88	2.05	2.22	2.39	2.57	2.74	2.91	3.08	3.25	3.42	3.59	3.76	3.93	4.10	4.28	20
11	0.16	0.33	0.49	0.65	0.81	0.98	1.14	1.30	1.47	1.63	1.79	1.95	2.12	2.28	2.44	2.60	2.77	2.93	3.09	3.26	3.42	3.58	3.74	3.91	4.07	19
12	0.15	0.31	0.46	0.62	0.77	0.93	1.08	1.24	1.39	1.55	1.70	1.85	2.01	2.16	2.32	2.47	2.63	2.78	2.94	3.09	3.24	3.40	3.55	3.71	3.86	18
13	0.15	0.29	0.44	0.58	0.73	0.88	1.02	1.17	1.32	1.46	1.61	1.75	1.90	2.05	2.19	2.34	2.49	2.63	2.78	2.92	3.07	3.22	3.36	3.51	3.65	17
14	0.14	0.28	0.41	0.55	0.69	0.83	0.96	1.10	1.24	1.38	1.52	1.65	1.79	1.93	2.07	2.21	2.34	2.48	2.62	2.76	2.89	3.03	3.17	3.31	3.45	16
15	0.13	0.26	0.39	0.52	0.65	0.78	0.91	1.04	1.16	1.29	1.42	1.55	1.68	1.81	1.94	2.07	2.20	2.33	2.46	2.59	2.72	2.85	2.98	3.11	3.24	15
16	0.12	0.24	0.36	0.48	0.60	0.73	0.85	0.97	1.09	1.21	1.33	1.45	1.57	1.69	1.81	1.94	2.06	2.18	2.30	2.42	2.54	2.66	2.78	2.90	3.02	14
17	0.11	0.22	0.34	0.45	0.56	0.67	0.79	0.90	1.01	1.12	1.24	1.35	1.46	1.57	1.69	1.80	1.91	2.02	2.14	2.25	2.36	2.47	2.59	2.70	2.81	13
18	0.10	0.21	0.31	0.42	0.52	0.62	0.73	0.83	0.94	1.04	1.14	1.25	1.35	1.46	1.56	1.66	1.77	1.87	1.98	2.08	2.18	2.29	2.39	2.49	2.60	12
19	0.10	0.19	0.29	0.38	0.48	0.57	0.67	0.76	0.86	0.95	1.05	1.14	1.24	1.34	1.43	1.53	1.62	1.72	1.81	1.91	2.00	2.10	2.19	2.29	2.39	11
20	0.09	0.17	0.26	0.35	0.43	0.52	0.61	0.69	0.78	0.87	0.96	1.04	1.13	1.22	1.30	1.39	1.48	1.56	1.65	1.74	1.82	1.91	2.00	2.08	2.17	10
21	0.08	0.16	0.23	0.31	0.39	0.47	0.55	0.63	0.70	0.78	0.86	0.94	1.02	1.10	1.17	1.25	1.33	1.41	1.49	1.56	1.64	1.72	1.80	1.88	1.96	9
22	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.70	0.77	0.84	0.90	0.97	1.04	1.11	1.18	1.25	1.32	1.39	1.46	1.53	1.60	1.67	1.74	8
23	0.06	0.12	0.18	0.24	0.30	0.37	0.43	0.49	0.55	0.61	0.67	0.73	0.79	0.85	0.91	0.97	1.04	1.10	1.16	1.22	1.28	1.34	1.40	1.46	1.52	7
24	0.05	0.10	0.16	0.21	0.26	0.32	0.37	0.42	0.47	0.52	0.57	0.63	0.68	0.73	0.78	0.84	0.89	0.94	0.99	1.05	1.10	1.15	1.20	1.25	1.31	6
25	0.04	0.09	0.13	0.17	0.22	0.27	0.31	0.35	0.39	0.44	0.48	0.52	9.57	0.61	0.65	0.70	0.74	0.78	0.83	0.87	0.92	0.96	1.00	1.05	1.09	5
26	0.03	0.07	0.10	0.14	0.17	0.21	0.24	0.28	0.31	0.35	0.38	0.42	0.45	0.49	0.52	0.56	0.59	0.63	0.66	0.70	0.73	0.77	0.80	0.84	0.87	4
27	0.03	0.05	0.08	0.10	0.13	0.16	0.18	0.21	0.24	0.26	0.29	0.31	0.34	0.37	0.39	0.42	0.44	0.47	0.50	0.52	0.55	0.58	0.60	0.63	0.65	3
28	0.02	0.03	0.05	0.07	0.09	0.10	0.12	0.14	0.16	0.17	0.19	0.21	0.23	0.24	0.26	0.28	0.30	0.31	0.33	0.35	0.37	0.38	0.40	0.42	0.44	2
29	0.01	0.02	0.03	0.04	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.17	0.18	0.19	0.20	0.21	0.22	1
III. 0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	III. 0

Table I.—For the Aberration of a Star, in North Polar Distance, or Right Ascension.

The Argument at top, is the Maximum of Aberration to each half-second.

The Argument at the sides, is the difference of Sun's Longitude, and the Star's annual number.

The Aberration is *minus*, if the difference is less than 3 signs; and *plus*, if the difference is more than 3 signs.

Diff. Long.	Maximum of Aberration.																									Diff. Long
S. D.	13.00	13.50	14.00	14.50	15.00	15.50	16.00	16.50	17.00	17.50	18.00	18.50	19.00	19.50	20.00	20.50	21.00	21.50	22.00	22.50	23.00	23.50	24.00	24.50	25.00	S. D.
O 0	13.00	13.50	14.00	14.50	15.00	15.50	16.00	16.50	17.00	17.50	18.00	18.50	19.00	19.50	20.00	20.50	21.00	21.50	22.00	22.50	23.00	23.50	24.00	24.50	25.00	VI. 0
1	13.00	13.50	14.00	14.50	15.00	15.50	16.00	16.50	17.00	17.50	18.00	18.50	19.00	19.50	20.00	20.50	21.00	21.50	22.00	22.50	23.00	23.50	24.00	24.50	25.00	29
2	12.99	13.49	13.99	14.49	14.99	15.49	15.99	16.49	16.99	17.49	17.99	18.49	18.99	19.49	19.99	20.49	20.99	21.49	21.99	22.49	22.99	23.49	23.99	24.49	24.98	28
3	12.98	13.48	13.98	14.48	14.98	15.48	15.98	16.48	16.98	17.48	17.98	18.47	18.97	19.47	19.97	20.47	20.97	21.47	21.97	22.47	22.97	23.47	23.97	24.47	24.97	27
4	12.97	13.47	13.97	14.46	14.96	15.46	15.96	16.46	16.96	17.46	17.96	18.45	18.95	19.45	19.95	20.45	20.95	21.45	21.95	22.44	22.94	23.44	23.94	24.44	24.94	26
5	12.95	13.45	13.95	14.45	14.94	15.44	15.94	16.44	16.94	17.43	17.93	18.43	18.93	19.43	19.92	20.42	20.92	21.42	21.92	22.41	22.91	23.41	23.91	24.41	24.90	25
6	12.93	13.43	13.92	14.42	14.92	15.42	15.91	16.41	16.91	17.40	17.90	18.40	18.90	19.39	19.89	20.39	20.88	21.38	21.88	22.38	22.87	23.37	23.87	24.37	24.86	24
7	12.90	13.40	13.90	14.39	14.89	15.38	15.88	16.38	16.87	17.37	17.87	18.36	18.86	19.35	19.85	20.35	20.84	21.34	21.84	22.33	22.83	23.32	23.82	24.32	24.81	23
8	12.87	13.37	13.86	14.36	14.85	15.35	15.84	16.34	16.83	17.33	17.82	18.32	18.82	19.31	19.81	20.30	20.80	21.29	21.79	22.28	22.78	23.27	23.77	24.26	24.76	22
9	12.84	13.33	13.83	14.32	14.82	15.31	15.80	16.30	16.79	17.28	17.78	18.27	18.77	19.26	19.75	20.25	20.74	21.23	21.73	22.22	22.72	23.21	23.70	24.20	24.69	21
10	12.80	13.29	13.79	14.28	14.77	15.26	15.76	16.25	16.74	17.23	17.73	18.22	18.71	19.20	19.70	20.19	20.68	21.17	21.67	22.16	22.65	23.14	23.64	24.13	24.62	20
11	12.76	13.25	13.74	14.23	14.72	15.22	15.71	16.20	16.69	17.18	17.67	18.16	18.65	19.14	19.63	20.12	20.61	21.11	21.60	22.09	22.58	23.07	23.56	24.05	24.54	19
12	12.72	12.21	13.69	14.18	14.67	15.16	15.65	16.14	16.63	17.12	17.61	18.10	18.58	19.07	19.56	20.05	20.54	21.03	21.52	22.01	22.50	22.99	23.48	23.96	24.45	18
13	12.67	13.15	13.64	14.13	14.62	15.10	15.59	16.08	16.56	17.05	17.54	18.03	18.51	19.00	19.49	19.97	20.46	20.95	21.44	21.92	22.41	22.90	23.38	23.87	24.36	17
14	12.61	13.10	13.58	14.07	14.55	15.04	15.52	16.01	16.50	16.98	17.47	17.95	18.44	18.92	19.41	19.89	20.38	20.86	21.35	21.83	22.32	22.80	23.28	23.77	24.26	16
15	12.56	13.04	13.52	14.01	14.49	14.97	15.45	15.94	16.42	16.90	17.39	17.87	18.35	18.84	19.32	19.80	20.28	20.77	21.25	21.73	22.22	22.70	23.18	23.67	24.15	15
16	12.50	12.98	13.46	13.94	14.42	14.90	15.38	15.86	16.34	16.82	17.30	17.78	18.26	18.74	19.23	19.71	20.19	20.67	21.15	21.63	22.11	22.59	23.07	23.55	24.03	14
17	12.43	12.91	13.39	13.87	14.34	14.82	15.30	15.78	16.26	16.74	17.21	17.69	18.17	18.65	19.13	19.60	20.08	20.56	21.04	21.52	21.99	22.47	22.95	23.43	23.91	13
18	12.36	12.84	13.31	13.79	14.27	14.74	15.22	15.69	16.17	16.64	17.12	17.59	18.07	18.55	19.02	19.50	19.97	20.45	20.92	21.40	21.87	22.35	22.83	23.30	23.78	12
19	12.29	12.76	13.24	13.71	14.18	14.66	15.13	15.60	16.07	16.55	17.02	17.49	17.96	18.44	18.91	19.38	19.86	20.33	20.80	21.27	21.75	22.22	22.69	23.17	23.64	11
20	12.22	12.69	13.16	13.63	14.10	14.57	15.04	15.50	15.97	16.44	16.91	17.38	17.85	18.32	18.79	19.26	19.73	20.20	20.67	21.14	21.61	22.08	22.55	23.02	23.49	10
21	12.14	12.60	13.07	13.54	14.00	14.47	14.94	15.40	15.87	16.34	16.80	17.27	17.74	18.20	18.67	19.14	19.61	20.07	20.54	21.01	21.47	21.94	22.41	22.87	23.34	9
22	12.05	12.52	12.98	13.44	13.91	14.37	14.83	15.30	15.76	16.23	16.69	17.15	17.62	18.08	18.54	19.01	19.47	19.93	20.40	20.86	21.33	21.79	22.25	22.72	23.18	8
23	11.97	12.43	12.89	13.35	13.81	14.27	14.73	15.19	15.65	16.11	16.57	17.03	17.49	17.95	18.41	18.87	19.33	19.79	20.25	20.71	21.17	21.63	22.09	22.55	23.01	7
24	11.88	12.33	12.79	13.25	13.70	14.16	14.62	15.07	15.53	15.98	16.44	16.90	17.36	17.81	18.27	18.73	19.18	19.64	20.10	20.55	21.01	21.47	21.93	22.38	22.84	6
25	11.78	12.24	12.69	13.14	13.59	14.05	14.50	14.95	15.41	15.86	16.31	16.77	17.22	17.67	18.13	18.58	19.03	19.49	19.94	20.39	20.85	21.30	21.75	22.20	22.66	5
26	11.68	12.13	12.59	13.04	13.48	13.93	14.38	14.83	15.28	15.73	16.18	16.63	17.08	17.53	17.98	18.42	18.87	19.32	19.77	20.22	20.67	21.12	21.57	22.02	22.47	4
27	11.58	12.03	12.47	12.92	13.37	13.81	14.26	14.70	15.15	15.59	16.04	16.48	16.93	17.37	17.82	18.27	18.71	19.16	19.60	20.05	20.49	20.94	21.38	21.83	22.28	3
28	11.48	11.92	12.36	12.80	13.24	13.69	14.13	14.57	15.01	15.45	15.89	16.33	16.78	17.22	17.66	18.10	18.54	18.98	19.42	19.87	20.31	20.75	21.19	21.63	22.07	2
29	11.37	11.81	12.24	12.68	13.12	13.56	13.99	14.43	14.87	15.31	15.74	16.18	16.62	17.06	17.49	17.93	18.37	18.80	19.24	19.68	20.12	20.55	20.99	21.43	21.87	1
I. 0	11.26	11.69	12.12	12.56	12.99	13.42	13.86	14.29	14.72	15.16	15.59	16.02	16.45	16.89	17.32	17.75	18.19	18.62	19.05	19.49	19.92	20.35	20.78	21.22	21.65	V. 0
1	11.14	11.57	12.00	12.43	12.86	13.29	13.71	14.14	14.57	15.00	15.43	15.86	16.29	16.71	17.14	17.57	18.00	18.43	18.86	19.29	19.71	20.14	20.57	21.00	21.43	29
2	11.02	11.45	11.87	12.30	12.72	13.14	13.57	13.99	14.42	14.84	15.26	15.69	16.11	16.54	16.96	17.39	17.81	18.23	18.66	19.08	19.51	19.93	20.35	20.78	21.20	28
3	10.90	11.32	11.74	12.16	12.58	13.00	13.42	13.84	14.26	14.68	15.10	15.52	15.93	16.35	16.77	17.19	17.61	18.03	18.45	18.87	19.29	19.71	20.13	20.55	20.97	27
4	10.78	11.19	11.61	12.02	12.44	12.85	13.26	13.68	14.09	14.51	14.92	15.34	15.75	16.17	16.58	17.00	17.41	17.82	18.24	18.65	19.07	19.48	19.90	20.31	20.73	26
5	10.65	11.06	11.47	11.88	12.29	12.70	13.11	13.52	13.93	14.34	14.74	15.15	15.56	15.97	16.38	16.79	17.20	17.61	18.02	18.43	18.84	19.25	19.66	20.07	20.48	25
6	10.52	10.92	11.33	11.73	12.14	12.54	12.94	13.35	13.75	14.16	14.56	14.97	15.37	15.78	16.18	16.58	16.99	17.39	17.80	18.20	18.61	19.01	19.42	19.82	20.23	24
7	10.38	10.78	11.18	11.58	11.98	12.38	12.78	13.18	13.58	13.98	14.38	14.77	15.17	15.57	15.97	16.37	16.77	17.17	17.57	17.97	18.37	18.77	19.17	19.57	19.97	23
8	10.24	10.64	11.03	11.43	11.82	12.21	12.61	13.00	13.40	13.79	14.18	14.58	14.97	15.37	15.76	16.15	16.55	16.94	17.34	17.73	18.12	18.52	18.91	19.31	19.70	22

9	10.10	10.49	10.88	11.27	11.66	12.05	12.43	12.82	13.21	13.60	13.99	14.38	14.77	15.15	15.54	15.93	16.32	16.71	17.10	17.49	17.87	18.26	18.65	19.04	19.43	21
10	9.96	10.34	10.72	11.11	11.49	11.87	12.26	12.64	13.02	13.41	13.79	14.17	14.55	14.94	15.32	15.70	16.09	16.47	16.85	17.24	17.62	18.00	18.39	18.77	19.15	20
11	9.81	10.19	10.57	10.94	11.32	11.70	12.08	12.45	12.83	13.21	13.58	13.96	14.34	14.72	15.09	15.47	15.85	16.23	16.60	16.98	17.36	17.74	18.11	18.49	18.87	19
12	9.66	10.03	10.40	10.78	11.15	11.52	11.89	12.26	12.63	13.00	13.38	13.75	14.12	14.49	14.86	15.23	15.61	15.98	16.35	16.72	17.09	17.46	17.84	18.21	18.58	18
13	9.57	9.87	10.24	10.60	10.97	11.34	11.70	12.07	12.43	12.80	13.16	13.53	13.90	14.26	14.63	14.99	15.36	15.72	16.09	16.46	16.82	17.19	17.55	17.92	18.28	17
14	9.35	9.71	10.07	10.43	10.79	11.15	11.51	11.87	12.23	12.59	12.95	13.31	13.67	14.03	14.39	14.75	15.11	15.47	16.83	16.19	16.54	16.90	17.26	17.62	17.98	16
15	9.19	9.55	9.90	10.25	10.61	10.96	11.31	11.67	12.02	12.37	12.73	13.08	13.44	13.79	14.14	14.50	14.85	15.20	15.56	15.91	16.26	16.62	16.97	17.32	17.68	15
16	9.03	9.38	9.73	10.07	10.42	10.77	11.11	11.46	11.81	12.16	12.50	12.85	13.20	13.55	13.89	14.24	14.59	14.94	15.28	15.63	15.98	16.32	16.67	17.02	17.37	14
17	8.87	9.21	9.55	9.89	10.23	10.57	10.91	11.25	11.59	11.94	12.28	12.62	12.96	13.30	13.64	13.98	14.32	14.66	15.00	15.35	15.69	16.03	16.37	16.71	17.05	13
18	8.70	9.03	9.37	9.70	10.04	10.37	10.71	11.04	11.38	11.71	12.04	12.38	12.71	13.05	13.38	13.72	14.05	14.39	14.72	15.06	15.39	15.72	16.06	16.39	16.73	12
19	8.53	8.86	9.18	9.51	9.84	10.17	10.50	10.82	11.15	11.48	11.81	12.14	12.47	12.79	13.12	13.45	13.78	14.11	14.43	14.76	15.09	15.42	15.75	16.07	16.40	11
20	8.36	8.68	9.00	9.32	9.64	9.96	10.28	10.61	10.93	11.25	11.57	11.89	12.21	12.54	12.86	13.18	13.50	13.82	14.14	14.46	14.78	15.11	15.43	15.75	16.07	10
21	8.18	8.50	8.81	9.13	9.44	9.75	10.07	10.38	10.70	11.01	11.33	11.64	11.96	12.27	12.59	12.90	13.22	13.53	13.84	14.16	14.47	14.79	15.10	15.42	15.73	9
22	8.00	8.31	8.62	8.93	9.24	9.54	9.85	10.16	10.47	10.77	11.08	11.39	11.70	12.01	12.31	12.62	12.93	13.24	13.54	13.85	14.16	14.47	14.78	15.08	15.39	8
23	7.82	8.12	8.43	8.73	9.03	9.33	9.63	9.93	10.23	10.53	10.83	11.13	11.43	11.74	12.04	12.34	12.64	12.94	13.24	13.54	13.84	14.14	14.44	14.74	15.05	7
24	7.64	7.94	8.23	8.52	8.82	9.11	9.40	9.70	9.99	10.29	10.58	10.87	11.17	11.46	11.76	12.05	12.34	12.64	12.93	13.23	13.52	13.81	14.11	14.40	14.69	6
25	7.46	7.74	8.03	8.32	8.60	8.89	9.18	9.46	9.75	10.04	10.32	10.61	10.90	11.18	11.47	11.76	12.05	12.33	12.62	12.91	13.19	13.48	13.77	14.05	14.34	5
26	7.27	7.55	7.83	8.11	8.39	8.67	8.95	9.23	9.51	9.79	10.07	10.35	10.62	10.90	11.18	11.46	11.74	12.02	12.30	12.58	12.86	13.14	13.42	13.70	13.98	4
27	7.08	7.35	7.62	7.90	8.17	8.44	8.71	8.99	9.26	9.53	9.80	10.08	10.35	10.62	10.89	11.17	11.44	11.71	11.98	12.25	12.53	12.80	13.07	13.34	13.62	3
28	6.89	7.15	7.42	7.68	7.95	8.21	8.48	8.74	9.01	9.27	9.54	9.80	10.07	10.33	10.60	10.86	11.13	11.39	11.66	11.92	12.19	12.45	12.72	12.98	13.25	2
29	6.70	6.95	7.21	7.47	7.73	7.98	8.24	8.50	8.76	9.01	9.27	9.53	9.79	10.04	10.30	10.56	10.82	11.07	11.33	11.59	11.85	12.10	12.36	12.62	12.88	1
II. 0	6.50	6.75	7.00	7.25	7.50	7.75	8.00	8.25	8.50	8.75	9.00	9.25	9.50	9.75	10.00	10.25	10.50	10.75	11.00	11.25	11.50	11.75	12.00	12.25	12.50	IV. 0
1	6.30	6.54	6.79	7.03	7.27	7.51	7.76	8.00	8.24	8.48	8.73	8.97	9.21	9.45	9.70	9.94	10.18	10.42	10.67	10.91	11.15	11.39	11.64	11.88	12.12	29
2	6.10	6.34	6.57	6.81	7.04	7.28	7.51	7.75	7.98	8.22	8.45	8.69	8.92	9.15	9.39	9.62	9.86	10.09	10.33	10.56	10.80	11.03	11.27	11.50	11.74	28
3	5.90	6.13	6.36	6.58	6.81	7.04	7.26	7.49	7.72	7.94	8.17	8.40	8.62	8.85	9.08	9.31	9.53	9.76	9.99	10.21	10.44	10.67	10.90	11.12	11.35	27
4	5.70	5.92	6.14	6.36	6.58	6.79	7.01	7.23	7.45	7.67	7.89	8.11	8.33	8.55	8.77	8.99	9.21	9.43	9.64	9.86	10.08	10.30	10.52	10.74	10.96	26
5	5.49	5.71	5.92	6.13	6.34	6.55	6.76	6.97	7.18	7.40	7.61	7.82	8.03	8.24	8.45	8.66	8.88	9.09	9.30	9.51	9.72	9.93	10.14	10.35	10.57	25
6	5.29	5.49	5.69	5.90	6.10	6.30	6.51	6.71	6.91	7.12	7.32	7.52	7.73	7.93	8.13	8.34	8.54	8.74	8.95	9.15	9.36	9.56	9.76	9.97	10.17	24
7	5.08	5.27	5.47	5.67	5.86	6.06	6.25	6.45	6.64	6.84	7.03	7.23	7.42	7.62	7.81	8.01	8.21	8.40	8.60	8.79	8.99	9.18	9.38	9.57	9.77	23
8	4.87	5.06	5.24	5.43	5.62	5.81	5.99	6.18	6.37	6.56	6.74	6.93	7.12	7.30	7.49	7.68	7.87	8.05	8.24	8.43	8.62	8.80	8.99	9.18	9.37	22
9	4.66	4.84	5.02	5.20	5.38	5.55	5.73	5.91	6.09	6.27	6.45	6.63	6.81	6.99	7.17	7.35	7.53	7.71	7.88	8.06	8.24	8.42	8.60	8.78	8.96	21
10	4.45	4.62	4.79	4.96	5.13	5.30	5.47	5.64	5.81	5.99	6.16	6.33	6.50	6.67	6.84	7.01	7.18	7.35	7.52	7.70	7.87	8.04	8.21	8.38	8.55	20
11	4.23	4.40	4.56	4.72	4.89	5.05	5.21	5.37	5.53	5.70	5.86	6.02	6.19	6.35	6.51	6.67	6.84	7.00	7.16	7.33	7.49	7.65	7.81	7.98	8.14	19
12	4.02	4.17	4.33	4.48	4.64	4.79	4.94	5.10	5.25	5.41	5.56	5.72	5.87	6.03	6.18	6.33	6.49	6.64	6.80	6.95	7.11	7.26	7.42	7.57	7.73	18
13	3.80	3.95	4.09	4.24	4.39	4.53	4.68	4.82	4.97	5.12	5.26	5.41	5.56	5.70	5.85	5.99	6.14	6.29	6.43	6.58	6.72	6.87	7.02	7.16	7.31	17
14	3.58	3.72	3.86	4.00	4.13	4.27	4.41	4.55	4.69	4.82	4.96	5.10	5.24	5.37	5.51	5.65	5.79	5.93	6.06	6.20	6.34	6.48	6.62	6.75	6.89	16
15	3.36	3.49	3.62	3.75	3.88	4.01	4.14	4.27	4.40	4.53	4.66	4.79	4.92	5.05	5.18	5.31	5.44	5.56	5.69	5.82	5.95	6.08	6.21	6.34	6.47	15
16	3.14	3.27	3.39	3.51	3.63	3.75	3.87	3.99	4.11	4.23	4.35	4.48	4.60	4.72	4.84	4.96	5.08	5.20	5.32	5.44	5.56	5.69	5.81	5.93	6.05	14
17	2.92	3.04	3.15	3.26	3.37	3.49	3.60	3.71	3.82	3.94	4.05	4.16	4.27	4.39	4.50	4.61	4.72	4.84	4.95	5.06	5.17	5.29	5.40	5.51	5.62	13
18	2.70	2.81	2.91	3.01	3.12	3.22	3.33	3.43	3.53	3.64	3.74	3.85	3.95	4.05	4.16	4.26	4.37	4.47	4.57	4.68	4.78	4.89	4.99	5.09	5.20	12
19	2.48	2.58	2.67	2.77	2.86	2.96	3.05	3.15	3.24	3.34	3.43	3.53	3.63	3.72	3.82	3.91	4.01	4.10	4.20	4.30	4.39	4.48	4.58	4.67	4.77	11
20	2.26	2.34	2.43	2.52	2.60	2.69	2.78	2.87	2.95	3.04	3.13	3.21	3.30	3.39	3.47	3.56	3.65	3.73	3.82	3.91	3.99	4.08	4.17	4.25	4.34	10
21	2.03	2.11	2.19	2.27	2.35	2.42	2.50	2.58	2.66	2.74	2.82	2.89	2.97	3.05	3.13	3.21	3.29	3.36	3.44	3.52	3.60	3.68	3.75	3.83	3.91	9
22	1.81	1.88	1.95	2.02	2.09	2.16	2.23	2.30	2.37	2.44	2.51	2.57	2.64	2.71	2.78	2.85	3.92	2.99	3.06	3.13	3.20	3.27	3.34	3.41	3.48	8
23	1.58	1.65	1.71	1.77	1.83	1.89	1.95	2.01	2.07	2.13	2.19	2.25	2.32	2.38	2.44	2.50	2.56	2.62	2.68	2.74	2.80	2.86	2.92	2.99	3.05	7
24	1.36	1.41	1.46	1.52	1.57	1.62	1.67	1.72	1.78	1.83	1.88	1.93	1.99	2.04	2.09	2.14	2.20	2.25	2.30	2.35	2.40	2.46	2.51	2.56	2.61	6
25	1.13	1.18	1.22	1.26	1.31	1.35	1.39	1.44	1.48	1.53	1.57	1.61	1.66	1.70	1.74	1.79	1.83	1.87	1.92	1.96	2.00	2.05	2.09	2.14	2.18	5
26	0.91	0.94	0.98	1.01	1.05	1.08	1.12	1.15	1.18	1.22	1.26	1.29	1.33	1.36	1.40	1.43	1.46	1.50	1.53	1.57	1.60	1.64	1.67	1.71	1.74	4
27	0.68	0.71	0.73	0.76	0.79	0.81	0.84	0.86	0.89	0.92	0.94	0.97	0.99	1.02	1.05	1.07	1.10	1.03	1.15	1.18	1.20	1.23	1.26	1.28	1.31	3
28	0.45	0.47	0.49	0.51	0.52	0.54	0.56	0.58	0.59	0.61	0.63	0.65	0.66	0.68	0.70	0.71	0.73	0.75	0.77	0.79	0.80	0.82	0.84	0.86	0.87	2
29	0.23	0.24	0.24	0.25	0.26	0.27	0.28	0.29	0.30	0.31	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.37	0.38	0.40	0.40	0.41	0.42	0.43	0.44	1
III. 0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	III. 0

Proportional Logarithms to each

Radius of the Table, two

Minutes.	h ′ ° ′ 0.0	h ′ ° ′ 0.5	h ′ ° ′ 0.10	h ′ ° ′ 0.15	h ′ ° ′ 0.20	h ′ ° ′ 0.25	h ′ ° ′ 0.30	h ′ ° ′ 0.35	h ′ ° ′ 0.40	h ′ ° ′ 0.45	h ′ ° ′ 0.50	h ′ ° ′ 0.55
0.0		1.3802	1.0792	9031	7782	6812	6021	5351	4771	4260	3802	3388
0.1	3.0792	1.3716	1.0749	9002	7760	6795	6006	5339	4760	4250	3793	3380
0.2	2.7782	1.3632	1.0706	8973	7738	6778	5992	5326	4750	4240	3785	3372
0.3	2.6021	1.3549	1.0663	8945	7717	6761	5977	5314	4739	4231	3776	3365
0.4	2.4771	1.3468	1.0621	8917	7696	6743	5963	5302	4728	4221	3768	3357
0.5	2.3802	1.3388	1.0580	8888	7674	6726	5949	5390	4717	4212	3759	3349
0.6	2.3010	1.3310	1.0539	8861	7653	6709	5935	5277	4707	4202	3750	3341
0.7	2.2341	1.3233	1.0498	8833	7632	6692	5920	5265	4696	4193	3742	3333
0.8	2.1761	1.3158	1.0458	8805	7611	6676	5906	5253	4685	4183	3733	3325
0.9	2.1249	1.3083	1.0418	8778	7590	6659	5892	5241	4675	4174	3725	3318
1.0	2.0792	1.3010	1.0378	8751	7570	6642	5878	5229	4664	4164	3716	3310
1.1	2.0378	1.2939	1.0339	8724	7549	6625	5864	5217	4653	4155	3708	3302
1.2	2.0000	1.2868	1.0300	8697	7528	6609	5850	5205	4643	4145	3699	3294
1.3	1.9652	1.2798	1.0261	8670	7508	6592	5836	5193	4632	4136	3691	3287
1.4	1.9331	1.2730	1.0223	8643	7488	6576	5823	5181	4622	4127	3682	3279
1.5	1.9031	1.2663	1.0185	8617	7467	6559	5809	5169	4611	4117	3674	3271
1.6	1.8751	1.2596	1.0147	8591	7447	6543	5795	5157	4601	4108	3665	3264
1.7	1.8487	1.2531	1.0110	8565	7427	6527	5781	5145	4590	4099	3657	3256
1.8	1.8239	1.2467	1.0073	8539	7407	6510	5768	5133	4580	4089	3649	3248
1.9	1.8004	1.2403	1.0036	8513	7387	6494	5754	5122	4570	4080	3640	3241
2.0	1.7782	1.2341	1.0000	8487	7368	6478	5740	5110	4559	4071	3632	3233
2.1	1.7570	1.2279	9964	8462	7348	6462	5727	5098	4549	4062	3623	3225
2.2	1.7368	1.2218	9928	8437	7328	6446	5713	5086	4539	4052	3615	3218
2.3	1.7175	1.2159	9893	8411	7309	6430	5700	5075	4528	4043	3607	3210
2.4	1.6990	1.2099	9858	8386	7289	6414	5686	5063	4518	4034	3598	3203
2.5	1.6812	1.2041	9823	8361	7270	6398	5673	5051	4508	4025	3590	3195
2.6	1.6642	1.1984	9788	8337	7251	6383	5660	5040	4498	4016	3582	3188
2.7	1.6478	1.1927	9754	8312	7232	6367	5646	5028	4488	4007	3574	3180
2.8	1.6320	1.1871	9720	8288	7212	6351	5633	5017	4477	3998	3565	3173
2.9	1.6168	1.1816	9686	8263	7193	6336	5620	5005	4467	3988	3557	3165
3.0	1.6021	1.1761	9652	8239	7175	6320	5607	4994	4457	3979	3549	3158
3.1	1.5878	1.1707	9619	8215	7156	6305	5594	4983	4447	3970	3541	3150
3.2	1.5740	1.1654	9586	8191	7137	6289	5580	4971	4437	3961	3533	3143
3.3	1.5607	1.1601	9553	8167	7118	6274	5567	4960	4427	3952	3525	3135
3.4	1.5477	1.1549	9521	8144	7100	6259	5554	4948	4417	3943	3516	3128
3.5	1.5351	1.1498	9488	8120	7081	6243	5541	4937	4407	3934	3508	3120
3.6	1.5229	1.1447	9456	8097	7063	6228	5528	4926	4397	3925	3500	3113
3.7	1.5110	1.1397	9425	8073	7044	6213	5516	4915	4387	3917	3492	3105
3.8	1.4994	1.1347	9393	8050	7026	6198	5503	4903	4377	3908	3484	3098
3.9	1.4881	1.1298	9362	8027	7008	6183	5490	4892	4367	3899	3476	3091
4.0	1.4771	1.1249	9331	8004	6990	6168	5477	4881	4357	3890	3468	3083
4.1	1.4664	1.1201	9300	7981	6972	6153	5464	4870	4347	3881	3460	3076
4.2	1.4559	1.1154	9269	7959	6954	6138	5452	4859	4338	3872	3452	3069
4.3	1.4457	1.1107	9238	7936	6936	6123	5439	4848	4328	3863	3444	3061
4.4	1.4357	1.1061	9208	7914	6918	6108	5426	4837	4318	3855	3436	3054
4.5	1.4260	1.1015	9178	7891	6900	6094	5414	4826	4308	3846	3428	3047
4.6	1.4164	1.0969	9148	7869	6882	6079	5401	4815	4298	3837	3420	3039
4.7	1.4071	1.0924	9119	7847	6865	6064	5389	4804	4289	3828	3412	3032
4.8	1.3979	1.0880	9089	7825	6847	6050	5376	4793	4279	3820	3404	3025
4.9	1.3890	1.0835	9060	7803	6830	6035	5364	4782	4269	3811	3396	3018
5.0	1.3802	1.0792	9031	7782	6812	6021	5351	4771	4260	3802	3388	3010

tenth part of one Minute.

Hours, or two Degrees.

h ′ ° ′ 1.0	h ′ ° ′ 1.5	h ′ ° ′ 1.10	h ′ ° ′ 1.15	h ′ ° ′ 1.20	h ′ ° ′ 1.25	h ′ ° ′ 1.30	h ′ ° ′ 1.35	h ′ ° ′ 1.40	h ′ ° ′ 1.45	h ′ ° ′ 1.50	h ′ ° ′ 1.55	Minutes.
3010	2663	2341	2041	1761	1498	1249	1015	0792	0580	0378	0185	0.0
3003	2656	2335	2035	1755	1493	1245	1010	0787	0576	0374	0181	0.1
2996	2649	2328	2030	1750	1487	1240	1005	0783	0572	0370	0177	0.2
2989	2643	2322	2024	1745	1482	1235	1001	0779	0568	0366	0174	0.3
2981	2636	2316	2018	1739	1477	1230	0996	0774	0563	0362	0170	0.4
2974	2629	2310	2012	1734	1472	1225	0992	0770	0559	0358	0166	0.5
2967	2623	2304	2007	1728	1467	1221	0987	0766	0555	0354	0162	0.6
2960	2616	2298	2001	1723	1462	1216	0983	0762	0551	0350	0158	0.7
2953	2610	2291	1995	1718	1457	1211	0978	0757	0547	0346	0155	0.8
2946	2603	2285	1989	1712	1452	1206	0974	0753	0543	0342	0151	0.9
2939	2596	2279	1984	1707	1447	1201	0969	0749	0539	0339	0147	1.0
2931	2590	2273	1978	1702	1442	1197	0965	0744	0535	0335	0143	1.1
2924	2583	2267	1972	1696	1437	1192	0960	0740	0531	0331	0140	1.2
2917	2577	2261	1967	1691	1432	1187	0956	0736	0526	0327	0136	1.3
2910	2570	2255	1961	1686	1427	1182	0951	0731	0522	0323	0132	1.4
2903	2564	2249	1955	1680	1422	1178	0947	0727	0518	0319	0129	1.5
2896	2557	2243	1950	1675	1417	1173	0942	0723	0514	0315	0125	1.6
2889	2551	2237	1944	1670	1412	1168	0938	0719	0510	0311	0121	1.7
2882	2544	2231	1938	1664	1407	1163	0933	0714	0506	0307	0117	1.8
2875	2538	2225	1933	1659	1402	1159	0929	0710	0502	0304	0114	1.9
2868	2531	2218	1927	1654	1397	1154	0924	0706	0498	0300	0110	2.0
2861	2525	2212	1921	1648	1392	1149	0920	0702	0494	0296	0106	2.1
2854	2518	2206	1916	1643	1387	1145	0915	0697	0490	0292	0103	2.2
2847	2512	2200	1910	1638	1382	1140	0911	0693	0486	0288	0099	2.3
2840	2505	2194	1904	1633	1377	1135	0906	0689	0482	0284	0095	2.4
2833	2499	2188	1899	1627	1372	1130	0902	0685	0478	0280	0091	2.5
2826	2492	2182	1893	1622	1367	1126	0897	0680	0474	0276	0088	2.6
2819	2486	2176	1888	1617	1362	1121	0893	0676	0470	0273	0084	2.7
2812	2480	2170	1882	1612	1357	1116	0888	0672	0466	0269	0080	2.8
2805	2473	2165	1876	1606	1352	1112	0884	0668	0462	0265	0077	2.9
2798	2467	2159	1871	1601	1347	1107	0880	0663	0458	0261	0073	3.0
2792	2460	2153	1865	1596	1342	1102	0875	0659	0454	0257	0069	3.1
2785	2454	2147	1860	1591	1337	1098	0871	0655	0450	0253	0066	3.2
2778	2448	2141	1854	1585	1332	1093	0866	0651	0446	0250	0062	3.3
2771	2441	2135	1849	1580	1327	1088	0862	0647	0442	0246	0058	3.4
2764	2435	2129	1843	1575	1322	1084	0857	0642	0438	0242	0055	3.5
2757	2429	2123	1838	1570	1317	1079	0853	0638	0434	0238	0051	3.6
2750	2422	2117	1832	1565	1313	1074	0849	0634	0430	0234	0047	3.7
2744	2416	2111	1827	1559	1308	1070	0844	0630	0426	0230	0044	3.8
2737	2410	2105	1821	1554	1303	1065	0840	0626	0422	0227	0040	3.9
2730	2403	2099	1816	1549	1298	1061	0835	0621	0418	0223	0036	4.0
2723	2397	2094	1810	1544	1293	1056	0831	0617	0414	0219	0033	4.1
2716	2391	2088	1805	1539	1288	1051	0827	0613	0410	0215	0029	4.2
2710	2384	2082	1799	1534	1283	1047	0822	0609	0406	0211	0025	4.3
2703	2378	2076	1794	1528	1278	1042	0818	0605	0402	0208	0022	4.4
2696	2372	2070	1788	1523	1274	1037	0814	0601	0398	0204	0018	4.5
2689	2366	2064	1783	1518	1269	1033	0809	0596	0394	0200	0015	4.6
2683	2359	2059	1777	1513	1264	1028	0805	0592	0390	0196	0011	4.7
2676	2353	2053	1772	1508	1259	1024	0801	0588	0386	0192	0007	4.8
2669	2347	2047	1766	1503	1254	1019	0796	0584	0382	0189	0004	4.9
2663	2341	2041	1761	1498	1249	1015	0792	0580	0378	0185	0000	5.0

Table L. (No. 1.)—Decimal numbers of each Degree of the Sun's Longitude (and their complements) for multiplying the Annual Precession of a Star in North Polar Distance.

Degr.	0		I.		II.		III.		IV.		V.		VI.		VII.		VIII.		IX.		X.		XI.		Degr.
	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	
0	.000	1·000	.084	.916	.168	.832	.254	.746	.340	.660	.426	.574	.511	.489	.594	.406	.675	.325	.756	.244	.837	.163	.918	.082	0
1	.003	.997	.087	.913	.171	.829	.257	.743	.343	.657	.429	.571	.513	.487	.596	.404	.678	.322	.759	.241	.839	.161	.920	.080	1
2	.006	.994	.089	.911	.174	.826	.260	.740	.346	.654	.432	.568	.516	.484	.599	.401	.681	.319	.761	.239	.842	.158	.923	.077	2
3	.008	.992	.092	.908	.177	.823	.263	.737	.349	.651	.435	.565	.519	.481	.602	.398	.683	.317	.764	.236	.845	.155	.926	.074	3
4	.011	.989	.095	.905	.180	.820	.266	.734	.352	.648	.438	.562	.522	.478	.605	.395	.686	.314	.767	.233	.847	.153	.929	.071	4
5	.014	.986	.098	.902	.183	.817	.269	.731	.355	.645	.440	.560	.524	.476	.607	.393	.689	.311	.770	.230	.850	.150	.931	.069	5
6	.017	.983	.101	.899	.186	.814	.272	.728	.358	.642	.443	.557	.527	.473	.610	.390	.691	.309	.772	.228	.853	.147	.934	.066	6
7	.019	.981	.103	.897	.189	.811	.275	.725	.361	.639	.446	.554	.530	.470	.613	.387	.694	.306	.775	.225	.855	.145	.937	.063	7
8	.022	.978	.106	.894	.192	.808	.278	.722	.364	.636	.449	.551	.533	.467	.615	.385	.697	.303	.777	.223	.858	.142	.940	.060	8
9	.025	.975	.109	.891	.194	.806	.281	.719	.366	.634	.452	.548	.536	.464	.618	.382	.699	.301	.780	.220	.861	.139	.942	.058	9
10	.028	.972	.112	.888	.197	.803	.283	.717	.369	.631	.454	.546	.538	.462	.621	.379	.702	.298	.783	.217	.864	.136	.945	.055	10
11	.031	.969	.115	.885	.200	.800	.286	.714	.372	.628	.457	.543	.541	.459	.624	.376	.705	.295	.785	.215	.866	.134	.948	.052	11
12	.033	.967	.118	.882	.203	.797	.289	.711	.375	.625	.460	.540	.544	.456	.626	.374	.708	.292	.788	.212	.869	.131	.950	.050	12
13	.036	.964	.120	.880	.206	.794	.292	.708	.378	.622	.463	.537	.547	.453	.629	.371	.710	.290	.791	.209	.872	.128	.953	.047	13
14	.039	.961	.123	.877	.209	.791	.295	.705	.381	.619	.466	.534	.549	.451	.632	.368	.713	.287	.794	.206	.874	.126	.956	.044	14
15	.042	.958	.126	.874	.211	.789	.297	.703	.383	.617	.468	.532	.552	.448	.635	.365	.716	.284	.796	.204	.877	.123	.959	.041	15
16	.044	.956	.129	.871	.214	.786	.300	.700	.386	.614	.471	.529	.555	.445	.637	.363	.718	.282	.799	.201	.880	.120	.961	.039	16
17	.047	.953	.132	.868	.217	.783	.303	.697	.389	.611	.474	.526	.558	.442	.640	.360	.721	.279	.802	.198	.882	.118	.964	.036	17
18	.050	.950	.135	.865	.220	.780	.306	.694	.392	.608	.477	.523	.561	.439	.643	.357	.724	.276	.804	.196	.885	.115	.967	.033	18
19	.053	.947	.137	.863	.223	.777	.309	.691	.395	.605	.480	.520	.563	.437	.645	.355	.726	.274	.807	.193	.888	.112	.970	.030	19
20	.055	.945	.140	.860	.226	.774	.312	.688	.398	.602	.483	.517	.566	.434	.648	.352	.729	.271	.810	.190	.891	.109	.972	.028	20
21	.058	.942	.143	.857	.229	.771	.315	.685	.401	.599	.485	.515	.569	.431	.651	.349	.732	.268	.812	.188	.893	.107	.975	.025	21
22	.061	.939	.146	.854	.232	.768	.318	.682	.404	.566	.488	.512	.572	.428	.654	.346	.734	.266	.815	.185	.896	.104	.978	.022	22
23	.064	.936	.149	.851	.235	.765	.321	.679	.406	.594	.491	.509	.574	.426	.656	.344	.737	.263	.818	.182	.899	.101	.981	.019	23
24	.067	.933	.152	.848	.237	.763	.324	.676	.409	.591	.494	.506	.577	.423	.659	.341	.740	.260	.820	.180	.901	.099	.983	.017	24
25	.070	.930	.154	.846	.240	.760	.326	.674	.412	.588	.497	.503	.580	.420	.662	.338	.743	.257	.823	.177	.904	.096	.986	.014	25
26	.072	.928	.157	.843	.243	.757	.329	.671	.415	.585	.499	.501	.583	.417	.664	.336	.745	.255	.826	.174	.907	.093	.989	.011	26
27	.075	.925	.160	.840	.246	.754	.332	.668	.418	.582	.502	.498	.585	.415	.667	.333	.748	.252	.828	.172	.910	.090	.992	.008	27
28	.078	.922	.163	.837	.249	.751	.335	.665	.421	.579	.505	.495	.588	.412	.670	.330	.751	.249	.831	.169	.912	.088	.994	.006	28
29	.081	.919	.166	.834	.252	.748	.338	.662	.423	.577	.508	.492	.591	.409	.672	.328	.753	.247	.834	.166	.915	.085	.997	.003	29
30	.084	.916	.168	.832	.254	.746	.340	.660	.426	.574	.511	.489	.594	.406	.675	.325	.756	.244	.837	.163	.918	.082	1.000	.000	30

Table L. (No. 2.)—Decimal numbers of each Day (and their complements) for multiplying the Annual Precession of a Star in North Polar Distance.

Days.	January.		February.		March.		April.		May.		June.		July.		August.		Sept.		October.		Nov.		Decembe		Days.
	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	Dec.	Comp.	
1	.000	1.000	.085	.915	.162	.838	.247	.753	.329	.671	.414	.586	.496	.504	.581	.419	.666	.334	.748	.252	.833	.167	.915	.085	1
2	.003	.997	.088	.912	.164	.836	.249	.751	.332	.668	.416	.584	.499	.501	.584	.416	.668	.332	.751	.249	.836	.164	.918	.082	2
3	.005	.995	.090	.910	.167	.833	.252	.748	.334	.666	.419	.581	.501	.499	.586	.414	.671	.329	.753	.247	.838	.162	.921	.079	3
4	.008	.992	.093	.907	.170	.830	.255	.745	.337	.663	.422	.578	.504	.496	.589	.411	.674	.326	.756	.244	.841	.159	.923	.077	4
5	.011	.989	.096	.904	.173	.827	.258	.742	.340	.660	.425	.575	.507	.493	.592	.408	.677	.323	.759	.241	.844	.156	.926	.074	5
6	.014	.986	.099	.901	.175	.825	.260	.740	.342	.658	.427	.573	.510	.490	.595	.405	.679	.321	.762	.238	.847	.153	.929	.071	6
7	.016	.984	.101	.899	.178	.822	.263	.737	.345	.655	.430	.570	.512	.488	.597	.403	.682	.318	.764	.236	.849	.151	.932	.068	7
8	.019	.981	.104	.896	.181	.819	.266	.734	.348	.652	.433	.567	.515	.485	.600	.400	.685	.315	.767	.233	.852	.148	.934	.066	8
9	.022	.978	.107	.893	.184	.816	.268	.732	.351	.649	.436	.564	.518	.482	.603	.397	.688	.312	.770	.230	.855	.145	.937	.063	9
10	.025	.975	.110	.890	.186	.814	.271	.729	.353	.647	.438	.562	.521	.479	.605	.395	.690	.310	.773	.227	.858	.142	.940	.060	10
11	.027	.973	.112	.888	.189	.811	.274	.726	.356	.644	.441	.559	.523	.477	.608	.392	.693	.307	.775	.225	.860	.140	.942	.058	11
12	.030	.970	.115	.885	.192	.808	.277	.723	.359	.641	.444	.556	.526	.474	.611	.389	.696	.304	.778	.222	.863	.137	.945	.055	12
13	.033	.967	.118	.882	.195	.805	.279	.721	.362	.638	.447	.553	.529	.471	.614	.386	.699	.301	.781	.219	.866	.134	.948	.052	13
14	.036	.964	.121	.879	.197	.803	.282	.718	.364	.636	.449	.551	.532	.468	.616	.384	.701	.299	.784	.216	.868	.132	.951	.049	14
15	.038	.962	.123	.877	.200	.800	.285	.715	.367	.633	.452	.548	.534	.466	.619	.381	.704	.296	.786	.214	.871	.129	.953	.047	15
16	.041	.959	.126	.874	.203	.797	.288	.712	.370	.630	.455	.545	.537	.463	.622	.378	.707	.293	.789	.211	.874	.126	.956	.044	16
17	.044	.956	.129	.871	.205	.795	.290	.710	.373	.627	.458	.542	.540	.460	.625	.375	.710	.290	.792	.208	.877	.123	.959	.041	17
18	.047	.953	.132	.868	.208	.792	.293	.707	.375	.625	.460	.540	.542	.458	.627	.373	.712	.288	.795	.205	.879	.121	.962	.038	18
19	.049	.951	.134	.866	.211	.789	.296	.704	.378	.622	.463	.537	.545	.455	.630	.370	.715	.285	.797	.203	.882	.118	.964	.036	19
20	.052	.948	.137	.863	.214	.786	.299	.701	.381	.619	.466	.534	.548	.452	.633	.367	.718	.282	.800	.200	.885	.115	.967	.033	20
21	.055	.945	.140	.860	.216	.784	.301	.699	.384	.616	.468	.532	.551	.449	.636	.364	.721	.279	.803	.197	.888	.112	.970	.030	21
22	.058	.942	.142	.858	.219	.781	.304	.696	.386	.614	.471	.529	.553	.447	.638	.362	.723	.277	.805	.195	.890	.110	.973	.027	22
23	.060	.940	.145	.855	.222	.778	.307	.693	.389	.611	.474	.526	.556	.444	.641	.359	.726	.274	.808	.192	.893	.107	.975	.025	23
24	.063	.937	.148	.852	.225	.775	.310	.690	.392	.608	.477	.523	.559	.441	.644	.356	.729	.271	.811	.189	.896	.104	.978	.022	24
25	.066	.934	.151	.849	.227	.773	.312	.688	.395	.605	.479	.521	.562	.438	.647	.353	.732	.268	.814	.186	.899	.101	.981	.019	25
26	.068	.932	.153	.847	.230	.770	.315	.685	.397	.603	.482	.518	.564	.436	.649	.351	.734	.266	.816	.184	.901	.099	.984	.016	26
27	.071	.929	.156	.844	.233	.767	.318	.682	.400	.600	.485	.515	.567	.433	.652	.348	.737	.263	.819	.181	.904	.096	.986	.014	27
28	.074	.926	.159	.841	.236	.764	.321	.679	.403	.597	.488	.512	.570	.430	.655	.345	.740	.260	.822	.178	.907	.093	.989	.011	28
29	.077	.923			.338	.762	.323	.677	.405	.595	.490	.510	.573	.427	.658	.342	.742	.258	.825	.175	.910	.090	.992	.008	29
30	.079	.921			.241	.759	.326	.674	.408	.592	.493	.507	.575	.425	.660	.340	.745	.255	.827	.173	.912	.088	.995	.005	30
31	.082	.918			.244	.756			.411	.589			.578	.422	.663	.337			.830	.170			.997	.003	31

Table M. (No. 1.)—For the Nutation of a Star in North Polar Distance, and Right Ascension in Time.

Argument.—Right Ascension of Star + Long. Moon's Node.

The sum of the two Equations found in these Tables, is the Nutation of a Star in North Polar Distance.

For the Nutation in Right Ascension, call the Right Ascension of the Star plus 3 Signs, if the Declination is North; and minus 3 Signs, if the Declination is South: the sum of the two Equations so found, multiplied by $\frac{1}{15}$th of the natural Tangent of the Declination of the Star, produces the Nutation in Right Ascension in Time.

Signs.	0. VI.	− +	I. VII.	− +	II. VIII.	− +	Signs.
D	"	Dif. 100′	"	Dif. 100′	"	Dif. 100′	D
0	0.000	0.036	0.615	0.031	1.066	0.018	30
1	0.022	0.036	0.634	0.031	1.076	0.017	29
2	0.043	0.036	0.652	0.030	1.086	0.017	28
3	0.064	0.036	0.670	0.030	1.096	0.016	27
4	0.086	0.036	0.688	0.030	1.106	0.016	26
5	0.107	0.036	0.706	0.029	1.115	0.015	25
6	0.129	0.036	0.723	0.029	1.124	0.014	24
7	0.150	0.036	0.740	0.029	1.133	0.014	23
8	0.171	0.036	0.757	0.028	1.141	0.013	22
9	0.193	0.035	0.774	0.028	1.149	0.013	21
10	0.214	0.035	0.791	0.027	1.156	0.012	20
11	0.235	0.035	0.807	0.027	1.163	0.011	19
12	0.256	0.035	0.823	0.026	1.170	0.011	18
13	0.277	0.035	0.839	0.026	1.177	0.010	17
14	0.298	0.035	0.855	0.026	1.183	0.010	16
15	0.318	0.035	0.870	0.025	1.188	0.009	15
16	0.339	0.035	0.885	0.025	1.194	0.008	14
17	0.360	0.034	0.900	0.024	1.199	0.008	13
18	0.380	0.034	0.914	0.024	1.203	0.007	12
19	0.401	0.034	0.929	0.023	1.208	0.007	11
20	0.421	0.034	0.943	0.023	1.212	0.006	10
21	0.441	0.033	0.956	0.022	1.215	0.005	9
22	0.461	0.033	0.970	0.022	1.218	0.005	8
23	0.481	0.033	0.983	0.021	1.221	0.004	7
24	0.500	0.033	0.995	0.021	1.224	0.004	6
25	0.520	0.032	1.008	0.020	1.226	0.003	5
26	0.539	0.032	1.020	0.020	1.227	0.002	4
27	0.559	0.032	1.032	0.019	1.229	0.002	3
28	0.578	0.032	1.043	0.019	1.230	0.001	2
29	0.597	0.031	1.055	0.018	1 230	0.000	1
30	0.615		1.066		1.230		0
Signs.	V. XI.	− +	IV. X.	− +	III IX.	− +	Signs.

*** See the Note to Table E.

Table M. (No. 2.)—For the Nutation of a Star in North Polar Distance, and Right Ascension in Time.

Argument.—Right Ascension of Star — Long. Moon's Node.

The sum of the two Equations found in these Tables, is the Nutation of a Star in North Polar Distance.

For the Nutation in Right Ascension, call the Right Ascension of the Star plus 3 Signs if the Declination is North; and minus 3 Signs if the Declination is South: the sum of the two Equations so found, multiplied by $\frac{1}{15}$th of the natural Tangent of the Declination of the Star, produces the Nutation in Right Ascension in Time.

Signs.	O. VI.	− +	I. VII.	− +	II. VIII.	− +	Signs.
D	"	Dif. 100′	"	Dif. 100′	"	Dif. 100′	D
0	0.000	0.245	4.200	0.211	7.274	0.120	30
1	0.147	0.244	4.326	0.208	7.346	0.117	29
2	0.293	0.244	4.451	0.206	7.416	0.113	28
3	0.440	0.244	4.575	0.204	7.484	0.109	27
4	0.586	0.244	4.697	0.201	7.550	0.105	26
5	0.732	0.243	4.818	0.199	7.613	0.101	25
6	0.878	0.243	4.937	0.197	7.673	0.098	24
7	1.024	0.242	5.055	0.194	7.732	0.094	23
8	1.169	0.242	5.171	0.191	7.788	0.090	22
9	1.314	0.241	5.286	0.189	7.842	0.086	21
10	1.459	0.240	5.399	0.186	7.893	0.082	20
11	1.603	0.240	5.511	0.183	7.942	0.078	19
12	1.746	0.239	5.620	0.180	7.989	0.073	18
13	1.890	0.238	5.729	0.177	8.033	0.070	17
14	2.032	0.237	5.835	0.174	8.074	0.065	16
15	2.174	0.236	5.939	0.171	8.113	0.061	15
16	2.315	0.234	6.042	0.168	8.150	0.057	14
17	2.456	0.233	6.143	0.165	8.184	0.053	13
18	2.596	0.232	6.242	0.162	8.216	0.049	12
19	2.735	0.230	6.339	0.159	8.245	0.044	11
20	2.873	0.229	6.435	0.156	8.272	0.040	10
21	3.010	0.227	6.528	0.152	8.296	0.036	9
22	3.147	0.226	6.619	0.149	8.318	0.032	8
23	3.282	0.224	6.708	0.145	8.337	0.028	7
24	3.416	0.222	6.795	0.142	8.354	0.024	6
25	3.550	0.221	6.881	0.138	8.368	0.019	5
26	3.682	0.219	6.964	0.135	8.379	0.015	4
27	3.813	0.217	7.045	0.131	8.388	0.011	3
28	3.943	0.215	7.123	0.128	8.395	0.007	2
29	4.072	0.213	7.200	0.124	8.398	0.003	1
30	4.200		7.274		8.400		0
Signs.	V. XI.	− +	IV. X.	− +	III. IX.	− +	Signs.

*** See the Note to Table E.

Table K.—For the Annual Precession of a Star in North Polar Distance.

Argument.—Right Ascension of the Star.

Signs.	O. VI.	− +	I. VII.	− +	II. VIII.	− +	Signs.
D		Dif. 100′	″	Dif. 100′	″	Dif. 100′	D.
0	20.0095	0.0052	17.3287	0.2953	10.0048	0.5067	30
1	20.0064	0.0152	17.1515	0.3042	9.7008	0.5115	29
2	19.9973	0.0253	16.9690	0.3127	9.3939	0.5163	28
3	19.9821	0.0355	16.7814	0.3213	9.0841	0.5208	27
4	19.9608	0.0458	16.5886	0.3297	8.7716	0.5253	26
5	19.9333	0.0557	16.3908	0.3380	8.4564	0.5297	25
6	19 8999	0.0660	16.1880	0.3462	8.1386	0.5338	24
7	19.8603	0.0758	15.9803	0.3543	7.8183	0.5377	23
8	19.8148	0.0862	15.7677	0.3623	7.4957	0.5415	22
9	19.7631	0.0960	15.5503	0.3702	7.1708	0.5452	21
10	19.7055	0.1060	15.3282	0.3780	6.8437	0.5487	20
11	19.6419	0.1162	15.1014	0.3858	6.5145	0.5520	19
12	19.5722	0.1258	14.8699	0.3932	6.1833	0.5552	18
13	19.4967	0.1360	14.6340	0.4007	5.8502	0.5580	17
14	19.4151	0.1487	14.3936	0.4080	5.5154	0.5610	16
15	19.3277	0.1555	14.1488	0.4150	5.1788	0.5635	15
16	19.2344	0·1653	13.8998	0.4223	4.8407	0.5658	14
17	19.1352	0.1750	13.6464	0.4290	4.5012	0.5683	13
18	19.0302	0.1847	13.3890	0.4360	4.1602	0.5703	12
19	18.9194	0.1943	13.1274	0.4425	3.8180	0.5723	11
20	18.8028	0.2038	12.8619	0.4492	3.4746	0.5740	10
21	18.6805	0.2133	12.5924	0.4555	3.1302	0.5757	9
22	18.5525	0.2228	12.3191	0.4618	2.7848	0.5772	8
23	18.4188	0.2320	12.0420	0.4678	2.4385	0.5782	7
24	18.2796	0.2413	11.7613	0.4738	2.0916	0.5795	6
25	18·1348	0.2507	11.4770	0.4797	1.7439	0.5802	5
26	17.9844	0.2597	11.1892	0.4853	1.3958	0.5810	4
27	17.8286	0.2688	10.8980	0.4910	1.0472	0.5815	3
28	17.6673	0.2777	10.6034	0.4963	0.6983	0.5818	2
29	17.5007	0.2867	10.3056	0.5013	0.3492	0.5820	1
30	17.3287		10.0048		0.0000		0
Signs.	V. XI.	+ −	IV. X.	+ −	III. IX.	+ −	Signs.

XVII. *Observation of the Solar Eclipse which took place on Sept. 7, 1820, at Naples. Communicated in a Letter from M. Piazzi to the Foreign Secretary.*

Read November 9, 1821.

THE eclipse of the 7th of September 1820 was observed at the Royal Observatory of Naples at Capodimonti, by CARLO BRIOSCHI, with the telescope annexed to the repeater of REICHENBACH, placed in the western turret, about 4 feet in focus and 36 lines aperture.

	Mean time.
	h ′ ″
Beginning of the eclipse (uncertain about ½ second)	1 58 39,0
A very feeble thread of light begins to appear between the lucid horns, which approach rapidly	3 23 23−
The lucid horns unite and form the annulus	3 23 27,0
The annulus breaks, and a thread of feeblest light, similar to that above mentioned, remains, which gradually diminishes in breadth	3 27 20,7
The abovementioned thread of light disappears	3 27 39±
End of the eclipse (uncertain about 1 second, the limb of the sun fluctuating greatly)	4 43 25,0

ERRATUM.

Page 11, lines 18 and 19, *for* stops *read* steps.

PRESENTS

RECEIVED BY

THE ASTRONOMICAL SOCIETY OF LONDON.

1820.	Presents.	Donors.
March 10.	Tables for determining the apparent time, from observed altitudes, of the sun or a star. Printed at the head quarters of the army in France. 1818. 8vo.	Major General Sir T. Brisbane.
	General Table of Equations of equal Altitude. No place or date. 8vo.	———
	Nouvelles Tables de Jupiter et Saturne. Par M. Bouvard. Paris 1808. 4to.	C. Stokes, Esq.
	A Philosophical and Mathematical Dictionary. By C. Hutton, LL.D. London 1815. 2 vols. 4to.	Dr. Hutton.
	Tracts on Mathematical and Philosophical Subjects. By C. Hutton, LL.D. London 1812. 3 vols. 8vo.	———
	Astronomical Lectures read in the Astronomical School of the University of Oxford. By John Keill, M.D. London 1748, 4th edition, 1 vol. 8vo.	Major Colby.
	Petri Gassendi Institutio astronomica, juxta hypotheses tam veterum quam recentiorum. London 1683, third edition, 1 vol. 8vo.	———
	Tractatus primus instrumentorum mechanicorum Levini Hulsii. Francfort 1605. 1 vol. 4to.	———
	Galilæi Galilæi Lyncæi, academiæ Pisanæ mathematici, systema cosmicum. Lugduni 1641. 1 vol. 4to.	———
	Description et usage du cercle de réflection, avec différentes méthodes pour calculer les observations nautiques. Par le Chevalier de Borda. Paris 1787. 1 vol. 4to.	———

1820.	Presents.	Donors.
March 10.	Recherches expérimentales sur les mouvemens des molécules de la lumière, autour de leur centre de gravité. Par M. Biot. Paris 1814. 1 vol. 4to.	Major Colby.
	Astronomy, in five books. By Roger Long, D.D. Cambridge 1742. 2 vols. 4to.	———
	Ephemerides des mouvemens célestes	———
	1715 à 1734 par Desplaces. Paris 1716 4to.	———
	1745 — 1755 par Lacaille. Paris 1744 4to.	———
	1755 — 1765 par Lacaille. Paris 1755 4to.	———
	1765 — 1775 par Lacaille. Paris 1763 4to.	———
	1775 — 1784 par Lalande. Paris 1774 4to.	———
	Récueil d'Observations faites en plusieurs voyages, par ordre de sa Majesté, pour perfectionner l'astronomie et la geographie. Par Messieurs de l'Academie Royale des Sciences. Paris 1693. 1 vol. folio.	F. Baily, Esq.
	Observationes Astronomicæ, annis 1781, 1782, 1783, institutæ in observatorio regio Hauniensi, et cum tabulis astronomicis comparatæ. T. Bugge. Hauniæ 1784. 1 vol. 4to.	———
	Præcipuarum stellarum inerrantium positiones mediæ ineunte seculo XIX, ex observationibus habitis in specula Panormitana, ab anno 1792 ad annum 1802. Auctore Josepho Piazzi. Panormi 1803. 1 vol. folio.	———
	Della specola astronomica de Regj studj di Palermo, di Giuseppe Piazzi. Libri Quattro. Palermo 1792. 1 vol. folio.	———
	Della specola astronomica de Regj studj di Palermo, di Giuseppe Piazzi. Libri Quinto. Palermo 1794. 1 vol. folio.	———
	Del Reale osservatorio di Palermo. Libri Sesto. Palermo 1806. 1 vol. folio.	———
	Præcipuarum stellarum inerrantium positiones mediæ ineunte seculo XIX, ex observationibus habitis in specula Panormitana, ab anno 1792 ad annum 1813. Auctore Josepho Piazzi. Panormi 1814. 1 vol. 4to.	———
	Almagestum Novum, astronomiam veterem novamque complectens. Auctore P. J. B. Ricciolo. Bononiæ 1653. 2 vols. folio.	———

1820.	Presents.	Donors.
Nov. 10.	Astronomical observations made at the Royal Observatory of Greenwich, in the years 1811 to 1816, both inclusive. By John Pond, Esq. Astronomer Royal. London 1815-1818. 2 vols. folio.	John Pond, Esq.
	Discours sur Nicolas Kopernik. Par Jean Sniadecki. Imprimé à Varsovie en. 1803 et 1818. Reimprimé à Paris 1820. 1 vol. 8vo.	Dr. Slawinski.
	A bronze medal, bearing the head of Copernicus.	———
Dec. 8.	A collection of examples of the application of the differential and integral calculus. By George Peacock, Esq. Cambridge 1820. 8vo.	G. Peacock, Esq.
	A collection of examples of the applications of the calculus of finite differences. By J. F. W. Herschel, Esq. Cambridge 1820. 8vo.	J. F. W. Herschel, Esq.
	Examples of the solutions of functional equations. By Charles Babbage, Esq. Cambridge 1820. 8vo.	C. Babbage, Esq.
	Astronomical observations. By S. Groombridge, Esq. Read before the Royal Society, June 29, 1820. 4to.	S. Groombridge, Esq.
1821.	Mémoires de l'Académie des Sciences, Arts et Belles Lettres de Dijon. Dijon 1820. 1 vol. 8vo.	The Academy of Dijon.
Jan. 12.	A method of computing astronomical refractions for objects near the horizon. By the Rev. J. Brinkley, D.D. Dublin 1820. 4to.	Rev. Dr. Brinkley.
	A method of correcting the approximate elements of the orbit of a comet, and the application of the same to observations made at the observatory of Trinity College Dublin on the comet of July 1819. By the Rev. J. Brinkley, D.D. Dublin 1820. 4to.	———
	Elementi di Trigonometria Sferoidica. Di Barnaba Oriani. Bologna 1806. 4to.	M. Oriani.
	Observations sur l'écrit de M. Laplace, lû le 29 Mars 1820 au bureau des longitudes, intitulé "Sur le perfectionnement de la theorie et des tables lunaires." Par MM. Carlini et Plana. Genev. Août 1820. 1 vol. 8vo.	MM. Carlini et Plana.
March 9.	On the lines that divide each semidiurnal arc into six equal parts. By W. A. Cadell, Esq. Edinburg 1816. 4to.	W. A. Cadell, Esq.

1821.	Presents.	Donors.
March 9.	On the reduction of certain classes of functional equations to equations of finite differences. By J. F. W. Herschel, Esq. Cambridge 1820. 4to.	J. F. W. Herschel, Esq.
	Supplementary table to that usually given of equations to equal altitudes. By Maj. Gen. Sir T. Brisbane.	Major General Sir T. Brisbane.
	Table to determine accurately the time of high water at any given port, dependent on the accelerated or retarded action of the moon on the tides, &c. By Major General Sir T. Brisbane.	———
	Mémoire sur la libration de la lune. Par M. Nicollet. Paris. 1 vol. 8vo.	M. Nicollet.
	Addition au mémoire sur la libration de la lune. Par M. Nicollet. Paris. 1 vol. 8vo.	———
	Nieueve Verhandelingen van het Zeeuwsch Genootschop der Wetenschappen. J. P. Bourje. Middelburg 1819. 8vo.	Professor Moll.
	Johannis Frederici Ludovici Schröder, Oratio de majoribus corporum cœlestium permutationibus, &c. Trajecti ad Rhenum 1816. 8vo.	———
	Gerardi Moll, Oratio de vitanda, in astronomico studio, fingendi temeritate, et cœli observatione quam diligentissime instituenda. Trajecti ad Rhenum 1819. 8vo.	———
	Copy of a portrait of Sir Isaac Newton, from the drawing in the possession of Dr. Hutton.	Dr. Kelly.
April 13.	Ephemerides des mouvemens célestes 1793 à 1800. Par J. Lalande, vol. 9. Paris 1792. 1 vol. 4to.	F. Baily, Esq.
	Astronomical Observations made at the Royal Observatory at Greenwich in the years 1799 and 1800. By the Rev. Nevil Maskelyne, D.D. London 1800 and 1801. 2 parts. folio.	Major Colby.
	On Pendulums, vibrating between cheeks. By B. Gompertz, Esq. London 1817. 1 vol. 8vo.	B. Gompertz, Esq.
May 11.	Method of determining the latitude, by a sextant or circle, with simplicity and accuracy, from circum-meridional observations taken near noon. By Maj. Gen. Sir T. Brisbane. Edinburgh 1821. 4to.	Major General Sir T. Brisbane.
June 8.	Transactions of the Cambridge Philosophical Society, vol. 1. part 1. Cambridge 1821. 4to.	Camb. Phil. Soc.

1821.	Presents.	Donors.
June 8.	Algebra, with Arithmetic and Mensuration, from the Sanscrit of Brahmegupta and Bháscara. Translated by H. T. Colebrooke. London 1817. 1 vol. 4to.	H. T. Colebrooke, Esq.
	Journal of a voyage for the discovery of a North-West passage from the Atlantic to the Pacific, performed in the years 1819 and 1820, in his majesty's ships Hecla and Griper, under the orders of Capt. W. E. Parry. London 1821. 4to.	Captain Parry, R.N.
Nov. 9.	Astronomische Beobachtungen auf der Königlichen Universitats—Sternwarte in Königsberg, von F. W. Bessel. From Nov. 12, 1813 to Dec. 31, 1818. Königsberg 1815, &c. folio.	M. Bessel.
	Account of the life and writings of Robert Simson, M.D. By the Rev. William Trail, LL.D. Bath 1812. 4to.	Dr. Hutton.
	Asiatick Researches, vol. 13. Calcutta 1820. 4to	Asiatic Society.
	Theoretische und Practische Astronomie, von J. J. Littrow. Wien 1821. 2 vols. 8vo.	M. Littrow.
	Ueber den erweiterten Gebrauch der Multiplikations Kreise von J. Littrow. Prag. 1820. 8vo.	———
	Astronomische Hülftafeln für 1821, herausgegeben von H. C. Schumacher. Copenhagen (no date). 8vo.	M. Schumacher.
	Ragguaglio del Reale Osservatorio di Napoli, eretto sulla Collina di Capodimonte. Napoli 1821. 4to.	M. Piazzi.
	Map of the Triangulation of the Canton of Berne. By Professor Treschel 1811-18.	M. Treschel.
	Three engravings of the sphere and hieroglyphics in the temple of Tentyra.	Baron Matolaij.
	Memoir relative to the annular eclipse of the sun, which will happen on September 7, 1820. By Francis Baily. London 1818. 8vo.	F. Baily, Esq.
	Memoir on a new and certain method of ascertaining the figure of the earth by means of occultations of the fixed stars. By A. Cagnoli. With notes and an appendix by Francis Baily. London 1819. 8vo.	———
Dec. 14.	Catalogue of stars taken from Mr. Flamsteed's observations, contained in the second volume of the *Historia Cœlestis*,	Major Colby.

1821.	Presents.	Donors.
Dec. 14.	and not inserted in the British Catalogue. With an appendix, to point out every observation in that volume belonging to the stars of the British catalogue. To which is added, a collection of *Errata* that should be noticed in the same volume, by Caroline Herschel. With introductory and explanatory remarks to each of them, by William Herschel, LL.D. F.R.S. Published by order and at the expense of the Royal Society. London 1798. folio.	
	Laws of the Astronomical Society of Edinburgh. Edinburgh 1815. 8vo.	Major Colby.
	Report of the nature and progress of the Glasgow Astronomical Society. Drawn up by order of the Directors. Glasgow 1809. 8vo.	———
	Astronomische Nachrichten. No. 1. Altona, Sept. 1821. 4to.	M. Schumacher.
	Ephemeris of the distances of the four planets, Venus, Mars, Jupiter and Saturn from the Moon's centre for 1823: together with their places for every day in the years 1822 and 1823. To which are annexed Tables for finding the latitude by the Pole-star for 1823. Calculated under the direction of H. C. Schumacher. Copenhagen 1821. 4to.	———
	Two tables, exhibiting the moon's equatorial semidiameter in time, or the interval of passage over the meridian from the eastern or western limb, to the moon's centre. By William Lambert. *Manuscript.*	W. Lambert, Esq.
1822. Jan. 11.	Introduction to a synoptical table of English and French lineal measures. 4to.	H. Goodwin, Esq.
	Astronomical tables and remarks for the year 1822. By Francis Baily. London 1822. 8vo.	F. Baily, Esq.

A LIST OF THE MEMBERS

OF

THE ASTRONOMICAL SOCIETY OF LONDON

On February 8, 1822.

Those members, to whose names a * is prefixed, have compounded for their annual contributions.

William	Allen, Esq. F.R.S. *Plough Court Lombard Street*
John Wilson	Anderson, M.D. *Glasgow*
Charles	Babbage, Esq. M.A. F.R.S. L. & E. No. 5 *Devonshire Street Portland Place*
* Arthur	Baily, Esq. *Gray's Inn*
* Francis	Baily, Esq. F.R.S. & L.S. *Gray's Inn*
Rev. Lawr. Palk	Baker, B.D. Fellow of St. John's College *Cambridge*
John	Barton, Esq. Deputy Comp. of His Majesty's Mint, *Tower Hill*
Captain F.	Beaufort, of the Royal Navy, F.R.S. *Grove*, *Epping*
Colonel Mark	Beaufoy, F.R.S. & L.S. *Bushey-heath near Stanmore*
Rev. Thomas	Belgrave, M.A. *North Kilworth*, *Leicestershire*
Richard	Best, Esq. *Greenwich*
George	Birkbeck, M.D. No. 25 *Cateaton Street*
Capt. Benjamin	Blake, *Bengal*
Major Thomas	Blanchard, of the Royal Engineers, No. 19 *New Ormond Street*
Rev. Miles	Bland, M.A. F.R.S. *St. John's College Cambridge*
W. H.	Booth, Esq. *Lincoln's Inn*
John	Bostock, M.D. F.R.S. *Upper Bedford Place*
* Timothy	Bramah, Esq. *Pimlico*

David	Brewster, LL.D. F.R.S. L. & E. *Edinburgh*
Rev. John	Brinkley, D.D. F.R.S. M.R.I.A. and Andrews Professor of Astronomy in the University of Dublin. *Dublin*
Maj. Gen. Sir Thomas	Brisbane, K.C.B. F.R.S. L. & E. *Brisbane near Greenock*
Edward F.	Bromhead, Esq. M.A. F.R.S. *Thurlby, Lincolnshire*
Charles F.	Bromhead, Esq. M.A. *Trinity College Cambridge*
George H.	Browne, Esq. *King Street Covent Garden*
William	Bullock, Esq. *Prittlewell, Essex*
* Rev. Chas. Parr	Burney, M.A. F.R.S. *Greenwich*
James	Bush, Esq. *Bishop's Teignton*
* Rev. George	Butler, D.D. F.R.S. & S.A. *Harrow*
James S.	Butt, Esq. *Pulteney Street Bath*
William	Cary, Esq. *Strand*
Rev. Thomas	Catton, B.D. F.R.S. *St. John's College Cambridge*
John	Clark, Esq. *Chatham Place Blackfriars*
John Alden	Clarke, Esq. *Battersea Rise*
William	Coffin, Esq. *Llandaff*
* Major Thomas	Colby, of the Royal Engineers, LL.D. F.R.S. L. & E. *Tower*
* Henry T.	Colebrooke, Esq. F.R.S. & L.S. *Albany, Piccadilly*
Richard	Comfield, Esq. *Northampton*
Samuel	Cooper, Esq. *Tranby, Yorkshire*
Capt. A. Lowry	Corry, of the Royal Navy, *Enniskillen, Ireland*
John	Cross, Esq. *Observatory, Glasgow*
Edmund Robert	Daniell, Esq. *Lincoln's Inn*
Rev. William	Dealtry, M.A. F.R.S. *Clapham*
* George	Dollond, Esq. F.R.S. *St. Paul's Churchyard*
William	Domville, Esq. *Lincoln's Inn*
Bryan	Donkin, Esq. *Grange, Bermondsey*
Daniel	Dowling, Esq. *Highgate*
Sir Robert L.	Dundas, No. 19 *Arlington Street Piccadilly*
Rev. J. C.	Ebden, M.A. *Trinity College Cambridge*
James	Edwards, Esq. *Putney*
Thomas F.	Ellis, Esq. M.A. No. 15 *Bedford Place*
Sir H. C.	Englefield, Bart. F.R.S. L. & E. F.S.A. & L.S. *Tilney Street*

Rev. Lewis	Evans, *Froxfield, Wilts*
Rev. Fearon	Fallows, M.A. F.R.S. *St. John's College Cambridge, Astronomer at the Cape of Good Hope*
Michael	Faraday, Esq. *Royal Institution*
John	Fardell, Esq. F.S.A. *Lincoln*
Rev. William	Farish, B.D. F.R.S. Professor of Experimental Philosophy in the University of Cambridge
Thomas	Forster, Esq. *East Grinstead, Surry*
John	Fuller, Esq. *Devonshire Place*
* Davies	Gilbert, Esq. Treas. & V.P.R.S. F.L.S. *East Bourne, Sussex*
Isaac Lyon	Goldsmid, Esq. *Clapham*
Benjamin	Gompertz, Esq. F.R.S. *Kennington Terrace, Vauxhall*
Right Honourable	Alexander, Duke of Gordon, F.R.S.
Olinthus G.	Gregory, LL.D. Professor of Mathematics, *Royal Military Academy, Woolwich*
His Excellency	Alexis Greig, Vice Admiral of the Imperial Russian Navy, *Odessa*
Rev. James	Grooby, M.A. *Cirencester, Gloucestershire*
* Stephen	Groombridge, Esq. F.R.S. S.R.A.Nap. *Blackheath*
Rev. Richard	Gwatkin, M.A. *St. John's College Cambridge*
Robert	Haldane, LL.D. Professor of Mathematics, *St. Andrews*
Capt. Basil	Hall, Royal Navy, F.R.S. L. & E.
Capt. Fran. Alex.	Halliday, of the Royal Navy, *Ham, Surry*
Rev. H. C.	Hamilton, M.A. *Trinity College Cambridge*
William	Handley, Esq. No. 12 *Winchester Place, Pentonville*
George	Harvey, Esq. *Plymouth*
* Sir William	Herschel, LL.D. F.R.S. *Slough*
J. F. W.	Herschel, Esq. M.A. F.R.S. L. & E. *Slough*
Rev. John P.	Higman, M.A. F.R.S. *Trinity College Cambridge*
Chamberlain	Hinchliff, Esq. *Park Street Southwark*
John	Hinde, Esq. M.A. Fellow of Sidney College Cambridge, and Mathematical Lecturer
Sir Benjamin	Hobhouse, Bart. F.R.S. *Whitton, Middlesex*
Charles	Holford, Esq. *Hampstead*
Rev. Thos. W.	Hornbuckle, *St. John's College Cambridge*

James	Horsburgh, Esq. F.R.S. *Herne Hill, Dulwich*
James	Hoy, Esq. *Gordon Castle*
Thomas	Jackson, LL.D. F.R.S. E. Professor of Natural Philosophy, *St. Andrews*
James	Jardine, Esq. F.R.S. E. *Edinburgh*
Rev. Thomas	Jephson, *St. John's College Cambridge*
Sir Alexander	Johnston, F.R.S. No. 19 *Great Cumberland Place*
Thomas	Jones, Esq. *Charing Cross*
Patrick	Kelly, LL.D. *Finsbury Square*
Joshua	King, Esq. *Queen's College Cambridge*
Lieut. Gen. A.	Kyd, F.R.S. No. 22 *Albemarle Street*
Thomas	Leybourn, Esq. *Royal Military College, Sandhurst*
Right Honourable George Earl of Macclesfield, F.R.S.	
* Rev. John	Maddy, D.D. F.R.S. & S.A. *Argyll Street*
Capt. Fred.	Marryatt, of the Royal Navy, F.R.S. & L.S. *Wimbledon*
J. S.	Memes, Esq. *Moulsey Park, Surry*
John	Millington, Esq. Prof. of Mechanics at the Royal Institution, No. 6 *Leigh Street, Burton Crescent*
William P.	Moffatt, Esq. *East Sheen, Surry*
Basil	Montagu, Esq. *Lincoln's Inn*
* Daniel	Moore, Esq. F.R.S. S.A. & L.S. *Lincoln's Inn*
W. M.	Mosely, Esq. *Winterdyne House near Bewdley, Worcestershire*
William	Mylne, Esq. *New River Head*
Sir Edward William Campbell Rich Owen, K.C.B. *Deal*	
Capt. W. F. W.	Owen, of the Royal Navy, *Deal*
Samuel	Parkes, Esq. F.L.S. Acad. Scien. Nat. Philadel. & Soc. Phil. Paris. Cor. Soc. Imp. Hist. Nat. Mosc. Socius, No. 30 *Mecklenburg Square*
Joseph	Parkinson, Esq. *Newman Street Oxford Road*
Capt. Edward	Parry, F.R.S. of the Royal Navy, *Bath*
Stephen	Pattisal, Esq. *Change Alley Cornhill*
William	Paul, Esq. Professor of Natural Philosophy, *Aberdeen*
Samuel	Peach, Esq. *Portland Place*
Rev. George	Peacock, M.A. F.R.S. *Trinity College Cambridge*

* Rev. William	Pearson, LL.D. F.R.S. *East Sheen, Surry*
H. F.	Pelerin, Esq. No. 10 *Canonbury Lane, Islington*
William Hasle.	Pepys, Esq. F.R.S. *Poultry*
Charles	Perkins, Esq. No. 67 *Mark Lane*
* Frederick	Perkins, Esq. *Lee, Kent*
* Henry	Perkins, Esq. *Springfield, Surry*
* Lewis Hayes	Pettit, Esq. F.R.S. *Lincoln's Inn*
* Rev. J. H.	Pinckney, D.D. *East Sheen, Surry*
John	Pond, Esq. Astronomer Royal, F.R.S. *Greenwich*
Rev. H.	Robinson, M.A. *St. John's College Cambridge*
Captain John	Ross, of the Royal Navy, *Stranraer*
Maj. Gen. John	Rowley, of the Royal Engineers, F.R.S. No. 90 *Pall Mall*
Charles	Rumker, Esq. *New South Wales*
Edward	Ryan, Esq. No. 3 *Hare Court, Temple*
John	Sanders, Esq. *Reigate, Surry*
Samuel	Scott, Esq. *Lower Grosvenor Street*
Capt. Wm. Hen.	Shireff, of the Royal Navy, No. 5 *Cumberland St. Portman Sq.*
* Capt. Wm. Hen.	Smyth, of the Royal Navy
* James	South, Esq. F.R.S. and L.S. No. 11 *Blackman Street Southwark*
Robert	Stevenson, Esq. F.R.S. E. *Edinburgh*
Charles	Stokes, Esq. F.R.S. S.A. & L.S. *Gray's Inn*
Edward	Strachy, Esq. F.R.S. 31 *Fitzroy Square*
William	Strutt, Esq. F.R.S. *Derby*
* Thomas	Telford, Esq. F.R.S.E. *Charing Cross*
Alexander	Tilloch, Esq. LL.D. M.R.I.A. *Picket Place Temple Bar*
John	Tomkyns, Esq. M.A. & Fellow of King's College Cambridge, *Inner Temple*
* Edward	Troughton, Esq. F.R.S. L. & E. *Fleet Street*
Henry	Tucker, Esq. No. 34 *Baker Street*
Charles	Tulley, Esq. *Islington*
Andrew	Ure, M.D. Professor of Chemistry and Natural Philosophy in the Andersonian Institution, *Glasgow*
* Joseph F.	Vandercom, Esq. *Bush Lane Cannon Street*
William	Vaughan, Esq. F.R.S. *Fenchurch Street*

Edward	Walker, Esq. *Nelson Square*
William	Wallace, Esq. Professor of Mathematics, *Edinburgh*
Rev. Michael	Ward, LL.B. *Tamworth*, *Staffordshire*
Sir William	Watson, F.R.S. *Great Pulteney Street Bath*
Richard	Watt, Esq. *Bishop Burton*, *Yorkshire*
Thackary	Wetherall, Esq. of the Royal Navy
William	Whewell, Esq. M.A. F.R.S. *Trinity College Cambridge*
Joseph	Whidby, Esq. F.R.S & L.S. *Plymouth*
Rev. J. W.	Whittaker, D.D. *St. John's College Cambridge*
Rev. G. B.	Wildig, M.A. *Liverpool*
William	Williams, Esq. No. 3 *Gray's Inn Place*
Daniel	Wilson, Esq. No. 6 *Mount Place Mile End*
Capt. John	Wilson, *Grove*, *Epping*
J. W.	Woollgar, Esq. *Lewes*, *Sussex*
John	Wrottesley, Esq. No. 10 *New Square Lincoln's Inn*
Sir Samuel	Young, Bart. F.R.S. *Formosa near Maidenhead*, *Berks*
Lieut. George	Young, R.N. *Formosa*

ASSOCIATES.

Français	Arago,	*Paris*
F. W.	Bessel,	*Königsberg*
Jean Bapt.	Biot,	*Paris*
John Elert	Bode,	*Berlin*
Alexis	Bouvard,	*Paris*
John B. J.	Delambre,	*Paris*
Charles F.	Gauss,	*Göttingen*
Alfred	Gautier,	*Geneva*
C. Louis	Harding,	*Göttingen*
Le Marquis	de Laplace,	*Paris*
Professor	Littrow,	*Vienna*
Gerard	Moll,	*Leyden*
William	Olbers,	*Bremen*
Barnabas	Oriani,	*Pavia*
Giuseppe	Piazzi	*Palermo*
H.	Schumacher,	*Copenhagen*
Peter	Slawinski,	*Wilna*
Frederick	Treschel,	*Berne*
J. D.	Vallot	*Dijon*
Henry John	Walbeck,	*Abö*
Baron	de Zach,	*Genoa*

LIST OF THE OFFICERS

OF

THE ASTRONOMICAL SOCIETY OF LONDON,

FOR THE YEAR 1822.

The Names, under each Office, are arranged alphabetically.

President.

Sir William Herschel, LL.D. F.R.S.

Vice-Presidents.

Major T. Colby, *Roy. Eng.* LL.D. F.R.S. L. &. E.
Sir H. C. Englefield, Bart. F.R.S. L. & E. F.S.A. & L.S.
Davies Gilbert, Esq. V.P.R.S. & F.L.S.
D. Moore, Esq. F.R.S. S.A. & L.S.

Treasurer.

Rev. W. Pearson, LL.D. F.R.S.

Secretaries.

C. Babbage, Esq. M.A. F.R.S. L. & E.
F. Baily, Esq. F.R.S. & L.S.
J. F. W. Herschel, Esq. M.A. F.R.S. L. & E. *(Foreign.)*

Council.

G. Birkbeck, M.D.
B. Gompertz, Esq. F.R.S.
O. G. Gregory, LL.D. *Professor of Mathematics.*
S. Groombridge, Esq. F.R.S.
J. Horsburgh, Esq. F.R.S.
Major General John Rowley, *Roy. Eng.* F.R.S.
J. South, Esq. F.R.S. & L.S.
E. Troughton, Esq. F.R.S. L. & E.

Trustees.

A. Baily, Esq.
D. Moore, Esq. F.R.S. S.A. & L.S.
C. Stokes, Esq. F.R.S. S.A. & L.S.
The Treasurer for the time being.

PRINTED BY R. AND A. TAYLOR, SHOE-LANE.

MEMOIRS

OF

THE ASTRONOMICAL SOCIETY

OF

LONDON.

VOL. I. PART II.

LONDON:
PUBLISHED BY BALDWIN, CRADOCK AND JOY,
PATERNOSTER-ROW.

1825.

PRINTED BY RICHARD TAYLOR,
SHOE-LANE, LONDON.

CONTENTS.

CONTENTS.

ERRATA.

Page 43, lines 18 and 19, *for* stops *read* steps.

185, *for* $I = G. \cos (\odot \hookleftarrow a)$ *read* $I = \text{Max.} \cos (\odot \hookleftarrow a)$.

265, line 5, *for* $\left(\dfrac{m\frac{2}{3}\sin^2 C - M\frac{2}{3}\sin^2 L}{M\frac{2}{3}\cos^2 L - m\frac{2}{3}\cos^2 L}\right)^{\frac{1}{2}}$ *read* $\left(\dfrac{m^{\frac{2}{3}}\sin^2 l - M^{\frac{2}{3}}\sin^2 L}{M^{\frac{2}{3}}\cos^2 L - m^{\frac{2}{3}}\cos^2 l}\right)^{\frac{1}{2}}$

N.B. The different terms of the latter expression are those developed in the calculations, and should be inserted in their places.

265, line 6, *for* axes L and *l*, *read* axes, L and *l*

267, lines 1 and 2, in the two formulæ there given, the sign $\times$ is inadvertently inserted instead of the sign $+$, both in the numerators and denominators. The error also occurs in the same terms inserted in the calculation.

277, line 34, *for* 180, *read* 180°.

279, — 23, — 180, — 180°.

403, — 24, — $(p-p)'$ — $(p-p')$.

416, No. 3, — 43100 — 43119.

— 4, — 44900 — 44881.

— 16, — 61700 — 61632.

433, line 15, — 90 *read* 90°.

— 22, — foot — head.

— 25, — 0 — 0°.

— 32, — 0 — 0°.

476, at the top, *for* $M''. (2\odot + N'')$ *read* $M''. \sin (2\odot + N'')$.

477, ... $M'''. (2☾ + N''')$ — $M'''. \sin (2\odot + N''')$.

P A P E R S.

XVIII. *Observations on the Collimation Adjustment of a Transit Instrument; together with some arguments in favour of certain Circumpolar Stars being added to our standard Catalogue, to facilitate a rigorous and frequent examination of the position of the instrument with regard to the Meridian, and of the altitude of the Pole, relative to the observer's station. By* JAMES SOUTH, *F.R.S. L. & E.*

Read January 11, 1822.

IN Mr. BRANDE's Journal of January last, a paper was published by me, wherein allusion was made to the adjustments of the Transit Instrument, and that of collimation was dismissed in these words: "The usual mode of detecting the error of it, as also of correcting it, is too well known to require comment:" subsequent experience however has shown me that such is not the case; and that, although the usual means have been resorted to, and apparently with satisfactory results, still other and more severe examination, will prove successful in detecting error; and as the least possible must be mischievous, it behoves us to do all in our power to counteract it.

The meridian mark (where there is one) is the object whereby the collimation of the instrument is examined and corrected, and when the central wire upon

reversion of the instrument bisects it, enough, it is usually supposed, has been achieved. Now there are two difficulties here to contend with; the first is, that of the mark being too near, and the other, that of its being too distant: if the former, then a slight lateral motion of the axis will render the experiments fallacious; if the latter, then indeed another difficulty occurs, namely, that of the frequent impossibility of defining it sufficiently, to warrant confidence in the accurate bisection of it: and perhaps to these might be added a third source of error, to be found in the eccentricity of the instrument. Under these circumstances, therefore, it has been proposed to observe sidereal transits with the axis in one position one day, and with it reversed on the day following: this mode however supposes reliance can be placed on the uniformity of the clock's rate; and when it is remembered that the adjustment of collimation is one of the first wanted, after the erection of a transit instrument, such confidence cannot reasonably be entertained. Added to this, also, the error of observation, and the different states of atmosphere at the time such comparative observations are made, go far to diminish confidence in the results: nor must it be forgotten that, in this climate at least, it is no unusual thing for us to be disappointed in being able to see, much more to observe, the wished-for star at the required time; hence also it not unfrequently happens, that the labours of one day are thrown away, through the lack of corresponding observations on the day following; or if not, then indeed a two days' or even a three days' rate of the clock must be confided in, a confidence which indeed few clocks will be found to merit. Hence therefore it becomes desirable to seek for some mode whereby these difficulties may be obviated; and for this purpose I can recommend the following, knowing from experience, that it is adequate to the purpose.

The stars I would select for the occasion, are those not far distant from the pole, whose slow motion over the wires renders them peculiarly fitted for the task; and, to save others the trouble of seeking out such, the table I myself employ is here subjoined.—It is not to be understood as giving accurately the mean places of the stars contained in it: such is not the case, nor would it be for this purpose in any respect useful.

Having therefore adjusted the collimation of the instrument by the land mark, as I consider approximately, I direct it to such an one of the stars as is nearly on the meridian, and the nearer to the pole the better; I note its transit over the 1st, over the 2d, and over the 3d wire. I then reverse the instrument, and note its transit over the 4th and 5th wires: now the three first wires may be represented by the three lines $\overset{A}{|}\ \ \overset{B}{|}\ \ \overset{C}{|}$, and it will be seen that the star has

travelled from A to C, and from B to C. Now on reversing the instrument it will have to return from C to B, and from C to A; and if it does this in the same intervals of time, it is evident that the error of collimation is equal to 0; but if any discordance be observed, an error of collimation exists*, and which, as in other instances, must be corrected one-half by the azimuthal motion of the instrument, and the other by the collimating screws. Now by the time this correction is made, probably another star of the list may present itself, which must be treated in the same manner, till on reversion every discordance between the observation vanishes; then indeed will the instrument be free from this source of error. The star of all others which I prefer is that about 54 minutes from the pole, when it can be observed at a convenient season.

The experienced astronomer need not however be informed, that such a star can only be employed where the instrument is large: accordingly he will find the catalogue afford him such as will be suited to his wants; he will however do well to select such, as are as near to the pole as he can detect the motion of, over the wires of his instrument.

Now by this mode—Any lateral motion of the instrument is rendered unimportant. The eccentricity of it is no longer a source of error. Great confidence in the clock's rate is not required. No corrections for the stars are needed. Many opportunities of examination and correction may be afforded in one evening, and each independent of the preceding. Material alteration in the state of the atmosphere will be usually avoided: And no possibility of sensible error will be left uncorrected.

*Thus in Blackman-street, Jan. 21, 1821, the transits of 81 δ Ursæ Minor over A B and C were observed at $6^h\ 16'\ 18''.5$—$23'\ 8''.0$ and $29'\ 59''.5$, and after reversing over B and A at $36'\ 51''.5$ and $43'\ 41''.0$ respectively.

Now $29'\ 59''.5 - 16'\ 18''.5 = 13'\ 41''.0 =$ A C.
And $43\ \ 41\ .0 - 29\ \ 59\ .9 = 13\ \ 41\ .5 =$ C A.

Dif. $= 0\ \ 0\ .5 =$ Error of Collimation.

Again $29'\ 59''.5 - 23'\ \ 8''.0 = 6'\ 51''.\ 5 =$ B C.
And $36\ \ 51\ .5 - 29\ \ 59\ .5 = 6\ \ 52\ .\ 0 =$ C B.

Dif. $= 0\ \ 0\ .05 =$ Error of Collimation.

* During the seexperiments the instrument is supposed to move in the plane of the meridian.

Catalogue of Circumpolar Stars reduced to Jan. 1st, 1821.

Star's Name.		Mag.	Right Ascension.	N. P. D.	Declination.
6 Ursæ Min.	S. P.	6	0h 14′ 39″	1° 18′ 29″	
5 ——	S. P.	6	0 15 6	2 29 4	
3 ——		6	0 56 5	2 43 39	87° 16′ 21″
44 ——	S. P.	6	3 10 56	5 21 41	
53 ——	S. P.	6	3 39 32	8 58 56	
57 ——	S. P.	6	3 40 39	2 6 48	
58 ——	S. P.	6	3 49 47	9 27 33	
59 ζ ——	S. P.	4	3 50 38	11 39 35	
76 ε ——	S. P.	4	5 4 39	7 41 10	
4 ——		6	5 51 48	0 54 31	89 5 29
81 δ ——	S. P.	3	6 30 4	3 24 54	
82 ——	S. P.	6	6 36 49	3 2 48	
83 ——	S. P.	6	6 47 33	6 58 30	
85 λ ——	S. P.	5	9 8 26	1 26 30	
86 ——	S. P.	6	9 32 14	3 43 8	
6 ——		6	12 14 39	1 18 29	88 41 31
5 ——		6	12 15 6	2 29 4	87 30 56
3 ——	S. P.	6	12 56 5	2 43 39	
44 ——		6	15 10 56	5 21 41	84 38 19
53 ——		6	15 39 32	8 58 56	81 1 4
57 ——		6	15 40 39	2 6 48	87 53 12
58 ——		6	15 49 47	9 27 33	80 32 27
59 ζ ——		4	15 50 38	11 39 35	78 20 25
76 ε ——		4	17 4 39	7 41 10	82 18 50
4 ——	S. P.	6	17 51 48	0 54 31	
81 δ ——		3	18 30 4	3 24 54	86 35 6
82 ——		6	18 36 49	3 2 48	86 57 12
83 ——		6	18 47 33	6 58 30	83 1 30
85 λ ——		5	21 8 26	1 26 30	88 33 30
86 ——		6	21 32 14	3 43 8	86 16 52

On reference to this Catalogue, it will be seen that several of the stars contained in it, might be appropriated to other useful purposes, provided their places were but accurately ascertained. It is almost needless to say, that I refer to the facilities which they would afford the practical astronomer in ascertaining two very important facts; namely, the precise position of his transit instrument with regard to the meridian, and of the pole with regard to his station.

Experience has decided, that of all the modes of placing a transit instrument in the meridian, or of ascertaining its deviation from it, that afforded by the transit of circumpolar stars above and below the pole, is by far the most accurate and independent; indeed, considering this circumstance, it seems not a little singular, whilst a piece of metal with a hole in its centre, perched upon bricks and mortar, should have been regarded almost with religious awe, that a process so perfect in itself should have been so little enforced, either by precept, or example. The cause of this apparent inconsistency may, perhaps, be traced to the circumstance of but few of the stars near to the pole, having as yet had their mean places accurately determined; and if so, surely it is high time that such an obstacle to their employment, should be removed.

On examining the Greenwich Catalogue, it will be found to contain only two stars at all fitted for the task; these are α and β Ursæ Minoris; and perhaps, where the instrument is large and capable of bearing a magnifying power of 3 or 400, the last is too distant from the pole to answer much purpose: indeed, with my own, I think its inferior transit not worth the trouble of taking, except, where others more appropriate cannot be procured. In truth, I consider the accuracy it affords, about equal to that obtained by means of a high and low star. Hence, therefore, the stars applicable to a service so important become reduced to one; and when it is remembered (as has been already stated) that here it is not unusual for us to be baffled in our attempts to observe its consecutive superior and inferior transits not only for days, but even for weeks, we are left to our only alternatives, either of being satisfied with others more distant, or with examination by the transits of high and low stars; but again, unfortunately, the latter of these are in our national Catalogue so few in number, that they sometimes from unfavourableness of weather are not easy to be procured.

Most of our transit instruments are supplied with five vertical wires, which by Mr. Troughton are so placed that a star in the Equator shall pass from

one to another in about 24 or 25 seconds; thus the observer can procure the transits of both limbs of the sun without either hurry or delay: the time however occupied by Polaris in travelling over the wires thus distant, renders it generally inconvenient to attend to its transit, except over the meridian wire; hence it frequently occurs that this transit from some untoward circumstance cannot be procured; either the star becomes invisible just at the time; or is perhaps so ill defined at the moment, that to bisect it, is impossible; or having marched up to the meridian wire very steadily, promising a highly satisfactory observation, lateral refraction, or some other cause, transports it suddenly to the other side. These are obstacles, which, even in the finest weather, will occasionally thwart the observer: as however the star is frequently visible and invisible, well defined and ill defined, steady and unsteady, many times in two or three minutes, it were to be wished that some further attempts should be made to get a satisfactory observation of it. I proceed as follows: the seven feet transit made for me by Mr. TROUGHTON is provided with a moveable wire, which by means of a micrometer screw admits of considerable motion on either side of the central one; this I usually place at a distance from the meridian wire equal (when Polaris is employed) to two minutes of time, having or not observed its passage over this station. I then advance the wire a quantity equal to one minute of time, and another attempt is made to observe it here: the meridian wire next presents itself, and if successful in procuring the transit satisfactorily over this, my business is completed; if not, the moveable wire is again recurred to, till it is no longer wanted. By these means it will be seen that in five minutes I have four chances of getting transits, each nearly, if not quite, equal to the meridian transit. In smaller instruments there may perhaps be some difficulty, and certainly considerable expense, in this micrometrical addition: I would therefore propose, that to these should be added two wires, one on each side of the meridian one, and at such a distance from it that Polaris should travel from either of them to it, in about two or perhaps three minutes; and were they of different fineness from the meridian wire, accidental confusion in observing stars of greater north polar distance would thereby be rendered impossible. It is needless to say, that when the moveable wire is employed, the correction for index error must be accurately ascertained, and its parallelism with the meridian wire will be indispensable.

Now although many transits of Polaris usually lost, might thus be procured, still disappointments are so frequent, that it would be highly desirable to add to our present Catalogue, those circumpolar stars, whose proximity to the pole ren-

ders them peculiarly applicable to a purpose so highly important; and that too in such numbers, as in tolerably good observing weather, would render the probability of procuring daily superior and inferior transits of some one or other of them, almost certain. When, however, this from unavoidable causes should be impracticable, very considerable accuracy would be obtained by comparing the superior transit of one star with the inferior transit of another; or even when only superior or only inferior have been observed, the average results of error afforded thereby will supply very sound materials, for ascertaining the amount of deviation from the meridian, as also data whereby correction may be applied; but for this purpose, the corrections in right ascension of the stars employed will be absolutely indispensable, and the error of the clock must be known to two- or three-tenths of a second. As to the selection of stars most proper for the purpose, what was said relative to it when the collimation was to be scrutinized, equally applies now that the position of the instrument is to be rigorously examined. The nearer to the pole is the star, the more severe will be the test, and the more accurate the result.

But if the transits of circumpolar stars above and below the pole, afford the most accurate information as to the position of the instrument relative to the meridian; it is equally to be acknowledged that their altitudes at the instant of such transit afford us the means of knowing the precise situation of the polar point, far superior to what can be obtained by any other process hitherto devised; and if β Ursæ Minoris was before objected to for ascertaining accuracy of instrumental position, in consequence of its distance from the pole, it will be here objectionable on the same grounds: there it was objected to, because its distance occasioned its too rapid motion over the wires; here because it will leave the star but a low altitude, during its inferior transit; for although perhaps at 36 degrees of altitude, (which it will have in this latitude,) we may consider the laws of refraction pretty well ascertained, still it must be remembered that to the traveller it will oftentimes prove not quite so fortunate, as he will occasionally want it in situations, where, its inferior transit, will plunge it in all the uncertainties of that phenomenon; hence, therefore, to him, it would be highly grateful to have a supply of others, whose proximity to the pole, would effectually preclude such a misfortune; and were also their daily corrections in north polar distance, as well as in right ascension given, it would be highly desirable.

The stars I would select from the Catalogue before alluded to, include all

those of the 3d, 4th and 5th magnitudes, and one of the 6th. We shall then, with the insertion of Polaris, have the following list, all of which, except 4 Ursæ Minoris are, under tolerably favourable circumstances, visible in the day time; and although the same cannot be said of it, still its extreme nearness to the pole perhaps compensates for that deficiency. It will be also some recommendation to the selection, that the periods of its star's transits, will be tolerably well diffused through the 24 hours,—a circumstance which will frequently obviate the necessity, of the observer leaving a comfortable bed, for a comfortless observatory, as is now frequently the case where the pole star at its superior and inferior transit, is regularly observed. Seeing, therefore, these circumstances, it is surely to be wished, that some industrious observer, with instruments adequate to the purpose, would undertake the task of furnishing their places with all possible accuracy: and were my own free from other engagements, I would gladly and heartily undertake it myself.

Star's Name.		Mag.	Right Ascension.	N. P. D.
α Ursæ Minor.		2	0^h 57′ 16″	1° 38′ 46″
59 ζ ——	S. P.	4	3 50 38	11 39 35
76 ε ——	S. P.	4	5 4 39	7 41 10
4 (Bode) ——		6	5 51 48	0 54 31
81 δ ——	S. P.	3	6 30 4	3 24 54
85 λ ——	S. P.	5	9 8 26	1 26 30
α ——	S. P.	2	12 57 16	1 38 46
59 ζ ——		4	15 50 38	11 39 35
76 ε ——		4	17 4 39	7 41 10
4 (Bode) ——	S. P.	6	17 51 48	0 54 31
81 δ ——		3	18 30 4	3 24 54
85 λ ——		5	21 8 26	1 26 30

In my allusions, however, to the non-employment of the superior and inferior transits of circumpolar stars, I would wish it to be understood, that I by no means include, our own great national establishment: here, as the observations will attest, the transit of Polaris is regularly taken below the pole as well as above it. Still, this is by no means general, nor will the principle be followed elsewhere as it ought, till further facilities are afforded. The time, however, is I hope not far distant when every instrument will break

the trammels which have so long disgraced it; and when, instead of noting as at present "Examined the instrument by the meridian mark, and finding it to deviate so and so, corrected it;"—we shall have the more philosophic and more rational memento,—"Examined the position of the instrument, by the superior and inferior transits of so and so, or so and so, and found it rigorously in the plane of the meridian."

J. South.

Blackman-street, Dec. 14, 1821.

In the absence of the necessary corrections in right ascension, of the 6 circumpolar stars more immediately proposed, for ascertaining accuracy in instrumental position, I have been favoured by Mr. Herschel, with the following formula and table, whereby the alteration in the apparent right ascension of either of them may be found, and applied to the preceding observed transit, whether superior or inferior; rendering therefore the immediate employment of them for this purpose, perfectly unobjectionable.

If $\odot$ represent the sun's longitude on any assigned day, and h, i, and N certain constant elements determined for any star by the following equations,

$$h = 0{\cdot}0042028 + 0{\cdot}0018304.\ \sin \mathrm{RA}.\ \tan \mathrm{Decl.}$$

$$\tan \mathrm{N} = \cos.\ \mathrm{obliq.} \times \cot.\ \mathrm{RA}.$$

$$i = -0{\cdot}01176304.\ \frac{\mathrm{Sin.\ RA}}{\cos.\ \mathrm{N}.\ \cos.\ \mathrm{Decl.}}$$

the semidiurnal variation in the apparent place of the star in right ascension will be represented by the formula

$$h + i.\ \cos.\ (\odot + \mathrm{N}).$$

The annexed table exhibits the values of h, i, and N, for each of the six circumpolar stars alluded to in this paper.

Star's Name.	Values of h	Values of i	Values of N
Polaris.	+0·019953	+0·377820	+ 2ˢ 14° 27′
4 Ursæ Minoris.	−0·119333	+0·742150	+11 26 20
59 ζ Urs. Min.	−0·003291	−0·056871	+10 29 51
76 ε Urs. Min.	−0·008966	−0·087551	+ 0 12 44
81 δ Urs. Min	−0·026206	−0·197210	+11 23 6
85 λ Urs. Min.	−0·045299	−0·447220	+10 15 22

Calculating upon these data, the following table has been constructed, in which the value of the expression $h + i.\ \cos.\ (\odot + \mathrm{N}.)$ is given for each of

the above stars for the first day of every month; which will be sufficient for all ordinary purposes, and which an easy interpolation will render applicable to any assigned day, and for any number of years past and to come.

Table of the Semidiurnal Variations of the Apparent Right Ascension of Circumpolar Stars, for the first Day of each Month, in seconds of time.

Date.	Polaris.	4 Urs. Min.	ζ Urs. Min.	ε Urs. Min.	δ Urs. Min.	λ Urs. Min.
January 1st.	+0.396	+0.209	−0.022	+0.026	−0.013	−0.296
February 1st.	+0.358	+0.581	+0.009	+0.063	+0.088	−00 5
March 1st.	+0.237	+0.801	+0.033	+0.078	+0.145	+0.149
April 1st.	+0.049	+0.854	+0.050	+0.071	+0.170	+0.328
May 1st.	−0.139	+0.713	+0.053	+0.043	+0.138	+0.401
June 1st.	−0.289	+0.412	+0.040	+0.002	+0.062	+0.358
July 1st.	−0.355	+0.020	+0.017	−0.041	−0.033	+0.215
August 1st.	−0.328	−0.305	−0.012	−0.077	−0.130	+0.002
September 1st.	−0.208	−0.551	−0.039	−0.095	−0.200	−0.226
October 1st.	−0.031	−0.620	−0.056	−0.091	−0.223	−0.403
November 1st.	+0.167	−0.489	−0.060	−0.064	−0.194	−0.490
December 1st.	+0.323	−0.192	−0.039	−0.022	−0.119	−0.453
January 1st.	+0.396	+0.209	−0.022	+0.026	−0.013	−0.296

XIX. *Tables of the Semidiameter of the Moon in Time, &c.* *By* William Lambert, *Esq.*

Read December 14, 1821.

Sir, City of Washington, October 8, 1821.

I HAVE the honour to transmit, for the acceptance of the useful and learned Society of which you are President, two Tables for ascertaining with sufficient accuracy the moon's semidiameter in time, or the interval of passage from either limb to the centre, in transits over the meridian. The tables of this kind hitherto published not being in my opinion so correct and extensive as they ought to be, the inclosed are submitted as an improvement on any I have yet seen, for the construction of which the rule has been given. This method is, perhaps, inferior to none yet devised for the determination of the longitude with due precision, especially when the distance between two meridians required to be found (considering the form of the earth to be that of a *perfect sphere*) shall have been reduced to the distance according to its *true* or *spheroidal* figure. The ratio of the equatorial diameter to the polar axis, which I have assumed in my calculations to ascertain our longitude from Greenwich and Paris Observatories, being as 320 to 319, I make the reduction in longitude = 2′ 13″.84 and 2′ 17″.91 respectively, admitting the distance to be from the capitol in this city—

$$\left.\begin{array}{lr} \text{To Greenwich} \ldots\ldots\ldots & 76^\circ\ 57'\ 45'' \\ \text{Paris Observatory} \ldots\ldots & 79\ \ 17\ \ 57\ \end{array}\right\}\text{E.}$$

I have the honour to be

Your most obedient Servant,

William Lambert.

To the President of

The Astronomical Society, London.

TABLE I.

Exhibiting the Moon's Equatorial Semidiameter in Time, or the Interval of Passage over the Meridian from the Western or Eastern Limb to the Moon's Centre.

☽'s equat. hor. Semidiam.	Argument—Variation in Right Ascension for 12 Hours.														
	° ′ 5 0	° ′ 5 15	° ′ 5 30	° ′ 5 45	° ′ 6 0	° ′ 6 15	° ′ 6 30	° ′ 6 45	° ′ 7 0	° ′ 7 15	° ′ 7 30	° ′ 7 45	° ′ 8 0	° ′ 8 15	° ′ 8 30
′ ″	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.	″ Dec.
14 30	59.611	59.692	59.772	59.853	59.933	60.014	60.094	60.175	60.255	60.336	60.417	60.497	60.578	60.658	60.739
35	59.954	60.035	60.116	60.197	60.278	60.359	60.440	60.521	60.602	60.683	60.764	60.845	60.926	61.007	61.088
40	60.296	60.377	60.459	60.540	60.622	60.703	60.785	60.866	60.948	61.030	61.111	61.193	61.274	61.355	61.437
45	60.639	60.721	60.803	60.885	60.967	61.048	61.130	61.212	61.295	61.376	61.458	61.540	61.622	61.704	61.786
50	60.981	61.064	61.146	61.229	61.311	61.393	61.476	61.558	61.641	61.723	61.806	61.888	61.970	62.053	62.135
55	61.324	61.407	61.489	61.572	61.655	61.738	61.821	61.904	61.987	62.070	62.153	62.236	62.319	62.401	62.484
15 0	61.666	61 750	61.833	61.916	62.000	62.083	62.166	62.250	62.333	62.416	62.500	62.583	62.666	62.750	62.833
5	62.009	62.093	62.177	62.261	62.344	62.428	62.512	62.596	62.680	62.763	62.847	62.930	63.014	63.098	63.182
10	62.352	62.436	62.520	62.605	62.689	62.773	62.857	62.941	63.025	63.110	63.195	63.279	63.363	63.447	63.531
15	62.695	62.779	62.864	62.948	63.033	63.118	63.203	63.287	63.372	63.457	63.542	63.626	63.711	63.796	63.880
20	63.037	63.122	63.207	63.292	63.378	63.463	63.548	63.633	63.718	63.803	63.889	63.974	64.059	64.144	64.229
25	63.379	63.465	63.551	63.636	63.722	63.808	63.893	63.979	64.065	64.150	64.236	64.322	64.407	64.493	64.578
30	63.722	63.808	63.894	63.980	64.066	64.153	64.239	64.325	64.411	64.497	64.583	64.669	64.755	64.842	64.928
35	64.065	64.151	64.237	64.324	64.410	64.497	64.584	64.670	64.757	64.843	64.930	65.016	65.103	65.190	65.277
40	64.408	64.494	64.581	64.668	64.755	64.842	64.929	65.016	65.103	65.190	65.277	65.363	65.451	65.538	65.626

45	64.751	64.837	64.924	65.012	65.099	65.187	65.274	65.362	65.449	65.537	65.624	65.711	65.799	65.886	65.975
50	65.093	65.180	65.268	65.356	65.444	65.532	65.619	65.708	65.795	65.884	65.971	66.059	66.147	66.235	66.324
55	65.436	65.523	65.611	65.700	65.788	65.877	65.965	66.054	66.142	66.231	66.318	66.407	66.495	66.584	66.673
16 0	65.778	65.866	65.955	66.044	66.133	66.222	66.311	66.400	66.489	66.578	66.666	66.755	66.844	66.933	67.022
5	66.120	66.209	66.298	66.388	66.477	66.567	66.657	66.745	66.835	66.924	67.013	67.102	67.192	67.281	67.371
10	66.462	66.552	66.641	66.732	66.821	66.912	67.002	67.091	67.181	67.270	67.360	67.450	67.540	67.629	67.720
15	66.804	66.895	66.984	67.076	67.165	67.257	67.347	67.437	67.527	67.617	67.707	67.798	67.888	67.978	68.069
20	67.147	67.238	67.328	67.420	67.510	67.602	67.692	67.783	67.873	67.964	68.054	68.146	68.236	68.327	68.418
25	67.490	67.581	67.672	67.764	67.855	67.947	68.037	68.129	68.219	68.311	68.402	68.494	68.584	68.676	68.767
30	67.833	67.925	68.016	68.108	68.200	68.292	68.383	68.475	68.566	68.658	68.750	68.842	68.933	69.025	69.117
35	68.175	68.268	68.359	68.452	68.544	68.636	68.728	68.820	68.912	69.004	69.097	69.189	69.281	69.373	69.466
40	68.517	68.611	68.702	68.796	68.888	68.981	69.073	69.166	69.258	69.351	69.444	69.536	69.629	69.721	69.815
45	68.860	68.954	69.046	69.140	69.232	69.326	69.418	69.512	69.604	69.698	69.791	69.884	69.977	70.070	70.164
50	69.203	69.297	69.390	69.484	69.576	69.671	69.763	69.858	69.950	70.045	70.138	70.232	70.325	70.419	70.513
55	69.546	69.640	69.734	69.828	69.921	70.016	70.109	70.204	70.297	70.392	70.485	70.580	70.673	70.768	70.862
17 0	69.889	69.983	70.078	70.172	70.266	70.361	70.455	70.550	70.644	70.739	70.833	70.928	71.022	71.117	71.211

The use of this Table will appear from the following example:—

Given the variation in Right Ascension 7° 12′ 30″, and the Moon's hor. Semidiameter 15′ 35″. Required the equatorial Semidiameter in Time, or the Interval of Passage from the Western Limb to the Moon's Centre?

For 7° 0′ against 15′ 35″, we find . . . 64″ 757, and for 7° 15′, 64″ 843, difference 86, which in the proportion of 15′ to 12′ 30″, gives 72 nearly . . . 72

Moon's Eq. Semid. in Time = 1′ 4″ 829dec. 64 .829

When the Semidiameter is not found in the Table, proportion is to be taken for its excess above the Tabular Number next preceding. Thus, in the foregoing Example, if the Semidiameter had been 15′ 37″.5, the Semidiameter in Time would be 65″.003 nearly.

As auxilliary to this, Table II., hereto annexed, has been constructed, showing the increase of the Equatorial Semidiameter in Time, arising from the Moon's declination north or south of the Equator.

TABLE II.

Showing the Increase, in Time, of the Moon's Semidiameter, arising from the Declination North or South of the Equator.

☽'s Declin. N. or S.	Argument.—Moon's Equatorial Semidiameter in Time.												
	″ Dec. 59.0	″ Dec. 60.0	″ Dec. 61.0	″ Dec. 62.0	″ Dec. 63.0	″ Dec. 64.0	″ Dec. 65.0	″ Dec. 66.0	″ Dec. 67.0	″ Dec. 68.0	″ Dec. 69.0	″ Dec. 70.0	″ Dec. 71.0
° ′ 0 0	″ 0.000	″ 0.000	″ 0.000	″ 0.000	″ 0.000	″ 0.000	″ 0.000	″ 0.000	″ 0.000	″ 0.000	″ 0.000	″ 0.000	″ 0.000
1 0	0.009	0.009	0.009	0.009	0.010	0.010	0.010	0.010	0.010	0.010	0.010	0.011	0.011
2 0	0.036	0.036	0.037	0.038	0.039	0.039	0.040	0.040	0.041	0.041	0.042	0.043	0.043
3 0	0.081	0.082	0.084	0.085	0.086	0.088	0.089	0.090	0.092	0.093	0.094	0.096	0.097
4 0	0.144	0.147	0.149	0.151	0.154	0.156	0.159	0.161	0.164	0.166	0.168	0.171	0.173
5 0	0.225	0.229	0.233	0.237	0.241	0.244	0.248	0.252	0.256	0.260	0.264	0.267	0.271
6 0	0.325	0.331	0.336	0.341	0.347	0.353	0.358	0.363	0.369	0.375	0.380	0.386	0.391
7 0	0.443	0.451	0.558	0.466	0.473	0.481	0.488	0.496	0.503	0.511	0.518	0.526	0.533
8 0	0.580	0.588	0.599	0.609	0.619	0.629	0.639	0.649	0.659	0.668	0.678	0.688	0.698
9 0	0.735	0.748	0.760	0.773	0.785	0.798	0.810	0.823	0.835	0.848	0.860	0.872	0.885
10 0	0.910	0.925	0.941	0.956	0.972	0.987	1.003	1.018	1.034	1.049	1.064	1.080	1.095
11 0	1.104	1.123	1.142	1.160	1.179	1.198	1.217	1.236	1.254	1.273	1.292	1.310	1.329
12 0	1.318	1.340	1.363	1.385	1.407	1.430	1.452	1.474	1.497	1.519	1.541	1.564	1.586
13 0	1.552	1.578	1.605	1.631	1.657	1.683	1.710	1.736	1.762	1.789	1.815	1.842	1.868
14 0	1.806	1.837	1.868	1.898	1.929	1.959	1.990	2.020	2.051	2.082	2.112	2.143	2.174

15 0	2.081	2.116	2.152	2.187	2.222	2.258	2.293	2.328	2.363	2.399	2.434	2.469	2.505
16 0	2.378	2.418	2.458	2.499	2.539	2.579	2.619	2.660	2.700	2.740	2.781	2.821	2.861
17 0	2.696	2.741	2.787	2.833	2.878	2.924	2.970	3.016	3.061	3.107	3.153	3.198	3.244
18 0	3.037	3.088	3.139	3.191	3.242	3.294	3.345	3.396	3.448	3.499	3.551	3.602	3.654
19 0	3.400	3.457	3.515	3.572	3.630	3.688	3.745	3.803	3.861	3.918	3.976	4.033	4.091
20 0	3.787	3.850	3.915	3.979	4.043	4.107	4.172	4.236	4.300	4.364	4.428	4.492	4.557
21 0	4.198	4.268	4.340	4.411	4.482	4.553	4.624	4.696	4.767	4.838	4.909	4.980	5.051
22 0	4.633	4.711	4.791	4.869	4.948	5.026	5.105	5.183	5.262	5.340	5.419	5.497	5.576
23 0	5.095	5.181	5.268	5.354	5.441	5.527	5.613	5.700	5.786	5.872	5.959	6.045	6.132
24 0	5.583	5.679	5.773	5.867	5.962	6.057	6.151	6.246	6.341	6.435	6.530	6.624	6.719
25 0	6.099	6.204	6.306	6.409	6.513	6.616	6.720	6.823	6.926	7.030	7.133	7.236	7.340
26 0	6.643	6.757	6.869	6.981	7.094	7.207	7.319	7.432	7.544	7.657	7.770	7.882	7.995
27 0	7.217	7.340	7.462	7.584	7.706	7.829	7.951	8.074	8.196	8.318	8.440	8.563	8.685
28 0	7.822	7.954	8.087	8.219	8.352	8.484	8.617	8.750	8.882	9.015	9.147	9.280	9.412
29 0	8.458	8.601	8.745	8.888	9.031	9.175	9.318	9.461	9.605	9.748	9.891	10.035	10.178

These Tables will be found useful to compute with sufficient accuracy the interval of time of the passage from the Moon's eastern or western Limb to the centre, in transits over the Meridian: in which case, it will be proper to augment the equatorial horizontal Semidiameter, according to the Moon's apparent altitude, and to deduct 3″ therefrom, as an allowance for the inflection of light. The variation in Right Ascension for 12 hours should also be corrected by the method of interpolation from successive differences, at the time of the transit, when great precision is required. It may be unnecessary to remark, that proportions are to be made for intermediate numbers not found in the Tables, as it will be evident to the practical Astronomer, to whom their aid may be necessary in his calculations.

The principles on which the Tables have been constructed will be clearly understood by the following rule, reduced to algebraical formulæ:—

Reduce the variation for 12 hours into time; add it to 12 hours, and call the sum, in seconds, A.
Call the Moon's *corrected* Semidiameter, in seconds, ... B.
Constant log. 4.1884250, + log. A, + log. B, = log. C, the ☽'s equat. semid. in time.
Log. C, + log. secant ☽'s decl. = ☽'s semid. in time, or interval of passage.

City of Washington,
October, 1821.

William Lambert.

XX. *Observations of the Planets during the Period of their respective Oppositions in 1820, 1821, and 1822: with the Computation of their Geocentric Longitudes and Latitudes, by means of the assumed Parallax therein mentioned, and of his own Tables of Refraction.*

By S. GROOMBRIDGE, ESQ. F.R.S.

Read April 12, 1822.

PALLAS.

1820.	Mean Time.	Observed Rt. Ascen.	Observed Declination.		Computed Longitude.	Computed Latitude.
			South.	Par.		
	h ′ ″	° ′ ″	° ′ ″		° ′ ″	° ′ ″
Feb. 1	9 42 57.4	96 45 6.6	25 32 13.7	−6.1	99 15 54.7	48 47 4.0 S.
4	9 30 22.0	96 33 10.2	24 30 48.1		98 53 18.1	47 46 32.0
6	9 22 8.6	96 27 45.0	23 49 10.4		98 41 59.8	47 5 57.5
10	9 6 4.5	96 22 35.4	22 19 58.0		98 26 41.6	45 36 37.1
16	8 43 0.4	96 30 28.2	20 1 36.7		98 24 49.8	43 18 4.7
17	8 39 16.4	96 33 26.7	19 37 58.7	−5.7	98 26 38.8	42 54 19.0

CERES.

1820.	Mean Time.	Observed Rt. Ascen.	Observed Declination.		Computed Longitude.	Computed Latitude.
			North.	Par.		
Feb. 1	11 49 37.6	128 30 18.9	31 29 3.2	+1.87	122 55 5.1	12 19 4.7 N.
4	11 34 57.2	127 47 5.7	31 44 49.4		122 14 33.3	12 25 4.3
10	11 5 53.7	126 24 49.8	32 11 3.4		120 58 39.7	12 33 19.8
16	10 37 25.5	125 11 27.3	32 29 41.0		119 52 25.4	12 36 37.2
17	10 32 45.5	125 0 23.1	32 32 1.8	+1.72	119 42 33.6	12 36 42·6

JUPITER.

1820.	Mean Time.	Observed Rt. Ascen.	Observed Declination.	Par.	Computed Longitude.	Computed Latitude.
			South.	Par.		
	h ′ ″	° ′ ″	° ′ ″		° ′ ″	° ′ ″
Sept. 9	12 4 9.5	349 58 38.4	5 59 57.1	−1.8	348 26 10.8	1 32 37.7 S.
10	11 59 44.1	349 51 15.4	6 3 8.9		348 18 10.2	1 32 41.1
11	11 55 18.8	349 43 56.1	6 6 18.5		348 10 13.7	1 32 44.6
12	11 50 53.8	349 36 36.0	6 9 30.9		348 2 15.5	1 32 50.0

SATURN.

			North.	Par.		
Oct. 1	11 59 27.5	10 29 0.0	1 33 6.2	+0.8	10 14 35.5	2 43 38.2 S.
2	11 55 14.2	10 24 39.6	1 31 13.8		10 9 51.7	2 43 39.6
3	11 51 0.8	10 20 19.8	1 29 23.1		10 5 9.1	2 43 39.6

VESTA.

1821.			North.	Par.		
Jan. 12	12 15 3.4	115 54 56.6	23 24 32.2	+1.6	113 39 41.2	2 2 58.0 N.
14	12 5 0.2	115 22 3.0	23 35 45.3		113 8 1.3	2 8 48.9
16	11 54 56.4	114 48 57.1	23 46 45.1		112 36 15.8	2 14 31.9
20	11 34 50.6	113 43 15.0	24 8 2.8		111 33 29.6	2 25 42.3
Feb. 2	10 30 47.3	110 28 36.0	25 7 13.8		108 29 28.9	2 57 40.9
5	10 16 28.5	109 50 43.2	25 18 23.6		107 53 56.9	3 4 1.7
7	10 7 4.2	109 27 31.2	25 25 14.6		107 32 13.4	3 8 0.3

PALLAS.

			North.	Par.		
May 8	13 28 22.5	248 37 52.8	24 11 9.3	+1.5	241 39 12.9	45 34 9.8 N.
12	13 9 34.1	247 51 33.6	24 44 15.0		240 30 6.5	45 57 35.1
15	12 55 21.8	247 15 18.0	25 5 33.0		239 37 7.9	46 11 9.7
16	12 50 36.5	247 2 54.0	25 12 5.1		239 19 12.2	46 15 1.4
18	12 41 4.8	246 37 53.4	25 24 20.3		238 43 18.4	46 21 49.9
20	12 31 32.3	246 12 39.4	25 34 59.5		238 7 36.0	46 26 56.0
23	12 17 10.8	245 34 5.4	25 48 32.5		237 13 54.0	46 31 53.2
26	12 2 49.5	244 55 35.8	25 59 3.9		236 21 21.0	46 33 42.2
30	11 43 41.7	244 4 27.6	26 8 24.4		235 13 17.0	46 31 18.1

CERES.

1821.	Mean Time.	Observed Rt. Ascen.	Observed Declination.		Computed Longitude.	Computed Latitude.
			South.	Par.		
	h ′ ″	° ′ ″	° ′ ″		° ′ ″	° ′ ″
May 12	12 48 21.0	242 32 26.1	14 44 17.8	−2.9	243 20 47.9	6 13 16.7 N.
15	12 33 50.2	241 51 30.9	14 46 0.3		242 42 1.2	6 4 11.5
16	12 28 58.6	241 37 33.6	14 46 37.4		242 28 48.9	6 1 1.7
18	12 19 14.9	241 9 31.8	14 47 57.6		242 2 19.7	5 54 31.2
20	12 9 30.4	240 41 16.3	14 49 22.1		241 35 39.9	5 47 49.3
23	11 54 52.6	239 58 36.9	14 51 46.7		240 55 31.6	5 37 17.7
26	11 40 15.6	239 16 12.4	14 54 29.1		240 15 45.2	5 26 21.0
30	11 20 49.2	238 20 23.2	14 58 52.4		239 23 40.5	5 10 53.2

GEORGIAN.

			South.	Par.		
June 18	12 18 5.5	271 25 26.8	23 42 35.2	−0.5	271 18 14.0	0 15 3.8 S.
20	12 9 52.7	271 20 9.0	23 42 40.9		271 13 23.0	0 15 6.4
23	11 57 32.8	271 12 7.5	23 42 48.3		271 6 2.1	0 15 10.2

SATURN.

			North.	Par.		
Oct. 13	12 2 14.2	22 46 13.8	6 32 16.0	+0.7	23 29 12.0	2 47 22.6 S.
14	11 58 0.7	22 41 49.2	6 30 27.9		23 24 27.5	2 47 26.5
15	11 53 47.1	22 37 23.4	6 28 43.3		23 19 43.1	2 47 26.6
16	11 49 33.5	22 32 57.6	6 26 57.9		23 14 58.4	2 47 27.3
20	11 32 39.2	22 15 15.6	6 20 3.4		22 56 3.1	2 47 23.7
22	11 24 11.7	22 6 17.7	6 16 37.5		22 46 29.5	2 47 17.6

JUPITER.

			North.	Par.		
Oct. 13	12 9 19.2	24 32 47.1	8 31 47.3	+1.5	25 51 16.5	1 34 38.4 S.
14	12 4 53.1	24 25 13.5	8 28 49.5		25 43 13.9	1 34 41.5
15	12 0 27.0	24 17 39.3	8 25 56.2		25 35 12.2	1 34 40.0
16	11 56 0.9	24 10 4.2	8 22 59.9		25 27 8.5	1 34 40.7
20	11 38 15.1	23 39 27.6	8 11 22.7		24 54 41.3	1 34 27.8
22	11 29 22.6	23 24 15.3	8 5 37.5		24 38 34.1	1 34 19.7

MARS.

1822.	Mean Time.	Observed Rt. Ascen.	Observed Declination. North.	Par.	Computed Longitude.	Computed Latitude.
Feb. 14	12h 43′ 58″.2	155° 27′ 13″.6	15° 0′ 33″.1	+7.5	151° 47′ 40″.1	4° 27′ 48″.2 N.
15	12 38 32.3	155 4 39.9	15 9 6.2		151 24 11.6	4 27 52.2
17	12 27 37.8	154 18 51.4	15 25 56.4		150 36 46.1	4 27 37.0
18	12 22 10.0	153 55 49.8	15 34 11.8		150 13 1.2	4 27 21.0
19	12 16 41.8	153 32 39.9	15 42 24.4		149 49 11.0	4 27 1.4
21	12 5 44.8	152 46 15.6	15 58 15.8		149 1 40.5	4 25 56.0
22	12 0 16.2	152 23 0.0	16 6 0.8		148 37 57.2	4 25 15.3

Stars observed with MARS for Parallax.

1822.	Star.	Rt. Ascen.	Declination. North.
Feb. 14	46 Leonis.	155 40 55.0	15 2 32.2
15	449 Mayer do.	154 49 50.7	15 14 44.0
17	449 Do.	154 49 53.1	15 14 43.3
19	42 Leonis.	153 4 26.1	15 51 57.8
21	42 Do.	153 4 28.6	15 51 56.6
22	42 Do.	153 4 28.0	15 51 56.4

VENUS.

1822.	Mean Time.	Rt. Ascen.	Declination. North.	Par.	Longitude.	Latitude.
May 14	21 1 22.1	7 51 57.0	2 1 5.8	+10.0	8 1 15.7	1 16 0.7 S.
16	21 0 41.2	9 40 0.0	2 37 4.6	9.7	9 54 41.5	1 25 26.2
18	21 0 5.6	11 29 19.9	3 14 10.1	9.3	11 49 43.4	1 34 3.1
19	20 59 49.9	12 24 31.5	3 33 3.9	9.2	12 47 51.1	1 38 8.1
20	20 59 36.0	13 20 10.5	3 52 6.8	9.0	13 46 26.9	1 42 9.0

VESTA.

1822.	Mean Time.	Rt. Ascen.	Declination. South.	Par.	Longitude.	Latitude.
June 12	12 17 57.5	265 14 20.7	18 50 0.0	−7.2	265 28 48.5	4 33 31.1 N.
13	12 13 0.5	264 59 2.2	18 53 39.5		265 14 24.6	4 29 22.2
14	12 8 3.4	264 43 42.6	18 57 21.3		265 0 0.6	4 25 9.4
17	11 53 11.3	263 57 28.8	19 8 31.1		264 16 39.7	4 12 16.9
18	11 48 13.9	263 42 2.4	19 12 19.3		264 2 13.0	4 7 51.4
19	11 43 16.5	263 26 39.0	19 16 15.6		263 47 50.4	4 3 16.3

GEORGIAN.

1822.	Mean Time.	Observed Rt. Ascen.	Observed Declination.		Computed Longitude.	Computed Latitude.
			South.	Par.		
	h ′ ″	° ′ ″	° ′ ″		° ′ ″	° ′ ″
June 25	12 9 38.5	275 58 2.7	23 39 40.7	−0.5	275 27 51.3	0 18 34.9 S.
27	12 1 25.4	275 52 43.5	23 39 51.4		275 22 58.7	0 18 33.7
29	11 53 12.2	275 47 22.5	23 40 4.2		275 18 4.4	0 18 34.7
30	11 49 6.0	275 44 48.6	23 40 11.0		275 15 43.3	0 18 35.7

JUNO.

1823.			North.	Par.		
Jan. 9	12 32 35.4	116 52 46.8	1 21 57.5	+5.3	118 38 41.0	19 27 39.9 S.
11	12 22 57.1	116 26 4.5	1 34 9.7	5.3	118 8 26.6	19 20 44.3
18	11 49 8.5	114 51 30.4	2 24 49.6	5.2	116 20 20.5	18 48 9.5
Feb. 4	10 29 27.9	111 38 28.8	5 1 13.9	4.7	112 33 36.7	16 45 43.1
21	9 16 47.7	110 10 48.0	7 51 34.8	4.1	110 38 14.0	14 10 4.8
26	8 57 4.7	110 9 55.8	8 38 53.8	3.9	110 30 28.2	13 23 22.1

BLACKHEATH. S. GROOMBRIDGE.

XXI. *On the Triangulation of the Cape of Good Hope. By Captain G. Everest, of the Bengal Artillery, &c.*

Read May 10, 1822.

Sir, East India House, Jan. 10, 1822.

THE Court of Directors of the East India Company having received from Captain George Everest, an officer in their service in the Artillery on the Bengal establishment, a copy of a letter which he lately had occasion to write to Colonel William Lambton, Superintendant of the Trigonometrical Survey of India, upon the subject of the geodetical operations which were carried on at the Cape of Good Hope during the last century, accompanied by the request of Captain Everest, that a copy thereof might be furnished to you as President of the Astronomical Society; I am accordingly commanded by the Court to transmit to you a copy of the said letter.

I have the honour to be,

Sir,

Your most obedient humble servant,

Joseph Dart, *Secretary*.

Sir William Herschel,
President of the Astronomical Society.

To Joseph Dart, Esq. Secretary to the Honourable the Court of Directors.

Cape Town, Cape of Good Hope,
September 3, 1821.

Sir,

I take the liberty to solicit that the inclosed paper may be submitted to the notice of the Honourable the Court of Directors by as early an opportunity as may be convenient; with a request from me that, should it be deemed unobjectionable, it may be forwarded, under their auspices, to Sir William Herschel, President of the Astronomical Society.

It is the copy of a letter which I have lately had occasion to write to Colonel W. Lambton upon the subject of those geodetical operations which were carried on at the Cape of Good Hope during the last century; and they are particularly important from their furnishing the only data the scientific world is in possession of respecting the compression of the southern hemisphere. We have in fact meridional arcs, determined in several different parallels of latitude in the northern hemisphere, but all leading to results very different from those of M. De la Caille; and as the most splendid of these, whether we regard its accuracy or extent, is beyond doubt that which owes its origin to the patronage of the Honourable the East India Company, it is a fair presumption that the Honourable the Court of Directors will not view the present, my humble attempt to clear up this serious anomaly, with indifference. I rest persuaded that I shall not be considered as unnecessarily intruding myself upon the time and attention of that Court, in whose service I have passed the prime of my life and youth, and under whose patronage, if the effects of climate do not blast my prospects, I look forward to the cheering hope of taking an ample share in the measurement of the arc now in progress towards Agra and Hurdwar; which, if it be completed, will exceed every other the world has yet witnessed, in the ratio of nearly 3 to 1, and be sufficient in itself to establish the compression of the globe independently of any other data.

The apparent informality of transmitting a copy of my letter directly to you, will I trust be overlooked by the Honourable Court. The matter of it, concerning the scientific world at large, has no more reference to India than to England, France, Italy, or America; it has been sent in this channel, therefore, purely to avoid the loss of time which would arise from travelling by a circuitous route: and I know the liberality of my friend Colonel Lambton

sufficiently well to be convinced that such a procedure will be attributed by him to the right motive, and cannot be unpleasing to him.

The matter and manner of my report are I believe perfectly original; as far as concerns me, they certainly are; for I am not aware that any attempt has hitherto been made to reconcile the contradictory conclusions which it relates to. A gentleman, who was formerly assistant to Lieut.-Colonel LAMBTON, (Captain WARREN, of His Majesty's 33d Foot,) was some years ago, when he passed the Cape of Good Hope, extremely desirous to obtain information relating to the subject, and left a memorandum to that effect with the Colonial Secretary, by which it appears, that he was authorized to communicate to the Royal Society any discovery he made: but the records of the Colonial Government and all the public and private libraries at the place were searched at my request in vain; and I know perfectly well, that the country now described has not, within the memory of any individual residing there, been subjected to a scientific examination since the year 1752.

With many apologies for intruding thus largely upon your valuable time, permit me to subscribe myself,

Sir,

Your most obedient and very faithful servant,

(Signed) GEORGE EVEREST,
Capt. Bengal Artillery, Chief Assistant of the Geod. Trig. Surv. of India.

To Lieutenant-Colonel William Lambton, H.M. 33d Foot.

Cape Town, Cape of Good Hope,
August 31, 1821.

Sir,

It will be in your recollection, that on my departure from Hyderabad in August 1820, you expressed a desire that I should, in the event of my proceeding to the Cape of Good Hope, examine the tract of country in which the geodetical operations of M. l'Abbé De la Caille were conducted; and I now proceed to acquaint you with the fulfilment of your wishes.

You are already aware that I arrived in Table Bay on the 25th of November 1820; immediately on which I commenced my inquiries; but as no information could be obtained on the spot, I directly applied to a correspondent in England for such papers as related to the object of my search; and, in effect, by the latter end of June of this year, a journal of M. De la Caille's Travels reached me, so that I was enabled by the 26th of July to visit the places described as the sites of his stations, and in fact to traverse the whole theatre of his labours.

It will appear from the plan in the margin*, that besides the two extremes of the base, there were four principal stations forming two large triangles united together by a common side; the celestial observation having been made at the two vertices, which are separated from each other by a distance of about 79 miles. The two middle stations terminating the common side are Capok Berg and Riebek's Castle, lying nearly east and west of each other. The station forming the northern termination of the arc is Klip Fonteyn, and that to the southward, was I conclude the site of M. l'Abbé's Observatory at Cape Town. The base appears to have passed in a direction a little northward of East, the southernmost point lying near Klip Berg, and the northernmost near an estate called Coggera. This being premised, I will now proceed to describe the existing condition of the station, and what decisive marks I have been enabled to trace on them.

Capok Berg is a rounded hill, of easy ascent, belonging to a range of granite, which runs at the back of the Moravian Missionary Settlement at Grœne Kloof; and I should estimate its height at about 600 feet. At the highest point of the level of the hill, and at the western side, is a solid rock of considerable area, with a smaller one on the northern side, as described in page 173 of the Journal; and as the angles taken from this correspond with those in the

* Plate III.

plan of triangles, I consider the identity of this station to be satisfactorily established, though it is to be regretted that no artificial marks now remain there. Its particular features are specified in page 173 of the Journal now in my possession, under the Mem. of 6 Aoust 1752.

Riebek's Castle is the next station in order to be described. The mountain so called, whose height I should think cannot be less than 1500 feet, is a lofty chain of sandstone, running nearly N.W. and S.E., of about two miles and a half in length at the summit, and divided into several different peaks, on the second of which, reckoning from the N.W., there are the remains of an old pile of stones, having (about five feet to its N.W.) a considerable quantity of half-burned wood in a state of partial decay. As this is the peak particularly adverted to in the Journal, and we know from other certain information that the signals were made by blazing fires, I think no doubt can remain of its identity.

In reference to the base, the following remark is extracted from page 174 of the Abbé De la Caille's Journal:—"Aoust 11, 1752. J'allai à cheval dans la plaine qui est au nord de la montagne appellée Contreberg, pour chercher un terrein propre à mesurer une base; cette plaine est très étendue et fort unie, mais un peu embarrassée de broussailles; j'ai pris pour terme sud de la mesure une roche qui paroit être de marbre blanc, et qui est sur un petit tertre; elle est très remarquable; j'ai trouvé qu'en élignant la base au nord on la pouvoit prolonger autant qu'il étoit nécessaire."—Hence, therefore, and from knowing nearly the angles subtended at the south end of the base between Capok Berg, Riebek's Castle, and the detached mountain at the back of Klip Fonteyn, with the distant appearances of which I had taken care to make myself acquainted, it became a work of no great difficulty to detect the approximate site of the point to be sought; and in effect, after frequent measurements, I at last came to a spot between the estate of Klip Berg and another estate bearing also the name of Klip Fonteyn, where a ridge of quartz rock, somewhat resembling marble in whiteness and lustre, protrudes itself through the soil, and where the angles answered the given conditions.

This therefore, it appears to me, is doubtless the spot alluded to in the description; for though neither the quartz rock in question is so remarkable as might be expected, nor are there any artificial marks remaining to denote it, yet a constant exposure to the atmosphere for a lapse of seventy years is probably sufficient to account for the obliteration of the latter, as well as any distinguishing features which the rock itself may formerly have borne.

The northern extremity of the base, viewed from the southern, is in the Plan at an angle of 27° with the station of Riebek's Castle; so that by assuming a distant point, and proceeding in a straight direction towards it, I was enabled, after several trials, to reach a small mound of raised earth in the vicinity of Coggera, where the angles between Riebek's Castle, Capok Berg, and the hill behind Klip Fonteyn, nearly corresponded with those taken from the Plan; and though here there are neither artificial nor natural marks to designate the site of the station, yet so little doubt remains in my mind of its being the point to be sought, that I consider myself warranted in offering the following remarks as drawn from an actual inspection of the tract of land through which the base line passed.

The soil is throughout a mixture of sand and clay, rather light than otherwise, but still sufficiently solid and substantial. The surface is undulated and irregular; and though there are neither rivers nor deep ravines, nor large forests, yet it is covered thickly with small bushes and brushwood; and there are at the present day inequalities enough to render such a measurement at least dubious, if it were performed by any of the means then in common use. The Abbé De la Caille's biographer does not in the Journal (and I regret to say this is the only work upon the subject yet in my possession) enter into any of the scientific details; but we know that the more refined corrections were introduced into practice at a period long subsequent to the year 1752; and the fact seems to me to be beyond doubt, that the plain to the north of Capok Berg and Contre Berg is no where sufficiently level and even to admit of an operation so delicate in its nature, unless indeed the chain were supported by Coffer's tripods and such other apparatus, as there is every reason to conclude M. De la Caille was not supplied with, since no hint leading to such a supposition is any where thrown out in the work before me. I speak of course of the face of the country as it at present stands; what it may have been seventy years ago is another affair, though I do not find on inquiry any individuals now in existence within whose remembrance any considerable change has taken place.

It would seem by the following extract from M. De la Caille's Journal, page 144, that he had originally intended to establish the northernmost station of his triangles at the summit of a mountain detached from the great range of Piquet Berg:—September 7, 1751. "J'ai été sur une des montagnes de la première chaine, dont je viens de parler. Cette montagne s'appelle Capok Berg, &c. &c. . . . . . ; j'ai vu une montagne fort éloignée dont une des extré-

mités étoit presque dans le nord, et fort propre pour terminer la mesure d'un degré. Depuis cette montagne en allant par l'est vers le sud, l'horizon est bordé de hautes montagnes." He however mentions afterwards, page 180, that his sector was erected in an old granary belonging to Klip Fonteyn for the following reasons:—14th September, 1752. "En général on y voit tout ce qu'on eut pu découvrir du sommet du Piquet Berg ou de la montagne voisine; c'est pour cela que je n'ai pas placé des signaux sur ces montagnes pour terminer mes triangles; mais que j'ai marqué un point pris à 36 toises à l'ouest de mon observatoire à fin d'y faire des feux pour former mon dernier triangle." Now in reference to this matter, it may not be amiss to mention, that the daughter of the quondam proprietor of Klip Fonteyn, now an aged lady named Letchie Schalkeveck, is still in existence, and not only gives a narration perfectly agreeing, but has pointed out the very platform on which the granary once stood; and states further, that the signal-fires were so large and brilliant, that those of Riebek's Castle were visible from Klip Fonteyn, a distance of more than forty-five miles, with the naked eye at night. The same lady relates also, that the Abbé De la Caille observed the stars with his instrument (the sector I suppose) in the granary (an aperture having been made in the thatched roof for the purpose), until the day when the fires were lighted; when, having previously sent M. Poitevin his assistant to make simultaneous observations at Riebek's Castle, he placed it, or some other, at some paces in front of the fire: and as this account tallies with what I have before observed in my remark respecting Riebek's Castle, it leads directly to the extraordinary conclusion, that not only the signals in these operations were ill defined, but that the instrument for measuring the horizontal angles was not placed over the centres of the stations. The above remarks relate particularly to the terrestrial measurements: we come now to the subject of M. De la Caille's celestial observations, which, from their superior delicacy and the important consequences attending any errors committed in them, will demand a more minute investigation.

The granary at Klip Fonteyn, which is beyond doubt the northern termination of the arc, is in the immediate vicinity of a very extensive range of sandstone mountains called Piquet Berg, some of the peaks in which I should think cannot be less than 2000 feet in height; and it is at the foot of the detached hill at the back of Klip Fonteyn, already so often alluded to, which rises, I should conclude, to the height of 700 feet, so that the whole horizon from the N.E. to the S.W. is completely inclosed by mountainous masses; whilst on

the opposite quarter is an open country without any counterbalancing matter whatever. The masses are so irregular, that it would be a work of extreme difficulty, if not of absolute impossibility, to calculate the attraction which they would be likely to exert upon the plumb-line; but it will be evident from what has preceded, that if any such lateral attraction did exist, it would lie all on one side, and principally in a N.W. direction.

The last station to be noticed is that at the southern termination of the arc, which I have been enabled satisfactorily to trace to a particular house in Strand Street, Cape Town, adjoining the one in which I reside; and as this may be an interesting subject for you hereafter, I shall particularize the grounds which have led me to this conclusion. The Abbé DE LA CAILLE's Journal, page 140, contains the following passage bearing immediately upon the question:—21st April, 1751. "Nous logeons chez M. BESTBIER, Capitaine de la Cavalerie Bourgeoise, chez qui je trouve un endroit propre pour observer, en y faisant batir un observatoire pour y placer mes instrumens." Now, as the house of M. BESTBIER was in the memory of a person* living in No. 7, Strand Street, the present residence of Mr. DE WITT, this circumstance would almost be sufficient to establish its identity with the point in question; but there is a mark in existence which furnishes another corroborating fact, namely, that a brass plate perforated with a small hole, and fixed horizontally in a vertical wall, with a black line traced immediately below it, for the obvious purpose of determining the sun's passage over the meridian, still stands at Mr. DE WITT's house, and is said to have been placed there by the Abbé DE LA CAILLE.

The erection of M. l'Abbé's observatory is stated in the Journal to have been commenced on Monday subsequent to the 24th of April, and to have been finished on the 17th of May following; and as it was performed by the Government workmen, it could not have been a very substantial fabric. The probability is, that he took advantage of Mr. BESTBIER's house having a flat roof; and if that were the case, it is very easy to imagine that extensive errors may have arisen from placing his instrument on so frail a basis.

But were it even admitted that the observatory had been unobjectionable in respect to stability, still its situation must have rendered it highly ineligible for the delicate operation depending on it. It is well known that Cape Town lies at the very base of Table Mountain, a vast formation of sandstone in hori-

* A female slave belonging to Mrs. HERTZOG is the person alluded to.

zontal strata rising above a sienitic or granitic base for the upper half of its height, like a mighty wall, quite bare, towering and precipitous; the lower half of the mountain being formed apparently of debris, which slopes gradually till it reaches the ocean. This immense mass, including the Devil's Hill, which forms in fact part of the same land as the Table, though the continuity of the upper strata is broken through by a considerable intervening chasm, occupies the horizon between S.E. 22¾° and S.W. 27°, and averages between 13° and 14° of elevation from the house in question; further southward it extends with various heights for the distance of about thirty-five miles, and ultimately forms the peninsula of the Cape of Good Hope. The mountain called The Lion occupies the horizon from N. 72¾° W. to S. 66° W., and is distinct from the Table; for though it was probably in the origin of the same formation, yet it is now quite different in its features, being rounded off, of easy ascent, and covered with soil; the small part, called The Lion's Head, is however formed of horizontal strata similarly disposed with those of Table Mountain, and the bases of the two run into each other, the uniting ridge being about 800 feet high, and apparently formed from the debris of each.

The Abbé De la Caille, when he was here in 1751 and 1752, had taken the heights of all the different eminences in the environs, and left a record at the request of the Colonial Government, in the Secretary's office, of which a copy being still in existence, I have made use of it to ascertain the relative approximate distances of the peaks by observing the angles of elevation and azimuth from the roof of the house adjoining that on which was the site of the observatory, considering the distances as radii to be determined by the heights as tangents. This method of course is rough, and admits of no great nicety; but it is sufficient to convey a general notion of the localities of this vicinity, and with that view I subjoin the following table.

Names.	Height.*	Elevation Angles.	Dist.*	Directions with the True Meridian.
Devil's Hill	3106	13° 55′	12,350	22¾° S.E.
Table { Eastern Extreme.	3302	13 12	13,880	2° S.E. } The surface included between these two rays is very solid and compact.
Table { Western Ditto .	3353	13 26	13,850	27° S.W. }
Lion's Head. . . .	2085	10 39	10,850	66° S.W.
Lion's Rump . . .	1102	10 24	5,760	72¾° N.W.

* The heights and distances are all in Paris feet, which are to English feet as 4.263 to 4.000.

Now it appears to me, that so vast a quantity of matter could not have existed so near to the site of the observatory without affecting the plumb-line of the instrument used by M. De la Caille in 1751, 1752. The lateral force exerted by mountains was certainly known to the scientific world at that period; for in the year 1738, MM. Bouguer and De la Condamine had attempted to calculate the attraction of Chimboraço: perhaps, indeed, M. De la Caille may have made some calculation to this effect; for I must candidly avow, that I have never been so fortunate as to read any of his papers in the Memoirs of the Academy of Sciences; but if he did, it is a great chance, considering the failure of the celebrated geodists who measured the arc near Quito, that he did not arrive at any accurate conclusion. For the present, therefore, I shall suppose that there was no correction applied on this account, but that the plumb-line of his sector was drawn out of the vertical by a lateral force at the southern extremity of the arc, in a direction of S.W. A°, and that the resolved part of this force parallel to the plane of the meridian was such as to cause the zero to stand at a'' too much to the southward; by which means his zenith would be affected in the opposite direction, and would appear more to the northward than was due by an equal arc. Similarly, if the resolved part of a lateral force exerted by the mountains at Klip Fonteyn caused the zero to stand b'' too much to the northward, the zenith of that place would have appeared b'' to the southward of the true place, and thus the whole apparent arc of amplitude would have been less than the true one by the sum $a''+b''$ of the two arcs of error, or, what is the same thing, the measure of the arc would have been too great.

If this reasoning be admitted (as I think it must), we may perhaps be able to reconcile some considerable discrepancies to which the geodetical operations in question have given rise; for if the earth be an ellipsoid of revolution, whose meridional arcs increase in a certain ratio as we advance from the equator to the poles, it is evident that, on comparing the measurement in this latitude with that made nearer the equator, the ratio of increase would be too great; and, on the contrary, if the comparison be made with a more northerly arc, the ratio of increase would be unduly diminished. In the first instance, the calculated figure of the elliptic meridian would have a greater compression; in the second, a less compression than would be found to result from a comparison of the two extreme measured arcs with each other. And that this really occurs in the case before us I shall now proceed to demonstrate.

To this end I shall first compare M. De la Caille's arc with that measured

by MM. BOUGUER and DE LA CONDAMINE in the year 1738; I shall then draw a comparison between M. DE LA CAILLE's arc, and that measured by M. CASSINI in the year 1740; and lastly, I shall compare the arcs of M. BOUGUER and M. CASSINI with each other. The formula I shall adopt is the common elliptic one of $\frac{c}{d} = \left(\frac{m\frac{2}{3}\sin^2 C - M\frac{2}{3}\sin^2 L}{M\frac{2}{3}\text{Cos}^2 L - m\frac{2}{3}\text{Cos}^2 L}\right)\frac{1}{2}$, where c and d represent the major and minor axes L and l, the middle points of latitude corresponding to the measures M and m; and the calculations here follow at full length.

1st Comparison, where L = 1° 30′ N.; M = 56749; l = 33° 18′ 30″ S.; m = 57037.

L = 1° 30′ Log.	Sin. $\bar{2}$.417919	....	Log. Cos. $\bar{1}$.999851
	2		2
M = 56749 Log. 4753958			
2	Sin.^{2} $\bar{4}$.835838		$\bar{1}$.999702
3)9507916	3.169305		3.169305
M $\frac{2}{3}$ Sin.^{2} L = 1.012 Log.	0.005143	M $\frac{2}{3}$ Cos.^{2} L = 1475.731	Log. 3.169007
l = 33° 18′ 30″ Log. Sin.	$\bar{1}$.739687	....	Log. Cos. $\bar{1}$.922064
	2		2
m = 57037 Log. 4.756157			
2	Sin.^{2} $\bar{1}$.479374		$\bar{1}$.844128
3)9.512314	3.170771		3.170771
m $\frac{2}{3}$ Sin.^{2} l 446.833	Log. 2.650145	m $\frac{2}{3}$ Cos.^{2} l 1034.902	Log. 3.014899

M $\frac{2}{3}$ sin.^{2} l — M $\frac{2}{3}$ sin.^{2} L = 445.821 $\qquad$ m $\frac{2}{3}$ Cos.^{2} L = m $\frac{2}{3}$ Cos.^{2} l = 440.829

1st Comparison continued.

445.821	Log. ...	2.649160
440.829	Log. ...	2.644270
		2)0.004890
$\frac{c}{d}$ = 1.0058	Log. ...	0.002445

$\frac{c}{d} - 1 = \frac{1}{172}$ Compression.

2d Comparison, where $L = 33^\circ\ 18'\ 30''$; $M = 57037$; $l = 49^\circ\ 22'$; $m = 57074$.

$l = 49^\circ\ 22'$ Log. Sin. $\bar{1}.880180$ . . . . Log. Cos. $\bar{1}.813725$

2   2

$m = 57074$ Log. 4756438   7.760360 $\bar{1}.627450$

2   3.170959   3.170959

3) 9.512876

$m\,\frac{2}{3}\,\sin.^2 l$ 853.726 Log. 2.931319 $m\,\frac{2}{3}\,\text{Cos.}^2 l$ 628.651 Log. 2.798409

$M\,\frac{2}{3}\,\sin.^2 L$ 446.833 $M\,\frac{2}{3}\,\text{Cos.}^2 L$ 1034.902

$M\,\frac{2}{3}\,\text{Cos.}^2 L - m\,\frac{2}{3}\,\text{Cos.}^2 l$

$m\,\frac{2}{3}\,\text{Sin.}^2 l - M\,\frac{2}{3}\,\text{Sin.}^2 L = 406.893$ Log. 2.609481

$M\,\frac{2}{3}\,\text{Cos.}^2 L - m\,\frac{2}{3}\,\text{Cos.}^2 l = 406.251$ Log. 2.608795

2) 0.000686

$\frac{c}{d} =$ 1.0008 Log. 0.000343 $\frac{c}{d} - 1 = \frac{1}{1250}$ Compression.

3d Comparison, where $L = 1^\circ\ 30'$; $M = 56749$; $l = 49^\circ\ 22'$; $m = 57074$.

$M\,\frac{2}{3}\,\text{Sin.}^2 L$ . . . . 1.012 $M\,\frac{2}{3}\,\text{Cos.}^2 L$ 1475.731

$m\,\frac{2}{3}\,\text{Sin.}^2 l$ . . . . 853.726 $m\,\frac{2}{3}\,\text{Cos.}^2 l$ 628.651

$m\,\frac{2}{3}\,\text{Sin.}^2 l - M\,\frac{2}{3}\,\text{Sin.}^2 L$ 852.714 Log. 2.930803 $M\,\frac{2}{3}\,\text{Cos.}^2 L - m\,\frac{2}{3}\,\text{Cos.}^2 l = 847.080$

$M\,\frac{2}{3}\,\text{Cos.}^2 L - m\,\frac{2}{3}\,\text{Cos.}^2 l$ 847.080 Log. 2.927925

2)0.002878

$\frac{c}{d} =$ 1.0033 Log. 0.001439 $\frac{c}{d} - 1 = \frac{1}{300}$ Compression.

Here, therefore, we have $\frac{1}{178}$, $\frac{1}{1250}$, and $\frac{1}{300}$, which vary *inter se* in a manner very similar to what might have been expected; but we may yet pursue the inquiry further, by considering the quantity $\frac{1}{300}$ as derived from the last comparison to be constant, since it is not very different from what has been derived from the best accredited modern operations. Making, therefore, always M and L apply to the lower latitude, m and l to the higher, we shall

have two well known formulæ $m = \left(\dfrac{\frac{c^2}{d^2}\,\text{Cos}^2\,L \times \sin^2 L}{\frac{c^2}{d^2}\,\text{Cos}^2\,l \times \sin^2 l}\right)^{\frac{3}{2}} \times M$ or else $M = \left(\dfrac{\frac{c^2}{d^2}\,\text{Cos}^2\,l \times \sin^2 l}{\frac{c^2}{d^2}\,\text{Cos}^2\,L \times \sin^2 L}\right)^{\frac{3}{2}} \times m$, in which, substituting as before, we shall know of what length the arc measured in this latitude must be in order to give a compression of $\frac{1}{300}$ when compared with each of the other two.

1st Comparison m determined for DE LA CAILLE'S Arc from BOUGUER'S.

Sin.² L 1° 30′ Log. $\bar{4}$.835838 N. N. 0.0006852	Sin.² l (33° 18′ 30″) Log. $\bar{1}$.479374 N.N. 0.3015604
$\frac{c^2}{d^2} = \overline{1.0033}\|^2$ Log. 0.002878 Cos.² L 1° 30′ Log. $\bar{1}$.999702	$\frac{c^2}{d^2} = \overline{1.0033}\|^2$ Log. 0.002878 Cos.² l (33° 18′ 30″) Log. $\bar{1}$.844128
$\frac{c^2}{d^2}$ Cos.² L . . Log. 0.002580 N. N. 1.0059583	$\frac{c^2}{d^2}$ Cos.² L . . . . . Log. $\bar{1}$.847006 N.N. 0.7030823
Sin.² L × $\frac{c^2}{d^2}$ Cos.² L. Log. 0.002875 N.N. 1.0066435	Sin.² l × $\frac{c^2}{d^2}$ Cos.² l Log. 0.002012 N.N. 1.0046427

Sin.² l × $\frac{c^2}{d^2}$ Cos.² l Log. 0.002012

0.000863
3

2) 0.002589

0.001295

M=56749 . . . . . Log. 4.753958

m=56918.4 . . . Log. 4.755253

These measures, excepting those of the Quito Arc, are taken from a very old edition of Dr. HUTTON'S Philosophical Dictionary; for the principal part of my library being in India, I have by me very few books of reference. The comparisons are sufficient for my present purpose; but I shall hereafter compare the more modern measurements of DE LAMBRE, Colonel LAMBTON and General MUDGE with DE LA CAILLE.

2d Comparison, M determined for DE LA CAILLE's Arc from CASSINI's.

$\text{Sin.}^2\, l$ 49° 22′	Log.	$\bar{1}.760360$	N. N. 0.5759173
$\frac{c^2}{d^2} = \overline{1.0033}\vert^2$	Log.	0.002878	
$\text{Cos.}^2\, l = 49° 22′$	Log.	$\bar{1}.627450$	
$\frac{c^2}{d^2}\,\text{Cos.}^2\, l$	Log.	$\bar{1}.630328$	N. N. 0.4269020
$\text{Sin.}^2\, l \times \frac{c^2}{d^2}\,\text{Cos.}^2\, l$	Log.	0.001223	N. N. 1.0028193
$\text{Sin.}^2\, L \times \frac{c^2}{d^2}\,\text{Cos.}^2\, L$	Log.	0.002012	
		$\bar{1}.999211$	
		3	
	2)	$\bar{1}.997633$	
		-1.998817	
$m = 57074$	Log.	4.756438	
$M = 56918.7$		4.755255	

1st Result	56918.4
2d Result	56918.7
Mean	56918.6

The mean of these results, or 56918.6 toises, is what would belong to a degree in latitude 33° 18′ 30″ south, upon the hypothesis that the earth was an ellipsoid of revolution with a compression of $\frac{1}{300}$; and had the arc measured in this latitude been found to be of that size, the three arcs introduced into these calculations would have led to the same conclusions. Now as the whole measure of the Abbé DE LA CAILLE's arc was 410814 feet* = 68469 toises, if we divide it alternately by 56918.6 and 57037 toises, the difference of our two quotients in degrees will be the approximate sum of the small angles of error, which I have above supposed to exist under the form of $a \times b$.

* Vide HUTTON's Philosophical Dictionary, page 233.

Now, $\frac{68469}{56918.6} = 1° \ 12' \ 10''.54$ on the meridian by hypothesis.

And $\frac{68469}{57037} = 1° \ 12' \ 1''.55$ Ditto by De la Caille. The difference therefore is $8''.99$, to be divided between both stations of observation; and perhaps this will not appear too great, when the localities described in the foregoing pages are duly taken into consideration. We know that Schehallien did exert a lateral attraction sufficient to cause a deviation of $5''.8$*. We know further, that MM. Bouguer and De la Condamine had found by experiment that $7''\frac{1}{2}$ was the effect of Chimboraço; and I think I have made out undeniably that in no part of the globe is this disturbing force more likely to have prevailed than at Cape Town, whilst the site of observation at Klip Fonteyn could hardly have been free from its effects. The irregularities which may have attended the terrestrial measurements cannot here of course be taken into consideration, because they are as likely to have tended to increase as to diminish the length of the arc; but still the very uncertainty under which they labour is sufficient to throw doubt upon the whole operations; and at least I think it must be admitted, that the measurement in this quarter of the globe is by far too dubious to establish the theory which would assign to the southern meridians a different ellipticity from that found to obtain in the northern hemisphere.

Were the operations in themselves of sufficient importance, it might indeed be an object to measure them again; but as it would be impossible perhaps to place an instrument over the centre of the site of the Observatory at Cape Town, or indeed at any one of the stations, excepting Riebek's Castle, I see not what would be gained by such a procedure. It might be interesting, no doubt, to ascertain the exact latitude of both extremes of the arc by a series of triangles connecting them with the Observatory now about to be erected in this neighbourhood; and this, which will doubtless be hereafter done, may in able hands furnish a *new* datum respecting the attraction of mountains; but as to the arc itself, it seems to me to be too small to be of any weight, even were all other objections removed; and the labour of correcting the old results, except for the mere curiosity of the matter, would therefore be much better expended on a perfectly new series of triangle.

Such a series, instead of terminating at Klip Fonteyn, might very easily be

---

* Vide Vince's Astronomy, pages 99 and 100.

carried through the country of the Namaquas to the northern boundary of the colony, which would furnish a very pretty arc of nearly four degrees in amplitude, and I doubt not set for ever at rest the anomalous hypothesis of the different form of the two opposite hemispheres of the globe.

In no country indeed could a datum of this nature and of equal importance be obtained with less personal toil and suffering to the individuals engaged in it; for the climate is perhaps without a parallel on earth, the face of the country presents no appalling difficulties, and there is a degree of hospitality and readiness to oblige on the part of the colonists in general, which would render a sojourn amongst them highly pleasing and satisfactory.

In the hope that the description above offered will meet your expectation, I shall now bring my report to a close; but I shall at all times be happy, after we meet, to enter into any further explanation of such points as you may consider to require elucidation. In the mean time, allow me to subscribe myself

Your very obedient servant and sincere friend,

George Everest,
Capt. Bengal Artillery, Chief Assistant to
the Geod. Trig. Surv. of India.

XXII. *The Right Ascension and Declination of the Comet of January* 1821. *By* J. N. NICOLLET, *of the Royal Observatory at Paris.*

Read December 13, 1822.

Tems moyen, compté de Minuit, à Paris.	Ascension Droit.	Déclin. Boréal.
1821.		
Janv. le 21 à 20^h 13′ 15″	0° 36′ 28″	16° 59′ 30″
23 à 19 19 43	0 20 2	16 44 48
25 à 21 0 45	0 2 11	16 31 6
29 à 18 44 9	359 34 3	16 9 57
30 à 20 1 56	359 27 4	16 4 26
Fevr. le 6 à 20 9 28	358 48 51	15 32 23
9 à 18 55 7	358 36 12	15 21 54
22 à 19 39 19	357 48 50	14 37 50
23 à 19 5 58	357 45 16	14 34 49
26 à 19 11 7	357 32 57	14 22 45
Mars le 1 à 19 6 47	357 18 30	14 8 47

XXIII. *On the Correction of the Transit Instrument. By* J. J. LITTROW, *Director of the Imperial Observatory in Vienna; Assoc. of the Astron. Soc. of London, &c. &c. Communicated in a Letter to* FRANCIS BAILY, *Esq. F.R.S. Dated August* 1, 1822.

Read November 8, 1822.

THE present paper is intended as a continuation of the excellent one which you published in the first part of the Memoirs of the Society (page 59). In that paper, it is true, you have noticed only one error of this instrument (but certainly one of the most important of those to which it is usually liable), yet that would be sufficient if, as you presume, the question is only about small instruments of this kind, which may be placed in a common window: the object, in such cases being merely to satisfy the most common wants, without pretending to any particular accuracy. But the transit instrument is at the same time one of the simplest and most perfect of modern instruments used in practical astronomy: and through the great perfection to which it is now brought by TROUGHTON, REICHENBACH, &c., it produces such excellent results that the professed astronomer can no longer be allowed to neglect any thing which he may correct by a careful calculation. It is therefore my present intention to describe, in a short and distinct manner, the perfect management of this instrument, as I am in the habit of using it in my own observations.

This instrument, in practice, is subject to three principal errors: 1°. the optical axis of the telescope may not be quite perpendicular to the horizontal axis of the instrument; which is called the *error of collimation:* 2°. the axis itself may not be perfectly horizontal; which is called the *error of inclination:* 3°. this same axis may not stand exactly east and west; which is called the *error in azimuth*.

There may in fact be several other errors; such as arise from the ends of the horizontal axis not being perfectly cylindrical, or from being of an unequal size, or from their centres not lying exactly in a straight line: or when anomalies present themselves on account of the bending of the instrument, &c. &c.

But, as errors like these are generally very small in the best instruments, (it being in the power of the artist to give to the ends of the axis their proper shape and direction,) I shall here only consider the three errors above mentioned, as the most essential. Although it would not be difficult to discover also the others (that of the bending perhaps excepted) by proper observations, and take a correct account of them: a subject to which I may perhaps recur at another time; my present object being merely to avoid the three principal errors in the observations with this excellent instrument.

In the first place I shall enter into a general analytical investigation of the question, in order that I may with the greater certainty apply those principles to practice.

Analytical investigation.

Let us imagine a plane, perpendicular to the horizontal axis of the instrument: but, since that axis is not supposed to be exactly horizontal, nor standing immediately east and west, it is evident that the perpendicular plane will not be in the meridian, neither will it be a vertical plane. It will however cut the horizon in a point, whose distance from the meridian we shall make $15\,a$: and a perpendicular arc from the zenith to this plane, we shall make $15\,b$: lastly, if a telescope be affixed to this plane, we shall call the angle, which its optical axis forms therewith, $15\,c$. It will thus be readily seen that a denotes the error in azimuth, b the error in inclination, and c the error of collimation, each expressed in time.

Let us now determine a given point in this plane by means of the three rectangular co-ordinates $x\,y\,z$; where the axis x lies in the line in which the meridian cuts the equator: so that $x\,y$ is the plane of the equator, $x\,z$ the plane of the meridian, and the beginning of the co-ordinates the centre of the earth. Since the telescope (when c is not $= o$) describes a small circle of the sphere, its equation will have the following form,

$$z = \mathrm{A}x + \mathrm{B}y + \mathrm{C}.$$

If therefore we make $s =$ the hour angle of a star at the moment of its passing through the plane, and $\delta =$ its declination, we shall have, by taking the distance of the star from the earth as unity,

$$x = \cos\delta.\cos s.$$
$$y = \cos\delta.\sin s.$$
$$z = \sin\delta.$$

and therefore the preceding equation becomes

$$\left.\begin{array}{l} \text{A.}\cos s + \text{B.}\sin s + \text{C.}\sec \delta - \tan \delta = o \\ \text{and, for any two other stars, we have} \\ \text{A.}\cos s' + \text{B.}\sin s' + \text{C.}\sec \delta' - \tan \delta' = o \\ \text{A.}\cos s'' + \text{B.}\sin s'' + \text{C.}\sec \delta'' - \tan \delta'' = o \end{array}\right\} \quad (1)$$

By means of these three equations, we may, by elimination, determine the value of the unknown quantities A, B, C, in values of $s, \delta, s', \delta', s'', \delta''$. But, if A, B, C are known, the position of the plane itself will be known also: and we may then determine the hour angle of every star, by means of the equation

$$\sin s = \frac{\text{BD}+\text{A}\sqrt{\text{A}^2+\text{B}^2-\text{D}^2}}{\text{A}^2+\text{B}^2}$$

where $\text{D} = (\tan \delta - \text{C.}\sec \delta)$: and thus the theorem would be resolved. Without however stopping at this very easy elimination, I would observe that the first of the equations (1) gives

$$\text{A.}\cos s + \text{B.}\sin s = \text{D}.$$

and as our plane is always near the meridian, and consequently s is always very small,

$$\text{A} + \text{B}s = \text{D} \qquad \text{or}$$

$$s = \frac{\text{D}-\text{A}}{\text{B}} \qquad (2)$$

It remains therefore only to express the values of A, B, C, D, by means of the a, b, c, above mentioned. But, we easily see that we have

$$^{*}\ \text{A} = \tan\left(\phi + \frac{b}{a}\right)$$

$$\frac{\text{A}}{\text{B}} = -\tan a.\sin\left(\phi + \frac{b}{a}\right)$$

$$\frac{\text{D}}{\text{B}} = -\tan a.\cos\left(\phi + \frac{b}{a}\right)\tan \delta + \frac{c}{\cos \delta}$$

* The respectable and learned author of this paper appears to have committed an oversight in the assigned approximate values of A, $\frac{\text{A}}{\text{B}}$, and $\frac{\text{D}}{\text{B}}$. For c, and a in the expression the tangent of a, should have been expressed in *arc* and not in *time*, and they all require b to be small in comparison of a; which may not be the case. These remarks might imply that the value given for s in the equation immediately following is incorrect; but in the reduction of this equation to the next the effect becomes cancelled by similar counteracting operations. For instance, by considering sine of $\frac{b}{a}$ to be sufficiently near $\frac{b}{a}$; which will only be so when b is small in comparison of a; and by the use of arc and time indiscriminately for each other. I much regret that time (since I had the pleasure to peruse this paper which was *committed* to me) and distance, would not permit me sufficiently to notify to the learned author my ideas, to enable him to make the needful

where ϕ denotes the latitude of the place of observation. If we substitute these values in the equation (2) and bear in mind that, according to the nature of the theorem, the quantities $a, b, c,$ can also be only very small quantities, we obtain

$$s = a. \sin\left(\phi + \frac{b}{a}\right) - a. \tan \delta. \cos\left(\phi + \frac{b}{a}\right) + \frac{c}{\cos \delta}$$

or, when reduced,

$$s = a. \frac{\sin(\phi - \delta)}{\cos \delta} + b. \frac{\cos(\phi - \delta)}{\cos \delta} + \frac{c}{\cos \delta}$$

If therefore $\alpha =$ the apparent right ascension of the star, $t =$ the time of observation by the clock, and $x =$ the correction for the error of the clock, we have

$$s = \alpha - t - x$$

and, consequently the last equation becomes

$$\alpha - t - x = a. \sin(\phi - \delta) \sec \delta + b. \cos(\phi - \delta). \sec \delta + c. \sec \delta. \quad (3)$$

But, I would remark that, as far as regards practice, this method of determining the three errors a, b, c, and the correction x of the clock, from the mere observation of the culmination of stars, is inconvenient and tedious : neither in fact is it even efficient for determining *every one* of these quantities. The reason of this latter observation will be best seen by introducing a' and b' instead of the quantities a and b, so that we may have

$$a' = a. \sin \phi + b. \cos \phi$$
$$b' = b. \sin \phi - a. \cos \phi$$

whereby the equation (3) becomes

$$\alpha - t - x = a' + b'. \tan \delta + c. \sec \delta$$

and we may see that, by having even several equations of the latter form, we can only determine, by elimination, the quantity $(x + a')$ but never x alone nor

corrections himself; as his method is both ample and ingenious. I ought also to remark, that the plan here proposed considers s so small that cos of s may be put equal to radius; but, if I may be allowed to express myself loosely for the purpose of using but few words, this would not be the case, unless the instrument is nearer to adjustment than the star is to the pole. If, for instance, the instrument were perfectly adjusted in every respect but azimuth, and the error in azimuth be z, the distance of the star from the pole $= \varepsilon$, the latitude of the place of observation $=$ L, and ε and z both small; we should have, except in certain cases, sine of $s = \frac{z}{\varepsilon}$ sec of L, nearly. This remark evidently affects the whole of the paper; but as I conceive errors in adjustment, when we do not use stars nearer to the pole than the polar star, may be avoided, which would render this remark of consequence, I do not consider it important.—BENJAMIN GOMPERTZ.

a' alone; because in all these equations the co-efficients of x and a' are always the same.

We might indeed determine x by the method of equal altitudes: but such a determination is seldom of the utmost accuracy; nor is it of equal weight with the transit instrument, whose great advantage lies in determining the time immediately, without any foreign assistance, and with the greatest accuracy. Lastly, I would remark that the equation (3) will be entirely useless if the values of a and b should accidentally be such that $\tan \phi = \frac{a}{b}$, and $c = o$; because, in such case, all the stars would give the same correction of the clock

$$\alpha - t - x = \sqrt{a^2 + b^2}$$

although the telescope should move out of the plane of the meridian.

It follows therefore, from the preceding remarks, that the method of determining the errors of the instrument and of the clock by mere observations of the culminations of the stars should be abandoned as unfit for practice. I shall therefore endeavour to determine some of the quantities a, b, c, in another manner, more adapted to general use.

Practical application.

I will endeavour to explain this second important part in the manner in which it should be practically applied in observatories. For this purpose, let us make

a = the azimuth of the telescope: *positive* when (pointing to the south of the zenith) it deviates towards the *east* of the meridian.

b = the inclination of the rotatory axis towards the horizon: *positive* when the west side of the axis stands too high.

c = the error of collimation: *positive* when the optical axis of the telescope (pointing to the south) deviates towards the east.

x = the correction of the clock: *positive* when the clock is too slow. And it is presumed that the clock is regulated to sidereal time; or that it can be reduced thereto by the usual and well-known methods.

t = the time shown by the clock at the moment of observation.

α = the apparent right ascension of the star: for the lower culminations we must add 12^h.

δ = the declination of the star. South declinations are negative: and, for lower culminations, $\delta = (180 - \text{declination})$.

ϕ = the latitude of the place of observation.

This being granted, we have, by a proper regard to the signs of the trigonometrical functions, agreeably to equation (3),

$$\alpha = t + x + a. \sin (\phi - \delta). \sec \delta + b. \cos (\phi - \delta). \sec \delta + c. \sec \delta$$

Let us admit, for the sake of brevity,

$$m = \sin (\phi - \delta). \sec \delta \qquad n = \cos (\phi - \delta). \sec \delta$$

and for another star

$$m' = \sin (\phi - \delta') \sec \delta' \qquad n' = \cos (\phi - \delta'). \sec \delta'$$

then we shall have

$$\alpha = t + x + am + bn + c. \sec \delta$$
$$\alpha' = t' + x + am' + bn' + c. \sec \delta'$$

and from these two equations we immediately deduce, for the azimuth,

$$a = \frac{(\alpha - t) - (\alpha' - t') - b\,(n - n') - c\,(\sec \delta - \sec \delta')}{m - m'} \qquad \text{(a)}$$

By this equation we are enabled to find the error in azimuth, a, if the quantities b and c are known.

An important question arises here, "What two stars should be selected in order to determine, in the most advantageous manner, the error in azimuth, a?" The common rule, usually given for this purpose, "That one star should be near the zenith and the other near the horizon," is not the best. We may readily see that both the stars should be as near the pole as possible; one being above and the other below it. One of them therefore will be the pole-star; since it is very near the pole, and since its right ascension is now so well ascertained in consequence of the many observations that have been made on it. It is therefore requisite that, in every active observatory, this star should be observed every day, as it was done by BRADLEY, one of the best observers in any age or country. It would be best to observe the pole-star in *both* its culminations: but, since the inclination b might easily vary in the space of 12 hours (which is not to be apprehended from the collimation c), let us consider b as the inclination at the time of the first culmination, and b' as that of the second; and D as the declination of the pole-star. Whence we have, from the preceding equations,

$$a = \frac{12^{h} - (t' - t) + b. \cos (\phi - D). \sec D - b'. \cos (\phi + D). \sec D + 2c. \sec D}{2 \cos \phi. \tan D} \qquad \text{(a')}$$

Another question here arises, "How are the values of b and c to be determined?" The value of b is most readily found by means of the level: an instrument which is now so accurately constructed by REICHENBACH that, according to his assertion, the radius of curvature is two hundred English miles.

The level should be suspended in the proper manner to the ends of the ho-

rizontal axis: in this position let its inclination west be denoted by w, and east by e.

Then let the level be turned so that the end which before was to the west should now point to the east, and vice versâ. In this position it will give w' for the west, and e' for the east. If k be the value of one of the divisions of the level (which value may be determined by several well-known methods), we shall have

$$b = \frac{k}{60}\left[(w+w')-(e+e')\right]$$

and this is the required value of b in seconds of time, which, with its proper sign, should be used in the preceding equations.

In order to determine c, we must observe a star near the pole on the first wire. The time of this observation, reduced to the middle wire by means of the known distance of the wires, we shall call θ. Then turn the telescope, so that the end of the horizontal axis, which before pointed to the east, will now point to the west; and observe the star on the last wire (which will evidently be the *same* wire that, in the former position of the telescope, was the first). The time of this observation, reduced to the middle wire, we shall call θ'. This being granted, we have

$$c = \frac{\theta'-\theta}{2}\cos\delta$$

If b should be the inclination before, and b' after the reversion, we should have

$$c = \frac{(\theta'-\theta)+(b'-b)\,n}{2}\cos\delta$$

observing to take $\delta=(180-\text{the declination})$ in the lower culmination.

Some observers endeavour, by means of the level and of the screw which moves the wires, to get rid of the quantities b and c. But, in the first place it is difficult, if not impossible, entirely to remove the two errors; and, secondly, they are each of them (and particularly the first) very variable: so that it is always safest to determine the quantities by immediate observation, and take them into account.

If therefore the quantities a, b, c are known, we may obtain from every observation the correction of the clock, by means of the equation

$$x = \alpha - t - am - bn - c.\ \sec\delta$$

and, this quantity (x) being known, the observation of every other star will give the right ascension of such star, by means of the equation

$$\alpha = t + x + am + bn + c.\ \sec\delta$$

It therefore only remains for us to show the practical application of the above formulæ, by a few examples.

The following observations have been made, at Buda in Hungary, with one of REICHENBACH's six-feet transit instruments.

On May 18, 1822, the following observations were made of the pole-star, at the time of its upper culmination, for the purpose of determining the error of collimation. After the observations of the first three wires (there being seven of them) in the usual position of the telescope, time (by the clock) of the middle wire,

$$\theta = 0^h\,57^m\,18^s.3 \qquad \begin{cases} e = 37.0 & e' = 32.5 \\ w = 34.5 & w' = 39.0 \end{cases}$$

and after the observation of the same three wires, in the reversed position of the telescope, time (by the clock) of the middle wire,

$$\theta' = 0^h\,57^m\,13^s.4 \qquad \begin{cases} e = 41.0 & e' = 38.3 \\ w = 30.5 & w' = 33.2 \end{cases}$$

the value of each division on the scale being $k = 0{,}71$ seconds in space.

On May 19, the following observations were made in the usual position of the telescope:

MIDDLE WIRE.

Pole-star	0^h	57^m	$13^s.50$	$e = 38.9$, $w = 34.2$	$e' = 39.0$, $w' = 34.5$
Sirius	6	37	55 .97	$e = 40.2$, $w = 33.1$	$e' = 37.7$, $w' = 35.4$
Procyon	7	30	37 .01		
α Hydræ	9	19	29 .01		
Regulus	9	59	32 .11		
Pole-star, S. P.	12	57	37 .97	$e = 36.0$, $w = 36.0$	$e' = 33.0$, $w' = 39.0$

the daily rate of the clock was $0^s.151$.

Calculation of these observations.

For the quantity b, we have

$$b = 0^s{,}01183\left[(w + w') - (e + e')\right] \text{ in seconds of time.}$$

For the pole-star, we have $\delta = 88^\circ\,21'\,32''$, $\phi = 47^\circ\,29'\,12''$, $n = 26.4$, $b = 0.047$, $b' = -0.184$, $c = \frac{(\theta'-\theta)+(b'-b)\,n}{2}\cos\delta = -5.4992\cos\delta = -0.1575$.

May 19. Determination of the azimuth. Apparent right ascension of the pole-

star ($= \alpha$) $0^h\ 56^m\ 51^s.60$ in the upper culmination, $D = \delta = 88° 21' 31''.7$, $12 - (t'-t) = -24.47$

upper culmin. $b = -0.109$

lower culmin. $b' = +0.071$

so that the equation (a′) becomes

$$a = \frac{(12-t'+t)\cos D + b.\cos(\varphi - D) - b'.\cos(\varphi + D) + 2c}{2\cos\varphi.\sin D}$$

$$= \frac{-0.70080 - 0.08242 + 0.05094 - 0.31}{2\cos\varphi.\sin D}$$

$$= -0''.77151 \text{ westerly.}$$

For the four other stars we have then $a = -0.77151$ $c = -0.1575$ and $b = -0.11$; and for the correction of the clock

	Sirius.	Procyon.	α Hydræ.	Regulus.
	h m s	h m s	h m s	h m s
α	6 37 18.49	7 29 59.77	9 18 51.61	9 58 54.82
t	6 37 55.97	7 30 37.01	9 19 29.01	9 59 32.11
$\alpha - t$	−37.48	−37.24	−37.40	−37.29
$-am$	0.72	0.52	0.64	0.45
$-bn$	0.05	0.08	0.06	0.09
$-c.\sec\delta$	0.16	0.16	0.16	0.16
corr ec. $x =$	−36.55	−36.48	−36.54	−36.59

From the mean of these few observations we deduce the correction of the clock to be $x = -36^s.54$ at $8^h\ 22^m$.

The values of m and n for the latitude $\varphi = 47° 29' 12''$ as above mentioned, are

	m	n	sec δ
Sirius	0.9371	0.4577	1.043
Procyon	0.6700	0.7490	1.006
α Hydræ	0.8308	0.5735	1.010
Regulus	0.5831	0.8437	1.025

This is, I believe, the best method of using the transit instrument, and it is desirable that it should be thus treated in all observatories. As we have but few practical guides for the use of such instruments as pretend to any degree of perfection, I have thought that I might be rendering an acceptable service by forwarding it to you.

XXIV. *On the Aberration of Light. By* BENJAMIN GOMPERTZ, *Esq. F.R.S. With a Plate.*

Read June 14, 1822.

AN accurate knowledge of the aberration of light, arising from the velocity of its particles, being indispensably necessary in theoretical and practical astronomy, I conceive that, notwithstanding the subject has been extremely well discussed by many learned men, a fresh essay on the theory may be written in the expectation of a favourable reception by astronomers. In the contemplation of the sciences, there is, besides the pleasure arising from the acquirement of knowledge of practical utility, a peculiar charm bestowed by the reasoning faculty in a well-directed pursuit of facts; and though the results shown by the arguments are frequently considered to be the only objects of value by the unlearned, the man of absolute scientific ardour will often, whilst he is enraptured with the argument, have not the least interest for the object for which his argument was instituted. But the subject before us is eminently useful in the results, and may be extremely interesting in the mode of attaining them. It has been treated with peculiar analytical beauty in a note in BIOT's Astronomy; but notwithstanding the elegance of that method, I am persuaded that some readers would not object to a less symbolic theory, though leading chiefly to the same results: and, at the same time, I believe I am correct in saying, that though the theory, as it is generally received, is of sufficient approximative accuracy as far as regards any part of the solar system, it may be found inadequate in its application to what have been termed the fixed stars, should they possess in reality, as it is generally supposed they do, a proper motion: for that part of the received theory which leads us to suppose that it is only necessary to compound the relative velocity of the eye to the object, or relative velocity of the object to the eye, with the motion of light, if I reason correctly, would lead us, at the same time, to suppose that it was impossible for a star which is continually visible to us, to revolve in a period not greater than several hundred years about a centre (with a velocity

which is not much less than the velocity of the earth in its orbit) without the motion of the star becoming perceptible in consequence of the aberration of light; but I think a proper investigation of the theory will prove the contrary.

Suppose B an eye at rest (Plate IV. Fig. 1.), PSMN the orbit of a star in which it always moves, and that the whole orbit subtends an imperceptible angle at B. When the star is at P, let its direction and velocity be VP perpendicular to PB, and let PK be the velocity of the particles of light emitted from a body at rest; and supposing PK large in proportion to PV, then if the aberration of light to an eye B at rest, depended on the relative velocity of the object from the eye, in the manner of the received theory, and OB were drawn parallel to VK, OBP would be nearly the angle of aberration caused by the proper motion of the emitting body; but according to my idea, the aberration from the true place of it is within the angle which the orbit PSMN subtends at the eye; and consequently, according to the hypothesis, is imperceptible. Again: if we suppose B to be the eye on the earth revolving about the sun, and that the star revolves in the orbit PSMN, in a different periodic time to the earth in its orbit, and with a greater absolute velocity, it would happen, according to the notion I am combating, that sometimes, whilst the earth is at a certain point of its orbit, the aberration would be one way; but at another time, when the earth is at the same point of its orbit, the aberration would be another way, sometimes nothing, and sometimes more than double the aberration that the star would have if it were really a fixed star; and nothing like this appearing in the heavens, may be itself almost a presumptive proof that the notion is incorrect.

Art. 1. Let n be the velocity of light or space described, in the time one, by a particle of light projected from a fixed point (Plate IV. fig. 2.): B the eye in motion moving in the direction tB with a velocity equal to tB: A the place of an object at the time at which it emitted the particle of light which meets the eye at B; A$k=v$ the direction of the motion and velocity of the object when at A; and E the place of the object when the particle of light emitted from it, whilst at A, meets the eye at B. Then as the particle described AB in virtue of two motions, the one arising from the velocity n of light as emitted from a fixed point, or as the mean velocity of light emitted in all directions from a body in motion, and the other from the motion Ak of the object emitting the light, if in AB we take Ar to represent the velocity with which the particle of light describes AB, in consequence of the two motions, and or

be drawn equal and parallel to Ak, and Ao be joined, Ao will represent the velocity and direction which the particle of light which now meets the eye would have had if the object had been at rest; and will therefore be equal to n; and if the angle kAr be put equal to μ, and supposed given, we shall have in the triangle Aor, Ao=n, $or=v$, and the angle Aro=μ, to find Ar, which we will call n'. Draw oh perpendicularly to AhB, and we shall have $oh=v$ sine of μ, $rh=v$ cos of μ, A$h=\sqrt{n^2-\overline{v \text{ sine of } \mu}|^2}$ and n' or A$r=\sqrt{n^2 \; \overline{v \text{ sine of } \mu}|^2}$ $-v$ cos of μ; and this, if n be very great in proportion to v, will be nearly equal to n. And we see that if the eye be at rest, and the object emitting the light be endowed with proper motion, it will appear to be in that place in which it was at the moment it emitted the particle of light which now meets the eye; and the aberration from this cause will be ∠ABE, and in that plane.

Art. 2. Between A and B, take Bp equal to Ar; and draw pw equal and parallel to tB and join wB; then the velocity pB of the particle of light may be considered as the result of the two velocities pw, wB: but w is the direction and velocity common also to the eye; it follows that the perceptible direction in which the light arrives at the eye is wB; and, supposing pB great in proportion to tB or its equal pw, we have the sine of ∠pBw equal to $\frac{pw \text{ sine of } wp\text{B}}{\text{B}p}$, that is, $\frac{\text{B}t}{\text{B}p}$ sine of ABt= $\frac{\varepsilon}{n'}$ sine of ABt.

Art. 3. Draw SB parallel to Ao, cutting Ak produced in S; then if the object A had continued to move uniformly and in the same direction Ak, during the time which the particle of light took in its passage from its emission at A to its arrival at B, the situation of the body would be S, and therefore ∠ABS will be the part of the aberration of light arising from the motion of the object when the eye at rest at B perceives it; and E and S will coincide; and considering n large in comparison of v, the angle ABS will be nearly $\frac{v}{n}$ sine of μ, and will be a very small angle. And when we are considering the aberration of light by which we are affected from any of the bodies in the solar system, from their proper motion, we may, without fear of error, consider the path described by the body during the time that a particle of light is moving from it to us to be rectilineal, and to be described by an uniform velocity; and in consequence we may, as far as regards our planetary system, consider the angle ABS as the real angle of aberration caused by the proper motion of the body emitting the light; but the case may be very different with respect to the aberration of

light caused by the proper motion of the stars at an immense distance. If, for instance, the star in question were in the ecliptic, and so near to us that its annual parallax of longitude amounted even to one second of a degree, and that all emitted particles of light moved always through a space equal to the radius of the earth's orbit in 8′ of time, then would the light in moving from the star to us be about $8\times60\times60\times60$ minutes, or 1200 days, and consequently the star may have a very different direction of motion, velocity and place when the light arrives at us, from that which it had when the light was emitted from it: and E its real place at the time of the eye at B perceiving it, may even not be in the line AS; so that neither the aberration caused by the proper motion of the star, nor the plane in which it is made, may be the same as this mode of investigating it would indicate. And the angle ABE of aberration depends on the place of the body when it emitted the particle of light, and when the said particle meets the eye. If the velocity of the body be small in proportion to the velocity of light, this angle will be small. And therefore I think we should admit, in estimating the aberration of light proceeding from the three causes, of which one is the velocity with which light is emitted from a fixed body, the second the velocity with which the body emitting the light moves in case that body be not fixed, and the third the velocity of the eye, that it is not rigorous to take the relative velocity of the eye from the emitted body conjointly with the velocity of light emitted from a fixed body, as the cause of such effective aberration: and though it may be considered to give the true result sufficiently near in estimating the aberration to our earth from any visible part of the solar system, it happens only from a particular state of the case, but not at all from a general coincidence of effect: and that this may appear more evident, I observe, that when the eye at rest receives a particle of light from a body, the direction in which it conceives the body to be is not at all altered by the velocity by which the particle of light arrives, but it perceives it to be in that direction in which the particle of light is moving when it meets the eye; and it is also quite unconnected with the true place of the body at the time that the light is received. If the body had no motion (the eye being at rest also, and *taking for granted that particles of light describe right lines*), it will then perceive it in the direction in which it is, provided it still have existence; but if the body had a motion, it will be perceived in the place where it was when it emitted the particle, whether it be there or not, or even should it be annihilated. The motion of the body emitting the light has the effect of sending the particles of a different beam to the eye, but does not alter the direction in which

the eye receives a beam from the place from whence that beam was sent; and the aberration from the proper motion of the moving body has no immediate connection deserving attention with the velocity of the body at the time of emitting the light, if that velocity be very small in proportion to the velocity of light; but the connection is with the ultimate change of place of the body.

Art. 4. But in the case of the star's not moving in a right line and with a constant velocity during the whole time that a particle of light is moving from the star to the eye, for the sake of analogy let us conceive a supposititious body to move with an uniform velocity V from the original place A of the star from whence the particle of light was emitted, and commencing its motion at the time that a particle of light is emitted from the star to arrive at the ultimate place E, at the same time that the star by its motion arrives there, that is, at the time the particle of light arrives at the eye, and we shall have AE : V :: AB : n'; and therefore, if $\angle$ EAB be put $=\mu'$, the sine of the $\angle$ ABE being equal to $\frac{\text{sine of } \mu' \times \text{AE}}{\text{AB}}$ will be $=\frac{\text{V}}{n'}$ sine of μ': so that the angle of aberration to an eye at rest, caused by the proper motion of the star and the velocity of emitted light, depends on five different data; the velocity of the star when it emits the particle of light, the direction of that velocity, the velocity of light emitted from a star at rest (*or the mean velocity of light emitted from a star in motion*), the velocity necessary for our supposititious body to move in a straight line, from the place where the star was when the particle of light was emitted, to the place of the star when the light is received, in the time the particle of light was moving from the star to the eye, and the angle μ. And the plane in which that aberration is made does not directly, and may not at all, depend on the direction in which the star was moving when the particle of light was emitted; but on the direction of the right line AE, or, in other words, the $\angle \mu'$: but if the velocity of light be great with respect both to v and V, as we have already intimated, for n' we may write n with sufficient accuracy: and we now observe also in that case, that for the sine of the angle ABE we may with sufficient accuracy take the angle itself; it follows, that the angle of aberration will become $\frac{\text{V}}{n}$ sine of μ' sufficiently near, in which v and μ, two of the five data, disappear. Draw NB parallel to EA, N and E being on the same side of AB; then, instead of supposing the supposititious body to have moved from its original position A, and the eye at rest, we suppose our moving supposititious body to have been at rest, and to have remained stationary in its ultimate position, whilst the eye be supposed (contrary to being in its real state

of rest) to have a velocity V, when the particle of light meets it in the contrary direction to that in which the supposititious body was conceived to have, it follows (from Art. 2.) that the aberration would be sufficiently near the same quantity $\frac{V}{n}$ sine of μ'.

Art. 5. And therefore, when the eye has a proper motion of its own (as in Art. 1.), and the star has also a proper motion (as in Art. 1.), we may suppose, instead of these data, that the eye has two motions, one in the direction tB, and the other V, the velocity of light being considered n' instead of n, in the direction NB, equal to the presupposed velocity of our supposititious body, but in a contrary direction, or that it has but one motion compounded out of the two; and if this be ZB in quantity and direction, the resulting aberration will be in the plane ZBA, and the sine of the resulting angle of aberration will be $\frac{ZB}{n'} \times$ sine of ZBA; and we may, for the sake of distinction, call tB the true velocity and direction of the eye, NB the supposititious velocity and direction of the eye; and ZB the efficient velocity and direction of the eye. And in conformity will ABt be the plane of aberration caused by the real velocity of the eye, and the plane ABZ the plane of efficient aberration. If the star has not changed its place, BZ will coincide with Bt; but if the eye be at rest, and the star has changed its place, BZ will coincide with Bn.

Art. 6. Fig. 3. Let T be the eye, Tt what we have termed the efficient velocity of the eye, TS the line passing through the eye and star or supposititious body, as the case may be, but of such a magnitude that it may represent the velocity of the particle of light issuing from the star, which we have represented by n', and which we have shown will be sufficiently nearly represented by n, the mean velocity of the particles of light emitted from a moving body, if the velocity of the moving body and that of the eye are small in proportion to the velocity of light. And the angle Tst will be the quantity of the angle of efficient aberration. About T as a centre and radius ST, let a sphere be described; let Tq cut the angle STt, so that $\angle$STq be $= \angle t$ST; and let Tq cut the sphere in q, then will q be the apparent place of the star or supposititious body in the sphere to the eye T at the centre. Draw the great circle SqN of the sphere cutting Tt in N; let Q be the equinoctial point, and draw the great circle QN, of which let P be the pole; and let PSM be a great circle cutting QN in M; and let π be the pole of some other great circle passing through Q, and draw the great circles PπW cutting QNM in W; draw πSZ cutting QNM in Z, SΣY perpendicularly to PS cutting PπW in Σ and

QNM in Y. Also draw the great circles Pq, πq, then is the sine of the absolute efficient aberration $Sq = \frac{Tt}{n'} \times$ sine of SN; and supposing n' immensely great in proportion to Tt, this may be taken for the angle itself: and this aberration may be divided into the two relative aberrations, the one the aberration measured by the angle SPq, which would be the aberration in longitude, or quantity by which the apparent longitude should be increased to obtain the true longitude if P were the pole of the ecliptic, but it would be the aberration in right ascension, if P were the pole of the equator; and the other the excess of Pq above PS, which would be the aberration in latitude, or quantity to be added to the apparent latitude to obtain the true latitude, if P were the pole of the ecliptic; but it would be the aberration in declination if P were the pole of the equator. But considering Sq small, the angle of aberration $SPQ = Sq \times \frac{\text{sine of } SqP}{\text{sine of PS}}$, that is, taking for Sq its value above, $\frac{Tt}{n'} \times$ (sine of $SqP) \times \frac{\text{sine of SN}}{\text{cosine of SM}}$; and this, considering sine of Psq as = sine of sqP, because the spherical triangle SMN is right-angled at M, is equal to $\frac{Tt}{n'} \cdot \frac{\text{sine of MN.}}{\text{cos of SM.}}$ Call this Expression 1. Also Sq being small, for $Pq - PS$ we may take $Sq \times \cos$ of PqS, or $\frac{Tt}{n'}$ sine of SN cos of PqS, or sufficiently near $\frac{Tt}{n'} \times$ sine of SN $\times$ cos of NSM; and this because cos of MSN = tan of MS. cot of SN, or $\frac{\text{sine of MS}}{\text{cos of MS}} \cdot \frac{\text{cos of SN}}{\text{sine of SN}}$, may be written $\frac{Tt}{n'} \cdot \frac{\text{sine of MS}}{\text{cos of MS}} \cdot$ cos of SN; and consequently, by the property of the right-angled spherical triangle SMN it is equal to $\frac{Tt}{n'}$ sine of MS cos of MN. Call this Expression 2. And, as above observed, Expression 1 will be the aberration in longitude, if QN be the ecliptic; but it will be the aberration in right ascension, if QN be the equator; and accordingly Expression 2 would be either the aberration in latitude or in declination. If the aberration be that caused only by the motion of the earth in the ecliptic, without considering any proper motion of the star, QN would be the ecliptic; if from the motion only of the earth about its axis, QN will be the equator. If the aberration in contemplation arises only from the proper motion of a planet in its orbit, S being the planet, and the eye at rest, it will be neither the equator nor the ecliptic; and consequently we see the advantage of introducing the pole π of some other great circle.

Art. 7. Sq being still considered small, the aberration Sq will cause an aberration about the pole $\pi = \frac{Sq. \text{ sine of } \pi Sq}{\text{sine of } S\pi}$, and an aberration or quantity to be added to

the apparent distance from the great circle, of which π is the pole, to give the true distance $=-\mathrm{S}q.\cos$ of $\pi\mathrm{S}q$; the former of these by putting $\frac{\mathrm{T}t}{n'}.$ sine of SN for Sq, will become $\frac{\mathrm{T}t}{n'}.\frac{\text{sine of }\pi\mathrm{S}q.\ \text{sine of SN}}{\text{sine of }\pi\mathrm{S}}$, or per spherics, $\frac{\mathrm{T}t}{n'}\ \frac{\text{sine of Z}}{\text{sine of }\pi\mathrm{S}}$ .sine of $\mathrm{NZ}=\frac{\mathrm{T}t}{n'}.\ \frac{\text{sine of Z}}{\text{sine of }\pi\mathrm{S}}.$ sine of (ZW+NW). Call this Expr. 3. And I observe that, because the angle W of the spherical triangle πWZ is right, we have cos of Z=sine of ZπW. cos of πW, and tangent of ZW=sine of πW .cot of ZπW. Again, the aberration $\pi q-\pi$S, or quantity to be added to the apparent distance from the great circle, of which π is the pole, to give the true distance being equal$-\mathrm{S}q.\cos$ of $\pi\mathrm{S}q$, or (by writing $\frac{\mathrm{T}t}{n'}$ sine of SN for SQ) $\frac{-\mathrm{T}t}{n'}$ sine of SN. cos of $\pi\mathrm{S}q$, that is, because ZSY is a right $\angle$ by construction, $\frac{-\mathrm{T}t}{n'}$ sine of SN. sine of NSY, or its equal $\frac{-\mathrm{T}t}{n'}$ sine of YN. sine of Y; that is, $\frac{-\mathrm{T}t}{n'}.$ sine of Y. sine of (WN$-$WY). Call this Expression 4; where, since the spherical triangle ΣWY is right-angled at W, and the spherical triangle $\pi\mathrm{S}\Sigma$ right-angled at S, we have tan of $\mathrm{WY}=\frac{\text{sine of }\Sigma\mathrm{W}.\ \text{tan of S}\pi}{\text{sine of S}\Sigma}=\frac{\text{sine of }\Sigma\mathrm{W}.\ \text{tan of }\Sigma\pi}{\text{sine of S}\pi\mathrm{W}.\ \text{sine of }\pi\Sigma}$; but sine of ΣW=sine of (πW$-\pi\Sigma$)=sine of πW. cos of $\Sigma\pi-$cos of πW .sine of $\Sigma\pi$; and therefore tan of $\mathrm{WY}=\frac{\text{tan of S}\pi}{\text{sine of S}\pi\mathrm{W}}\times$(sine of πW. cot of $\Sigma\pi$ $-$cos of πW)=per spherics $\frac{\text{sine of }\pi\mathrm{W}.\ \text{cos of S}\pi\Sigma-\text{tan of S}\pi.\ \text{cos of }\pi\mathrm{W}}{\text{sine of S}\pi\mathrm{W}}$. and because cos of Σ is equal to cos of WY .sine of Y, and also=cos of Sπ. sine of S$\pi\Sigma$; it follows that sine of $\mathrm{Y}=\frac{\text{cos of S}\pi.\ \text{sine of S}\pi\Sigma}{\text{cos of WY}}$.

Art. 8.* If the aberration be that of a fixed star caused by the earth's

---

* LEMMA. (Fig. 4. Plate IV.)

Let the motion of the earth T about the sun F be in the order $p\mathrm{T}\pi$, and in consequence this order will be from west to east: p being the perihelion, draw TQ from the earth to the vernal equinox, TH parallel to Fπ; H and π being both on the same side of FT, put the longitude of the sun or angle QTF=$\odot$; the longitude of the sun when the earth is in the perihelion $=\varphi$, that is, in our figure the $\angle$QTH formed on the contrary side of T that Fπ is situate, therefore $\angle p$FT being $=\angle$FTH, the angle being taken on the side in which Q lies, is $=360-(\varphi-\odot=360+\odot-\varphi$; let the mean distance of the earth from the sun $=1$. Eccentricity of the orbit equal to e, the earth's mean velocity in its orbit $=\varepsilon$, MTt the tangent of the earth's way, FM$\perp$MTt; then

motion about the sun, then will QNM be the ecliptic; and taking the mean distance of the earth from the sun unity; the earth's mean velocity$=\varepsilon$; eccentricity$=e$, $\odot$ the sun's geocentric longitude at the time in question, ϕ the sun's geocentric longitude when in the perigee; then will $Tt=\varepsilon.\Big[1+e.\cos \text{ of}$

---

since $TF=\frac{1-e^2}{1+e.\cos \text{ of } pFT}$ it is $=\frac{1-e^2}{1+e.\cos \text{ of } (\odot-\varphi)}$; and tangent of $\angle MFT=\frac{e.\text{ sine of } pFT}{1+e.\cos \text{ of } pFT}$ $=\frac{e.\text{ sine of } (\odot-\varphi)}{1+e.\cos \text{ of } (\odot-\varphi)}$: and the velocity of the earth at T, which is in the direction $Tt=\frac{\varepsilon}{MF}=\frac{\varepsilon}{TF.\cos \text{ of } MFP}$. Here it is plain, that if e be so small that all powers of e above the first may be neglected, we may take for TF the quantity $\frac{1}{1+e.\cos \text{ of } (\odot-\varphi)}$; and that for the angle MFT we may take e sine of $(\odot-\varphi)$; and for $\angle MTF$ we may take $90-e$ .sine of $(\odot-\varphi)$; and for cos of MFT we may take unity; which will make the velocity of T come out $\frac{\varepsilon}{TF}$, or $\varepsilon\big[1+e.\cos \text{ of } (\odot-\varphi)\big]$. And the angle MTQ of the earth's way with the line passing through the earth and vernal equinox $=90+\odot-e.$ sine of $(\odot-\varphi)$; or adding 180° we have the $\angle QTt$ in the order QMH, in which the longitude is reckoned $=270^\circ+\odot-e.$ sine of $(\odot-\varphi)$. Moreover, as the mean and true longitude of the sun is supposed the same when the earth is in the perihelion, if M be the mean longitude of the sun when the earth is at T, we shall have $\odot=M+2e.$ sine of $(M-\varphi)$, sufficiently near for our purpose; and therefore the above-formed $\angle QTt$ in the order QMH, in which the longitude is reckoned $=270^\circ+M+2e.$ sine of $\overline{M-\varphi}-e.$ sine of $(\odot-\varphi)$; or sufficiently near $270^\circ+M+e.$ sine of $(M-\varphi)$, and the velocity sufficiently near $\varepsilon.\big(1+e.\cos \text{ of } (M-\varphi)\big)$.

If T be some other planet, p its perihelion, and ΩFo cutting the tangent MTt in o be the intersection of the orbit Fpo of the planet revolving about the sun F, and the ecliptic, the eccentricity of the orbit e' to the mean distance unity and ε' the mean velocity of the planet; $v=\angle\Omega FT$ the heliocentric distance of the planet from the node, $\varpi=\Omega Fp$ the distance of the perihelion reckoned from west to east, from the node, and we have *mutatis mutandis* from above the velocity of the planet at $T=\varepsilon'.\big[1+e'\cos \text{ of } \overline{v-\varpi}\big]$ nearly; and the $\angle oTF=90^\circ-e'.$ sine of $\overline{v-\varpi}$ and $\therefore \angle MoF$ or the angle made by the direction contrary to the planet's motion with the line of the node $=v-180^\circ+oTF=v-90^\circ-e'.$ sine of $(v-\varpi)$. If K be put for the mean heliocentric distance of the planet from the node, we shall have $v=K+2e'$ sine of $(K-\varpi)$ nearly; and $\therefore \angle MoF=K-90^\circ+e'$ sine of $(K-\varpi)$ nearly, and the velocity$=\varepsilon'.\big[1+e'\cos \text{ of } \overline{K-\varpi}\big]$ nearly.

If T be a comet moving in a parabola from west to east, ε' the velocity at the perihelion p. $pF=a$, then as it is well known angle $MFT=\frac{1}{2}\angle pFT$, and $FT=\frac{2a}{1+\cos \text{ of } PFT}=\frac{a}{(\cos \text{ of } MFT)^2}$; $\therefore MF=\frac{a}{\cos \text{ of } MFT}=\frac{a}{\cos \text{ of } \frac{1}{2}pFT}$, $\therefore$ velocity at $T=\frac{a.\varepsilon'}{FT}=\varepsilon'\cos \text{ of } \frac{1}{2}pFT=\varepsilon'.\cos \text{ of } \frac{1}{2}(v-\omega)$; and the angle $OTF=90^\circ-\frac{1}{2}(v-\varpi)$; the angle $MOF=v-180^\circ+OTF=\frac{1}{2}(v+\omega)-90$ the angle the direction contrary to the comet's motion makes with the line of the node.

$(\odot-\phi)\big]$, e^2 and the higher powers being neglected. And QN or $\angle$ QTN $=$ $\odot-e$. sine of $(\odot-\phi)+270°$, neglecting the powers of e above the first: then if l be the longitude of the star, and λ the latitude; because MN $=l-$QN, we have the aberration in longitude (from Expression 1. Art. 6.) $=\frac{\varepsilon}{n'}$ $\cdot\frac{1+e.\ \text{cos of}\ (\odot-\phi)}{\text{cos of}\ \lambda}\times\text{cos of}\big[\odot-l-e.\text{sine of}\ (\odot-\phi)\big]$; and recollecting that when e is small, we may for $(1+e.\ \text{cos of A.}).\ \text{cos of}\big[\text{B}-e.\ \text{sine of A}\big]$ take cos of B$+e$. cos of (B$-$A), a theorem which may be easily demonstrated, the aberration in longitude becomes $\frac{\varepsilon}{n'\ \text{cos of}\ \lambda}\big[\text{cos of}\ (\odot-l)+e.\ \text{cos of}\ (\phi-l)\big]$. And for the aberration of latitude from Expression 2, we have $\frac{\varepsilon\ \text{sine of}\ \lambda}{n'}$ $.\big[1+e.\ \text{cos of}\ (\phi-\odot)\big].\ \text{cos of}\big[l-270°-\odot-e.\ \text{sine of}\ (\phi-\odot)\big]$ equal to $\frac{\varepsilon.\ \text{sine of}\ \lambda}{n'}.\big[\text{cos of}\ (l-270°-\odot)+e.\ \text{cos of}\ (l-270°-\phi)\big]=\frac{\varepsilon\ \text{sine of}\ \lambda}{n'}\big[\text{sine of}\ (\odot-l)+e.\ \text{sine of}\ (\phi-l)\big]$.

And if π be the pole of the equator, ω the obliquity of the ecliptic, then will Q be the pole of PπW; $\because$ if the right ascension of the star $=a$, and declination $=d$, since NW $=90-$QN $=-180-\odot+e$. sine of $(\odot-\phi)$; and therefore ZW$+$NW $=-180°+$ZW$-\odot+e$. sine of $(\odot-\phi)=-270°+$QZ$-\odot+e$ .sine of $(\odot-\phi)$ and conseq. sine of ZW$+$NW$=$cos of $(\odot-e$. sine of $(\odot-\phi)-$ZQ$)$; & $\because$ By Expression 3, Art. 7, we have the aberration in RIGHT ASCENSION $=\frac{\text{sine of Z}}{\text{cos of}\ d}\times\frac{\varepsilon}{n'}\big[1+e.\ \text{cos of}\ (\phi-\odot)\big]\times\big[\text{cos of}\ \odot-\text{ZQ}-e.\ \text{sine of}\ (\odot-\phi)\big]$; which from the observation above, e being small, is $=\frac{\varepsilon\ \text{sine of Z}}{n'\ \text{cos of}\ d}\big[\text{cos of}\ (\odot-\text{ZQ})+e.\ \text{cos of}\ (\phi-\text{ZQ})\big]$ where from Art. 7, cos of Z$=$sine of $(a-90°)$. sine of $\omega=-$cos of a. sine of ω; or we may find sine of Z directly, because it is equal to $\frac{\text{sine of}\ \pi\text{Q. sine of Z}\pi\text{Q}}{\text{sine of ZQ}}$ and therefore the ABERRATION IN RIGHT ASCENSION may be written $\frac{\varepsilon}{n'}\frac{\text{sine of}\ a}{\text{cos of}\ d.\ \text{sine of ZQ}}\big[\text{cos of}\ (\odot-\text{ZQ})+e.\ \text{cos of}\ (\phi-\text{ZQ})\big]$

And from expression 4, Art. 7, we have the ABERRATION IN DECLINATION $\frac{\text{T}t}{n'}$ sine of Y .sine of (WN$-$WY) $=-\frac{\text{T}t}{n'}$ sine of Y$\times$sine of (QY$-$QN) $=-$ $\frac{\varepsilon}{n'}$sine of Y.$\big[1+\varepsilon.\ \text{cos of}\ (\odot-\phi)\big]\times\text{sine of}\ \big[\text{QY}-270°-\odot+e\ \text{sine of}\ (\odot$

$-\phi)\Big]=-\frac{\varepsilon}{n'}$ sine of Y. $\Big[1+e$ cos of $(\odot-\phi)\Big]\times$ cos of $\Big[\odot-$ QY $-e.$ sine of $(\odot-\phi)\Big]$; that is by the observation, e being small, $-\frac{\varepsilon}{n'}$ sine of Y. $\Big[$cos of $(\odot-$ QY$)+e.$ cos of $(\phi-$QY$)\Big]=-\frac{\varepsilon}{n'}$sine of Y$\Big[$sine of $(\odot+$WY$)+e.$ sine of $(\phi+$ WY$)\Big]$; or because WY $=180^\circ-$supp. of WY, $\frac{\varepsilon}{n'}$ sine of Y $\times\Big[$sine of $(\odot-$ supplement of WY$)+e.$ (sine of $\phi-$ supplement of WY$)\Big]$; sine of Y being $=$ $\frac{\text{sine of } d.\ \text{sine of } (a-90^\circ)}{\text{cos of WY}}=\frac{-\text{sine of } d.\text{cos of } a}{\text{cos of WY}}=\frac{\text{sine of } d.\ \text{cos of } a}{\text{cos of supplement of WY}}$, and $-$tan of WY, or tangent of supplement of WY $=\frac{\text{cos of } \omega.\ \text{sine of } a-\text{cos of } d.\ \text{sine of } \omega}{\text{cos of } a}$.

If these quantities should be required in mean motions, we may transform them by substituting $M+2e.$ sine $(M-\phi)$, in the room of $\odot$ (see the note at the commencement of Art. 8): thus we have by substitution in the aberration in longitude, and neglecting the second and higher powers of e, that quantity $=$ $\frac{\varepsilon}{n' \text{ cos of } \lambda}\Big[$cos of $(M-l)-2e.$ sine of $\overline{M-l}\times$ sine of $\overline{M-\phi}+e.$ cos of $(\phi-l)\Big]$ $=\frac{\varepsilon}{n' \text{ cos of } \lambda}\times\Big[$cos of $(M-l)-e.$ (cos of $2M-l-\phi)\Big]$. The aberration in latitude by a similar reduction $=\frac{\varepsilon.\ \text{sine of } \lambda}{n'}\times\Big[$sine of $\overline{M-l}+e.$sine of $\overline{2M-l-\phi}\Big]$ The aberration in right ascension $=\frac{\varepsilon}{n'}$ sine of $a\Big[$cos of $(M-$ZQ$)+e.$ cos of $(2M-$ZQ$-\phi)\Big]$. And the aberration in declination $=\frac{\varepsilon}{n'}$sine of Y. $\Big[$ sine of $(M-$supp. of WY$)+e.$ sine of $(2M-$supplement of WY$-\phi)\Big]$; M representing the mean longitude of the sun. The same expressions may also be immediately derived from the expressions of Art. 6; thus for the aberration in longitude we have by the note just alluded to $\angle$ QTt of figure 4, Plate IV. that is, the angle measured by arc QN of figure 3, Plate IV. $270^\circ+M+e.$ sine of $(M-\phi)$ nearly, and the velocity of the earth nearly $=\varepsilon\Big[1+e.$ cos of $(M-\phi)\Big]$ consequently merely by writing these for their equivalents in the expression $\frac{Tt}{n'}.\frac{\text{sine of MN}}{\text{cos of SM}}$ the aberration in longitude of Art. 6, No. 1, and recollecting that whatever G may be, sine of G$-270^\circ=$ cos of $(\pm G)$, it will stand $\frac{\varepsilon}{n'.\ \text{cos of } \lambda}$ $.\Big(1+e.$ cos of $(M-\phi)\Big)\times$cos of$\Big(M-l+e.$ sine of $(M-\phi)\Big)=\frac{\varepsilon}{n'.\ \text{cos of } \lambda}\times$ $\Big(1+e.$ cos of $\overline{\phi-M}\Big)\times$cos of $\Big(M-l-e$ sine of $\overline{\phi-M}\Big)=$by the theorem

used above, neglecting e^2, &c., $\frac{\varepsilon}{n'.\cos\text{ of }\lambda} \times \left[\cos\text{ of }\overline{M-l}+e.\cos\text{ of }\overline{2M-l-\phi}\right]$ the same as above. And in the same way may the other expressions be immediately obtained.

Art. 9. To find the aberration owing to the motion of the earth about its axis (Fig. 5, Plate IV.), describe a sphere through the eye T of the observer passing through the star S. Let QKNM be a great circle of the sphere parallel to the earth's equator, and in this sphere let P be the north pole of the equator and Z the zenith of the place supposed to be north of the equator; let PZK, PSM respectively be the meridians of the place and of the star, Q the equinoctial point termed the first point of Aries, QKN being taken in the order of the signs, that is from west to east, and which is in the order in which the eye moves. And the right line TN being in the equator perpendicular to TK, and directed as in the figure, it follows that if ε'' represent the velocity of the earth at the equator about its axis, E the hour angle reckoned from west to east, and in TN we take $Tt=\varepsilon''$. sine of PZ, it will represent the velocity of the eye, and therefore expression 1, Art. 6, since NM=QKM−QKN=QKM−90°−QK=E−90°, will give the aberration in right ascension or quantity to be added to the observed right ascension to have the true$=\frac{\varepsilon''}{n'}.\frac{\text{sine of PZ}}{\cos\text{ of SM}}$ $\times$ sine of NM$=-\frac{\varepsilon''}{n'}$.cos of latitude of the place $\times \frac{\cos\text{ of E}}{\cos\text{ of star's declination}}$. And expression 2 of the same Article gives the aberration in declination or quantity to be added to the observed declination, to have the true$=\frac{\varepsilon''}{n'}$. sine of PZ .sine of MS. cos of $(90°-E)=\frac{\varepsilon''}{n'}$. sine of PZ. sine of MS$\times$sine of E$=\frac{\varepsilon''}{n'}$ .cos of latitude of the place$\times$sine of declination of the star$\times$sine of the hour angle.

Art. 10. If S (Fig. 3. Plate IV.) be a planet, its latitude λ, and longitude l, whose motion is in the direction parallel to NT and the velocity be equal in quantity and direction to tT; and T☊ be the line of the nodes; the plane NT☊ will be parallel to the plane of the planet's orbit. Let the great circle N☊ be the section of the sphere by the plane QT☊; TQ and T☊ being both in the ecliptic, and let πS cut the ecliptic, whose pole is π, in C; then Q☊ is the longitude of the node; put this equal to k; put ☊N or the angle the line of the planet's motion makes with the line of the node equal to h, and the ∠N☊C or the inclination of the planet's orbit to the plane of the ecliptic=☊. Now Art. 7,

the aberration in longitude arising from the planet's motion alone $=\frac{Tt}{n'}\frac{\text{sine of } \pi sq.\ \text{sine of SN}}{\text{sine of } \pi S}$; that is, drawing the great circle NB to cut πS at right angles at B, $\frac{Tt}{n'}\times\frac{\text{sine of NB}}{\text{sine of } \pi S}$. Let BN cut the ecliptic in D; then, because B and C are right angles, D is the pole of BC; therefore sine of NB=(cos of ND=) sine of NΩ. sine of ΩD. cos of Ω+cos of ΩN. cos of ΩD; and ΩD is equal to ΩC$-90°=$QC$-$Q$\Omega-90°$; whence the aberration in longitude $=\frac{Tt}{n'\cos\text{ of }\lambda}\Big[-\text{sine of } h.\ \cos\text{ of } (l-k).\ \cos\text{ of } \Omega + \cos\text{ of } h.\ \text{sine of } (l-k)\Big]$ or because $\cos\text{ of } \Omega=1-2\,\overline{\text{sine of } \tfrac{1}{2}\Omega}|^2$ it becomes $\frac{Tt}{\cos\text{ of } \lambda.\, n'}\times\Big[-\text{sine of} (h+k-l)$ $+2.\,\overline{\text{sine of } \tfrac{1}{2}\Omega}|^2\ \text{sine of } h\times\cos\text{ of } (l-k)\Big]$. And the aberration in latitude by the same Article, being $\frac{-Tt}{n'}$ sine of SN .cos of $\pi Sq=\frac{Tt}{n'}$ .sine of SN .cos of BSN$=\frac{Tt}{n'}$. sine of BN. cos of ZSN; but cot of ZSN=sine of BS. cot of BN; $\because$ sine of BN .cot of ZSN=cos of BN. sine of BS; but ND is the complement of NB, and the arc BC (or angle D, because D is the pole of BC) is equal to $\lambda-$BS; hence the aberration in latitude $\frac{Tt}{n'}$ sine of ND. sine of $(\lambda-\angle\text{NDC})=\frac{Tt}{n'}\Big[\text{sine of ND .cos of NDC .sine of } \lambda-\text{sine of ND .sine of NDC .cos of } \lambda\Big]$; but sine of ND .sine of NDC=sine of NΩ. sine of Ω; and $\because$ sine of ND. cos of NDC=cot of NDC .sine of NΩ .sine of Ω; but cot of NDC$=-\frac{\text{sine of } \Omega D}{\text{sine of } \Omega}$ cot of ΩN+cos of ΩD. cot of Ω; whence sine of ND .cos of NDC=$-$sine of ΩD .cos of ΩN+cos of ΩD .sine of NΩ .cos of Ω and $\because$ the aberration in latitude $=\frac{Tt}{n'}\Big[(-\text{sine of } \Omega D.\ \cos\text{ of } \Omega N + \cos\text{ of } \Omega D.\ \text{sine of } N\Omega.\ \cos\text{ of } \Omega)\ \text{sine of } \lambda - \text{sine of } N\Omega.\ \text{sine of } \Omega.\ \cos\text{ of } \lambda\Big]$ $=\frac{Tt}{n'}\Big[(\cos\text{ of } (l-k).\ \cos\text{ of } h + \text{sine of } (l-k)\ \text{sine of } h.\ \cos\text{ of } \Omega).\ \text{sine of } \lambda-\text{sine of } h.\ \text{sine of } \Omega.\ \cos\text{ of } \lambda\Big]=\frac{Tt}{n'}\Big[(\cos\text{ of } (h+k-l).\ \text{sine of } \lambda + 2.\,\overline{\text{sine of } \tfrac{1}{2}\Omega}|^2\ \text{sine of } (k-l)\ \text{sine of } h.\ \text{sine of } \lambda-\text{sine of } h.\ \text{sine of } \Omega.\ \cos\text{ of } \lambda\Big]$

If v represent the heliocentric distance of the planet from the ascending node, and ϖ the heliocentric distance of the perihelion from that node, e' the eccentricity to the mean distance unity of the orbit, ϵ' the mean velocity of the planet, we have from the note at the bottom of the page where Art. 8 commences, $Tt=\epsilon'.\Big[1+e'\cos\text{ of } (v-\varpi)\Big]$ nearly, and h or $\angle\,\Omega N=v-90°-$

e'. sine of $\overline{v-\varpi}$, nearly; sine of $(h-l+k)$ = sine of $(v+k-l-e'$. sine of $(v-\varpi)-90)=-$ cos of $(v+k-l-e'$. sine of $\overline{v-\varpi})$, and sine of $h=-$cos of $(v-e$ sine of $\overline{v-\varpi})$; therefore from above we have the aberration in longitude arising from the proper motion of the planet to an eye at rest, or quantity to be added to the apparent longitude to give the true longitude $=\frac{\varepsilon'}{n' \text{cos of } \lambda}\Big[1+e'$. cos of $\overline{v-\varpi}\Big]$ $\times$ cos of $(v-l+k-e'$. sine of $\overline{v-\varpi})-2$ cos of $\overline{l-k}$. $\overline{\text{sine of } \frac{1}{2}\,\Omega}^2\times$ cos of $(v-e'$ sine of $\overline{v-\varpi})\Big]$ = by the theorem mentioned in Art. 9. $\frac{\text{cos of } \lambda.\ n'}{\varepsilon'}\Big[$cos of $(v+l-k)+e'$. sine of $(\varpi-l+k.)-2.\overline{\text{sine of } \frac{1}{2}\,\Omega}^2$. $\overline{\text{cos of } v+e'.\ \text{cos of } \varpi}$. cos of $(l-k)\Big]$

And the aberration in latitude or quantity to be added to the apparent latitude, to give the true latitude from the proper motion of the planet, the eye being considered at rest $=\frac{\varepsilon'}{n'}\Big[1+e'$. cos of $\overline{v-\varpi}\Big]\times\Big[$cos of $(v+k-l-e'$ sine of $\overline{v-\varpi}-90°)\times$ sine of $\lambda-(2.\overline{\text{sine of } \frac{1}{2}\,\Omega}^2$. sine of $\overline{k-l}$. sine of $\lambda-$sine of Ω. cos of $\lambda)\times($cos of $v-e'$ sine of $\overline{v-\varpi})\Big]$ = (by the theorem in Art. 9) $\frac{\varepsilon'}{n'}\Big[$ cos of $(v+k-l-90°)$. sine of $\lambda+e'$ cos of $(\varpi+k-l-90°)$. sine of $\lambda-(2.\overline{\text{sine of } \frac{1}{2}\,\Omega}^2$ .sine of $\overline{k-l}$. sine of $\lambda-$sine of Ω. cos of $\lambda)\times($cos of $v+e$. cos of $\varpi)\Big]=$ $\frac{\varepsilon'}{n'}\Big[\Big\{$sine of $(v+k-l)+e'$. sine of $(\varpi+k-l)-2\,\overline{\text{sine of } \frac{1}{2}\,\Omega}^2$. sine of $(k-l)$. (cos of $v+e'$. cos of $\varpi)\Big\}$. sine of $\lambda+$sine of Ω. cos of λ. (cos of $v+e$. cos of $\varpi)\Big]$

And for a comet if ε' be the velocity at the perihelion, from the same node, we have $h=\frac{v+\varpi}{2}-90°$ and $Tt=\varepsilon'$. cos of $\frac{1}{2}(v-\varpi)$. And therefore the aberration of the comet supposed to move from west to east, arising from its motion in longitude which it is necessary to add to the apparent longitude to obtain the true, is equal to $\varepsilon'\frac{\text{cos of } \frac{1}{2}\,\overline{v-\omega}}{n'.\ \text{cos of } \lambda}\Big[-$sine of $(\frac{v+\varpi}{2}-90+k-l)+2.\overline{\text{sine of } \frac{1}{2}\,\Omega}^2$ $\times$ sine of $(\frac{v+\varpi}{2}-90°)$. cos of $(l-k)\Big]=\varepsilon.\frac{\text{cos of } \frac{v-\varpi}{2}}{\text{cos of } \lambda.\ n'}\Big[$ cos of $(\frac{v+\varpi}{2}+k-l)-$ $2.\overline{\text{sine of } \frac{1}{2}\,\Omega}^2$. cos of $(k-l)$. (cos of $\frac{v-\omega}{2})\Big]$. And the aberration of latitude or quantity to be added to the apparent latitude to have the true latitude $\varepsilon'.\frac{\text{cos of } \frac{1}{2}\,(v-\varpi)}{n'}\Big[$ sine of $(\frac{v+\varpi}{2}+k-l)$. sine of $\lambda-2$ sine of λ. $\overline{\text{sine of } \frac{1}{2}\,\Omega}^2$ sine of $(k-l)$. cos of $\frac{v+\varpi}{2}+$cos of $\frac{v+\varpi}{2}$. sine of Ω. cos of $\lambda\Big]$

To these aberrations of the planet or comet arising from its proper motion

there are to be added the aberrations owing to the motions of the eye, of which those due to the motions of the earth about the sun are given in Art. 8, and those owing to the motion of the earth about its axis are very small, and of which we have given the quantity in right ascension and declination, but not in longitude and latitude, which in fact would be here required, but those quantities are easily found by our general method.

As to the aberration connected with the place of the sun and moon, as the tables of their places contain the correction, except the slight alteration arising from their irregular motions, it is not necessary to take account of it in this place; unless it be thought proper to correct the place of the sun when the geocentric places of the planets are to be obtained from having the heliocentric place given from the tables.

Since the great extension of symbolic notation, mathematicians have almost universally dropped the preposition in expressions such as, sine *of* x, tangent *of* x, logarithm *of* x, fluent *of*, &c., either with a view to abbreviation, or in the idea that sin x, tan x, log x, &c. are more symbolic than sine *of* x, &c.: but I have always disapproved of that form of expression, as I consider, for instance, that sin x in reality rather expresses that x is the sine than that x is the arc. And the same mathematicians who most commonly use sin x to express the sine of the arc x, use it also to express that x is the sine; and so also of tan x, &c.: thus Euler *(Institutiones Calculi Integralis)*, vol. i. page 146, has "ang sin x;" page 148, "ang cos x;" page 93, "arc tan x;" in which x is put respectively for the sine, cosine, and tangent; and not for the arc, agreeably to his more common notation. Also Arbogast (*Derivations*, page 9,) has the expressions "arc sin $(\alpha+\beta+\gamma x^2+$ &c.), log sin $(\alpha+\beta x+\gamma x^2+$&c.)," &c. In the first of these two, 'sin' implies that $\alpha+\beta x+$&c. is a sine; but 'sin' in the second implies that it is an arc. La Croix in his *Traité Elémentaire de Calcul Diff.* p. 372, has the expression "arc tang $(\frac{x}{y})$;" where tang signifies that $\frac{x}{y}$ is a tangent, and not an arc as usual. These anomalies in notation have evidently been considered an inconvenience, as is apparent from the adoption by different authors of another mode of expressing the above or similar functions; thus on the same page to which we have just referred we have the two modes of notation, arc (tang $\frac{x}{y}$) and arc (tang$=\frac{x}{y}$), for the same thing. The latter

notation, unless, for '$=$' we are to read *being* or some similar word, appears to me awkward; as when such expressions occur in an equation, in reading it, the attention may be distracted by the different intents, and interruptions of the term *equal*; notwithstanding which, should 'tang' in the expression "tang $\frac{x}{y}$" be used, as it generally is, to express "tang of $\frac{x}{y}$," such, or a similar, notation as the above becomes necessary for the other meaning. But if the preposition be not dropped where in propriety it should be used, we shall have 'sine of x, sine x,' &c. put for what they in reality express. Emerson frequently in his table of fluxions and fluents retains this preposition; and Landen (see his *Lucubrations*) seems quite sensible of the necessity of retaining it. I have generally been careful not to omit it; and I cannot think, as I believe some mathematicians do for whose judgement and accuracy I have the greatest respect, that "sine(of x)," "sine^2(of x)," "sine^{-1}(of x)," are less analytical expressions than "sin x," "sine $^2(x)$," "sine$^{-1}(x)$." Strict notation appears to me of the greatest moment in complex analysis; and I much admire the present adopted distinction of expression, "sin $^n x$," "sine x^n," and "$\overline{\sin x}|^2$," &c. with the exception that, for the reasons above given, I require the preposition to be retained.

XXV. *On the Measurement of Altitudes by the Barometer. By Professor* Littrow, *of Vienna, &c. &c.*

Read December 13, 1822.

My object is to give a method of determining altitudes by barometrical measurements simpler than any hitherto known to me, and yet quite exact. Many attempts have already been made to accomplish this object, and they may be distributed into two classes; the former of which aim at embracing the analytical formulæ on which the research is grounded in all their generality, by which the tables become voluminous, and cumbersome to the traveller, who would frequently be desirous of knowing on the spot the result of his observations; the others, to avoid this evil, give, it is true, small and compendious tables, but which either presume the use of logarithms, which every traveller does not know, and which renders another huge book necessary; or abridge the original formulæ, and omit parts of it, which cannot but injure the accuracy of the results obtained.

The want therefore of very easy, very small, and yet perfectly exact tables is yet, to the best of my knowledge, unsatisfied; and what follows is an attempt to supply this deficiency.

I take for the basis of the whole research the well-known formula of Ramond, as given by Laplace in the 4th volume of the *Mécanique Céleste,* which is generally acknowledged as the best.

If we call b, t, T, the barometer, the external and interior thermometers (where, consequently, T represents the temperature of the mercury, and t that of the outward air,) for the upper station; and b', t', T$'$ the same things for the lower; φ the geographical latitude; and lastly H the difference of altitudes of the two stations in toises, then Ramond's formula is

$$\left.\begin{aligned} &\mathrm{N}=9436.966\,\big(1+0.00284\cos 2\varphi\big)\,.\,\big(1+0.0025\,(t+t')\big) \\ &\mathrm{H}=\mathrm{N}.\quad \log.\frac{b'}{\left[1+0.00023\,(\mathrm{T}'-\mathrm{T})\right]b} \end{aligned}\right\}\quad 1.$$

We must now endeavour to express this somewhat complicated formula by a small table so constructed as to require only the three first rules of arithmetic, without the need of any knowledge of proportions, of logarithms, &c. Besides which, the computation must be short and easy, and the result *perfectly exact.*

To attain this end, let us take the two magnitudes M, m, so as to have

$$\text{M}=9436.966\ \big(1+0.0025\ (t+t')\big)$$

$$m=1+0.00023\ (\text{T}'-\text{T}).$$

If then for a while we leave out the consideration of the factor dependent on φ, RAMOND's formula gives

$$\text{H}=\text{M}.\ \log.\frac{b'}{m\,b}\cdot$$

Suppose then we assume

$$\text{H}=\text{M}.\ \log.\frac{b'}{b}+x.$$

Equating the two values of H, and observing that $m-1$ is always a very small quantity compared with unity, we find

$$x=-0.4342945\ \text{M}\ (m-1).$$

Or, restoring the values of M and m,

$$x=-0.94264\ (\text{T}'-\text{T})-0.0023565\ (\text{T}'-\text{T})\ (t'+t).$$

After this introduction, it is easy to give the formula (I.) the following form, where $\theta=\frac{t'+t}{4}$.

$$\begin{aligned} \text{H}=&\left\{\text{M}.\ \log.\frac{28.16666}{b}+(0.943+0.0096\,\theta)\,\text{T}\right\}\\ &-\left\{\text{M}.\ \log.\frac{28.16666}{b'}+(0.943+0.0096\,\theta)\,\text{T}'\right\} \end{aligned} \qquad \text{II.}$$

an expression which it is very easy to reduce into a table having the above-mentioned properties. I assume, for instance, that the tables shall give for every value of b the quantity

$$\text{A}=9436.966.\ \log.\frac{28.16666}{b}$$

and then proceed as follows:

We first seek A for the upper station, entering the table with b; then place the decimal point in A two places forward, and multiply this new number by θ. We lastly seek $\text{B}=0.943+0.0096\,\theta$, and multiply this number by T. The

sum of these three quantities is an approximate value of the height h of the first station above a certain fixed point. In other words,

$$h = A + \frac{A\,\theta}{100} + B\,T.$$

In like manner we seek the quantity

$$h' = A' + \frac{A'\theta}{100} + B\,T';$$

and the difference of altitudes required will be in toises

$$H = h - h'.$$

We see that the quantities θ and B are the same for both the two stations, and need therefore only be computed once. The thermometer here is supposed to be REAUMUR's. If we use FAHRENHEIT's or any other scale, it must be reduced to REAUMUR's. The barometrical altitudes B, b', may however be taken in any scale, English, French, German, &c., and require no reduction.

If we would avoid computing B, we may use the following little table.

θ	B	θ	B
−10°	0.85	+ 0°	0.94
− 9	0.86	1	0.95
− 8	0.87	2	0.96
− 7	0.88	3	0.97
− 6	0.89	4	0.98
− 5	0.90	5	0.99
− 4	0.91	6	1.00
− 3	0.92	7	1.01
− 2	0.94	8	1.02
− 1	0.94	9	1.03
0	0.95	10	1.04
		11	1.05
		12	1.06
		13	1.07
		14	1.08
		15	1.09

For the factor in φ I have provided by a peculiar and very convenient table which follows from the equation

$$\text{Corrected } H = H\,(1 + 0.00284 . \cos 2\varphi).$$

To find the errors to which these interesting observations are usually liable, the equation (I) gives, making H, t and t' variable,

$$\frac{dH}{H} = 0.0025 . \frac{dt + dt'}{a}$$

putting $a = 1 + 0.0025\ (t + t')$, whence it follows that a degree of error in the construction or reading off of the two exterior thermometers renders H erroneous by 0.0025 part of its value, and thus produces errors as follows:

In an altitude of	400 toises an error of	1 toise.
	800	2 toises.
	1200	3 ....
	&c.	&c.

We must therefore take care to be provided with good thermometers when the altitudes are great, if we would obtain correct results.

If we make H, T, T′, variable, we have

$$\frac{dH}{H} = \frac{0.943}{H} \cdot a. \ (d\,T - d\,T').$$

Whence it follows that the interior thermometers, or those attached to the mounting of the barometer, exert a less hurtful influence in producing error, the greater the altitude is. For the levelling of rivers, streets, &c., the exterior thermometer requires to be particularly good.

Finally, if we make H, b, b', variable,

$$\frac{dH}{H} = \frac{4098.4224}{H}\ a. \left(\frac{db'}{b'} - \frac{db}{b}\right).$$

Whence it follows that an error in the barometer is the most important of all, and that this relative error $\frac{dH}{H}$ is so much greater the smaller is the altitude H. Hence, for the purposes last enumerated, the barometer should be used with great caution.

It remains still to show how one may conclude the altitude above the sea from one insulated observation. It is well known that this process does not give the altitude so exactly as the method of corresponding observations. We require first the height of the barometer b' on the level of the sea at the time of observation. By a great number of observations I find that we may take

	In.	Lin.		In.	
b' =	28	2	=	28.16666	Paris.

It is more difficult to estimate the temperature t' on the level of the sea at the time of observation. LINDENAU has employed a number of observations of SAUSSURE (in the 11th volume of the *Monatlichen Correspondenz* of Baron Von

ZACH, and more lately in his *Tables Barometriques*, Gotha 1809), and grounded thereon a table. I find that this table represented very well such observations as I could collect on this point, and that the numbers in it might be represented with sufficient accuracy for our purpose by the following simple equation:

$$t' \text{ or } T' = 53 + t - 2b;$$

when t and b represent the heights of the exterior thermometer, and the barometer at the upper station.

It is right to elucidate the use of the tables by an example by which their convenience will appear.

EXAMPLE I.

Upper Station.			Lower Station.	
	In.	Lin.	In.	Lin.
$b=$	22	5.6	$b'=27$	3.2
$t=$	8°.4 Reaum.		$t'=15.2$	
$T=$	9.3		$T'=16.4$	

$\varphi = 58°\ 0'$

This gives $\theta = 5.9$; B = 1.0.

		935.9	.....	135.6
		−9.2	.....	−2.5
A	=	926.7	.....	133.1
$\frac{A\theta}{100}$	=	54.7	.....	7.9
B T	=	9.3	.....	16.4
h	=	990.7	.... h' =	157.4
h'	=	157.4		
H	=	833.3		
Cor. for φ		−1.0		
H (cor[d].)	=	832.3 Toises.		

EXAMPLE II.

Upper Station.			Lower Station.	
	In.	Lin.	In.	Lin.
$b=$	19	8.7	$b'=26.$	4.5
$t=$	−12.3		$t'=2.3$	
$T=$	−10.5		$T'=3.5$	

$\varphi=50°\ 0'$; $\theta=-2.5$; B=0.919.

Consequently,

		1472.3	.....	275.9
		−12.1	.....	−6.4
		1460.2	.....	269.5
		−36.5	.....	−6.7
		− 9.6	.....	+3.2
h	=	1414.1	..... h' =	266.0
h'	=	266.0		
H	=	1148.1		
		−0.5		
H′	=	1147.6 Toises.		

EXAMPLE III.

Let $b = 19$ in. 10.14 lin. $t=3.2$, $T=7.6$, required the height above the sea.

This gives $t'=T=16.5$; $\theta=4.92$; $B=0.99$.

Consequently,

		1437.7
		− 2.4
A	=	1435.3
$\frac{A\,\theta}{100}$	=	70.7
BT	=	7.5
		1513.5
		16.3
		1497.2 Toises = Height required.

TABLE I.

Barom. In.	Barom. Lin.	A.	Diff. for $\frac{1}{10}$ line.	Barom. In.	Barom. Lin.	A.	Diff. for $\frac{1}{10}$ line.	Barom. In.	Barom. Lin.	A.	Diff. for $\frac{1}{10}$ line.
29	6	−189.5	1.16	26	7	237.2	1.28	23	6	742.4	1.46
	5	−177.9	1.16		6	250.0	1.30		5	757.0	1.46
	4	−166.3	1.16		5	263.0	1.29		4	771.6	1.46
	3	−154.7	1.17		4	275.9	1.29		3	786.2	1.47
	2	−143.0	1.18		3	288.8	1.30		2	800.9	1.48
	1	−131.2	1.17		2	301.8	1.32		1	815.7	1.48
29	0	−119.5	1.19		1	315.0	1.30	23	0	830.5	1.49
28	11	−107.6	1.18	26	0	328.0	1.33	22	11	845.4	1.50
	10	− 95.8	1.18	25	11	341.3	1.32		10	860.4	1.50
	9	− 84.0	1.20		10	354.5	1.31		9	875.4	1.51
	8	− 72.0	1.19		9	367.6	1.34		8	890.5	1.50
	7	− 60.1	1.19		8	381.0	1.33		7	905.5	1.51
	6	− 48.2	1.21		7	394.3	1.33		6	920.6	1.53
	5	− 36.1	1.20		6	407.6	1.36		5	935.9	1.52
	4	− 24.1	1.20		5	421.2	1.33		4	951.1	1.53
	3	− 12.1	1.21		4	434.2	1.35		3	966.4	1.53
	2	0.0	1.22		3	448.0	1.35		2	981.7	1.55
	1	12.2	1.21		2	461.5	1.36		1	997.2	1.55
28	0	24.3	1.23		1	475.1	1.37	22	0	1012.7	1.56
27	11	36.6	1.22	25	0	488.8	1.37	21	11	1028.3	1.56
	10	48.8	1.23	24	11	502.5	1.38		10	1043.9	1.58
	9	61.1	1.24		10	516.3	1.38		9	1059.7	1.57
	8	73.5	1.23		9	530.1	1.38		8	1075.4	1.58
	7	85.8	1.24		8	543.9	1.39		7	1091.2	1.57
	6	98.2	1.25		7	557.8	1.39		6	1106.9	1.60
	5	110.7	1.24		6	571.7	1.39		5	1122.9	1.60
	4	123.1	1.25		5	585.7	1.39		4	1138.9	1.60
	3	135.6	1.25		4	599.6	1.40		3	1154.9	1.60
	2	148.1	1.27		3	613.6	1.41		2	1170.9	1.62
	1	160.8	1.26		2	627.7	1.42		1	1187.1	1.63
27	0	173.4	1.27		1	641.9	1.42	21	0	1203.4	1.63
26	11	186.1	1.27	24	0	656.1	1.43	20	11	1219.7	1.64
	10	198.8	1.27	23	11	670.4	1.43		10	1236.1	1.65
	9	211.5	1.29		10	684.7	1.44		9	1252.6	1.65
	8	224.4	1.28		9	699.1	1.44				
					8	713.5	1.44				
					7	727.9	1.45				

TABLE I. (Continued.)

Barom.		A.	Diff. for $\frac{1}{10}$ line.	Barom.		A.	Diff. for $\frac{1}{10}$ line.
In.	Lin.			In.	Lin.		
20	8	1269.1	1.65	17	7	1931.2	1.94
	7	1285.6	1.65		6	1950.6	1.96
	6	1302.1	1.68		5	1970.2	1.97
	5	1318.9	1.67		4	1989.9	1.97
	4	1335.6	1.68		3	2009.6	1.97
	3	1352.4	1.68		2	2029.3	2.00
	2	1369.2	1.71		1	2049.3	2.01
	1	1386.3	1.70	17	0	2069.4	2.02
20	0	1403.3	1.72	16	11	2089.6	2.02
19	11	1420.5	1.72		10	2109.8	2.03
	10	1437.7	1.73		9	2130.1	2.04
	9	1455.0	1.73		8	2150.5	2.06
	8	1472.3	1.74		7	2171.1	2.06
	7	1489.7	1.74		6	2191.7	2.07
	6	1507.1	1.76		5	2212.4	2.09
	5	1524.7	1.77		4	2233.3	2.10
	4	1542.4	1.76		3	2254.3	2.10
	3	1560.0	1.77		2	2275.3	2.13
	2	1577.7	1.79		1	2296.6	2.13
	1	1595.6	1 80	16	0	2317.9	2.15
19	0	1613.6	1.80	15	11	2339.4	2.15
18	11	1631.6	1.81		10	2360.9	2.17
	10	1649.7	1.82		9	2382.6	2.17
	9	1667.9	1.83		8	2404.3	2.18
	8	1686.2	1.83		7	2426.1	2.19
	7	1704.5	1.83		6	2448.0	2.22
	6	1722.8	1.86		5	2470.2	2.22
	5	1741.4	1.86		4	2492.4	2.23
	4	1760.0	1.86		3	2514.7	2.24
	3	1778.6	1.87		2	2537.1	2.26
	2	1797.3	1.89		1	2559.7	2.27
	1	1816.2	1.89	15	0	2582.4	
18	0	1835.1	1.91				
17	11	1854.2	1.91				
	10	1873.3	1.93				
	9	1892.6	1.93				
	8	1911.9	1.93				

TABLE II.

LATITUDE.

H.	$+44°$ -46	$+42°$ -48	$+40°$ -50	$+38°$ -52	$+36°$ -54	$+34°$ -56	$+32°$ -58	$+36°$ -60
Toises. 50	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
100	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.2
200	0.0	0.1	0.1	0.1	0.2	0.2	0.2	0.3
300	0.0	0.1	0.1	0.2	0.3	0.3	0.4	0.4
400	0.0	0.1	0.2	0.2	0.3	0.4	0.5	0.6
500	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
600	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
700	0.1	0.2	0.3	0.4	0.6	0.7	0.9	1.0
800	0.1	0.2	0.4	0.5	0.7	0.8	1.0	1.1
900	0.1	0.3	0.4	0.6	0.8	0.9	1.1	1.2
1000	0.1	0.3	0.5	0.7	0.9	1.0	1.2	1.4
1100	0.1	0.4	0.5	0.7	1.0	1.2	1.4	1.6
1200	0.1	0.4	0.6	0.8	1.0	1.2	1.5	1.8
1300	0.1	0.4	0.6	0.8	1.1	1.3	1.6	1.9
1400	0.1	0.5	0.7	0.9	1.2	1.4	1.7	2.0
1500	0.2	0.5	0.7	1.0	1.3	1.6	1.9	2.2
1600	0.2	0.5	0.8	1.1	1.4	1.7	2.0	2.3
1700	0.2	0.5	0.8	1.1	1.5	1.8	2.1	2.4
1800	0.2	0.6	0.9	1.2	1.6	1.9	2.2	2.6
1900	0.2	0.6	0.9	1.3	1.7	2.0	2.4	2.8
2000	0.2	0.6	1.0	1.3	1.7	2.1	2.5	2.9
2100	0.2	0.7	1.1	1.4	1.8	2.2	2.6	3.1
2200	0.2	0.7	1.1	1.5	1.9	2.3	2.7	3.2
2300	0.2	0.7	1.2	1.6	2.0	2.4	2.9	3.4
2400	0.2	0.7	1.2	1.6	2.1	2.5	3.0	3.5
2500	0.3	0.8	1.3	1.7	2.2	2.6	3.1	3.6
2600	0.3	0.8	1.3	1.8	2.3	2.7	3.2	3.7
2700	0.3	0.8	1.4	1.9	2.4	2.9	3.4	3.9
2800	0.3	0.9	1.4	1.9	2.5	3.0	3.5	4.0
2900	0.3	0.9	1.5	2.0	2.5	3.1	3.6	4.1
3000	0.3	0.9	1.5	2.0	2.6	3.2	3.7	4 2

XXVI. *A Note respecting the Application of Machinery to the Calculation of Astronomical Tables.* By CHARLES BABBAGE, *Esq. F.R.S. Sec. Ast. Soc., &c. &c.*

Read 14th June, 1822.

IT is known to several of the members of this society that I have been engaged during the last few months in the contrivance of machinery, which by the application of a moving force may calculate any tables that may be required. I am now able to acquaint the society with the successful results at which I have arrived; and although it might at the first view appear a bold undertaking to attempt the construction of an engine which should execute operations so various as those which contribute to the formation of the numerous tables that are constantly required for astronomical purposes, yet to those who are acquainted with the method of differences the difficulty will be in a considerable degree removed.

I have taken the method of differences as the principle on which my machinery is founded; and in the engine which is just finished I have limited myself to two orders of differences. With this machine I have repeatedly constructed tables of square and triangular numbers, as well as a table from the singular formula x^2+x+41, which comprises amongst its terms so many prime numbers.

These, as well as any others which the engine is competent to form, are produced almost as rapidly as an assistant can write them down. The machinery by which these calculations are effected is extremely simple in its kind, consisting of a small number of different parts frequently repeated.

In the prosecution of this plan, I have contrived methods by which type shall be set up by the machine in the order determined by the calculation; and the arrangements are of such a nature that, if executed, there shall not exist the possibility of error in any printed copy of tables computed by this engine. Of several of these latter contrivances I have made models; and, from the experiments I have already made, I feel great confidence in the complete success of the plans I have proposed.

C. BABBAGE.

DEVONSHIRE-STREET, PORTLAND-PLACE,
June 2, 1822.

XXVII. *Observations on the Application of Machinery to the Computation of Mathematical Tables. By* CHARLES BABBAGE, *Esq. F.R.S., &c. &c.*

Read 13th December, 1822.

SINCE I had the honour of communicating to the Astronomical Society a short account of an arithmetical engine for the calculation of tables, which has been examined by several of the members of this society, I have not added much to the practical part of the subject. I have however paid some attention to the improvements of which the machinery is susceptible, and which will, if another engine is made, be greatly improved.

The theoretical inquiries to which it has conducted me are however of a singular nature; and I shall take this opportunity of briefly explaining to the society some of the principles on which they depend, as far as the nature of the subject will permit me to do this without the introduction of too many algebraic operations, which are rarely intelligible when read to a large assembly.

Of the variety of tables which are required in the present state of science, by far the larger portion are intimately connected with that department of it which it is the peculiar object of this society to promote.

The importance of astronomical science, whether viewed as the proudest triumph of intellectual power, or considered as the most valuable present of abstract science to the comfort and happiness of mankind, equally claims for it the first assistance from any new method for condensing the processes of reasoning or abridging the labour of calculation. Astronomical tables were therefore the first objects on which I turned my attention, when attempting to improve the power of the engine, as they had formed the first motive for constructing it.

I have already stated to the society, in my former communication, that the first engine I had constructed was solely destined to compute tables having constant differences. From this circumstance it will be apparent that after a certain number of terms of a table are computed, unless, as rarely happens, it has a constant order of differences, we must stop the engine and place in it

other numbers, in order to produce the next portion of the table. This operation must be repeated more or less frequently according to the nature of the table. The more numerous the order of differences, the less frequent will this operation become requisite. The chance of error in such computations arises from incorrect numbers being placed in the engine: it therefore becomes desirable to limit this chance as much as possible. In examining the analytical theory of the various differences of the sine of an arc, I noticed the property which it possesses of having any of its even orders of differences equal to the sine of the same arc increased by some multiple of its increment multiplied by a constant quantity. With the aid of this principle an engine might be formed which would require but little attendance, and I believe that it might in some cases compute a table of the form A sin θ from the 1st value of $\theta=0$ up to $\theta=90°$ with only one set of figures being placed in it.

It is scarcely necessary to observe what an immense number of astronomical tables are comprised under this form, nor the great accuracy which must result from having reduced to so few a number the preliminary computations which are requisite.

In pursuing into its detail the principle to which I have alluded, which lends itself so happily to numerical application, I have traced its application to other species of tables, and am enabled to point out a course of analytical investigation which will in all probability afford ready methods for constructing tables, even of the most complicated transcendent, in a manner equally easy.

I will now advert to another circumstance, which, although not immediately connected with astronomical tables, resulted from an examination of the engine by which they can be formed.

On considering the arrangement of its parts, I observed that a different mode of connecting them would produce tables of a new species altogether different from any with which I was acquainted. I therefore computed with my pen a small table such as would have been formed by the engine had it existed in this new shape, and I was much surprised at discovering that no analytical method was yet known for determining its n^{th} term. The following is the first series I wrote down:

Series.	Diff.	Series.	Diff.	Series.	Diff.
0 . . . 2	0	10 . . . 222	42	20 . . . 924	86
1 . . . 2	2	264	46	1010	86
4	6	310	46	1096	92
10	6	356	52	25 . . . 1188	100
16	12	15 . . . 408	60	1288	108
5 . . . 28	20	468	68	1396	114
48	28	536	74	1510	114
76	34	610	74	1624	118
110	34	684	78	30 . . . 1742	120
144	38	20 . . . 762	80	1862	122
10 . . 182	40	842	82	1984	

The equation of finite differences from which it is produced is

$$\Delta^2 u_z = \text{units fig. of } u_{z+1}$$

which is one of a class of equations never hitherto integrated. I succeeded in transforming this equation into a more analytical form: but still it presented great difficulties; I therefore undertook the investigation in a different manner, and succeeded in discovering a formula which represented its nth term. It is the following:

$$u_z = (\bar{a}) + 20b\,(10b + 2a - 1)$$

TABLE

0	2
1	2
2	4
3	10
4	16
5	28
6	48
7	76
8	110
9	144

where $(\bar{a})$ represents the number opposite a in the annexed subsidiary table, and a is the figure in the unit's place of z, and b is that number which arises from cutting off the last figure from z. Example: let the 17th term be required, then $z=17$, and $a=7$, $b=1$; the number opposite 7 in the table is

$$(\bar{7}) = \quad . \quad . \quad . \quad 76$$

$$10b + 2a - 1 = 10 + 14 - 1 = 23$$

$$20b = 20 \qquad 20b\,(10b+2a-1) = \underline{460}$$

$$536 = u_{17}$$

I have formed other series of the same class, and have succeeded in expressing any term independent of all the rest by two distinct processes. Thus I have incidentally been able to integrate the equations I have mentioned: I will just state one other of a simple form; it is the equation

$$\Delta \underset{z}{u} \text{ units fig. } \underset{z}{u}$$

whose integral is

$$\underset{z}{u} = 20b + 2^a$$

where a is that one of the numbers 1, 2, 3, 4, which taken from z leaves the remainder divisible by 4, and b is the quotient of that division.

The table is as follows:

1	—	2
		4
		8
4	—	16
		22
		24
		28
8	—	36
		42

One of the general questions to which these researches give rise is, supposing the law of any series to be known, to find what figure will occur in the kth place of the nth term. That the mere consideration of a mechanical engine should have suggested these inquiries, is of itself sufficiently remarkable; but it is still more singular, that amongst researches of so very abstract a nature I should have met with and overcome a difficulty which had presented itself in the form of an equation of differences, and which had impeded my progress several years since, in attempting the solution of a problem connected with the game of chess.

XXVIII. *On some new Tables for determining the Time, by means of Altitudes taken near the Prime Vertical. By* FRANCIS BAILY, *Esq. F.R.S. and L.S.*

Read January 10, 1823.

AS every method which tends to abridge the labour of computation must be desirable to the practical astronomer, I hope I need not apologize for drawing the attention of the Society to the subject of the following pages.

To those persons who are fortunately possessed of transit instruments the following observations are not addressed: but, since this is not the lot of all, and as many persons may be desirous of obtaining the correct time, who either have not the means or convenience of affixing a transit instrument, or who may be travelling with a view to the improvement of the connected sciences of astronomy and geography, I trust that the subject will be considered, if not generally, at least *partially* useful.

It is well known to the practical astronomer that all observations of altitude for determining the time should be taken as near the prime vertical as possible; since the motion of the star in altitude is the greatest at this point of its revolution: and, on the other hand, the least possible effect is produced from any error in the assumed latitude of the place. It is therefore necessary that those persons, who are travelling for the purposes above mentioned, or whose positions on the globe are not accurately ascertained, should attend particularly to this remark. To which I may add, as a further argument for its adoption, that the computations, to which I am about to allude, will by such means be rendered extremely simple and easy.

In order to determine the time correctly, by this method, the following quantities are required to be known.

$L =$ the latitude of the place
$D =$ the declination of the star
$Z =$ the zenith distance of the star

whence we deduce

$$\theta = \text{the hour angle}$$

by means of the following general expression

$$\sin^2\frac{\theta}{2} = \frac{\sin\left[\frac{Z+(L-D)}{2}\right].\sin\left[\frac{Z-(L-D)}{2}\right]}{\cos L.\ \cos D} \qquad (1)$$

and by applying θ to the apparent right ascension of the star ($=$ Æ) we shall have the correct sidereal time

$$T = Æ \pm \theta \qquad (2)$$

When a *series* of observations is taken, as is generally the case, the application of this formula becomes exceedingly troublesome. Nevertheless there is no effectual mode of abridging the labour, although some recent attempts have been made for that purpose. See BODE's *Astronomische Jahrbuch* for 1818, page 123: and *Connaissance des Tems* for 1820, pages 357 and 397. But, if the observations are made near the prime vertical, the formula may be rendered much more simple, and the computations much more easy, as I shall now proceed to show.

For, when the star is on the prime vertical we may deduce the altitude A ($= 90 - Z$) from the following formula

$$\sin A = \frac{\sin D}{\sin L} \qquad (3)$$

and θ from *either* of the two following formulæ

$$\sin\ \theta = \frac{\cos A}{\cos D} \qquad (4)$$

$$\cos\theta = \tan D.\ \cot L \qquad (5)$$

It will be evident, from the inspection of these formulæ, that the altitude of the star, and also the hour-angle depend wholly on the declination of the star and the latitude of the place: which may be considered as constant quantities during any series of observations. It will be easy therefore to compose a table, for any given latitude, containing the altitude and hour-angle (when on the prime vertical) of any number of stars proposed to be observed. But, since, in a *series* of observations, they cannot all be confined to this point, it will be necessary to make a correction for such deviation therefrom as may occur. This may be effected in the following manner.

The general equation for deducing the altitude of a star is

$$\sin A = \sin D.\ \sin L + \cos D.\ \cos L.\ \cos\theta.$$

If we difference this equation, assuming A and θ as variable, and the rest of the quantities as constant, we shall have

$$d\,A.\cos A = -d\,\theta.\sin\theta.\cos D.\cos L.$$

$$\frac{dA}{d\,\theta} = -\frac{\sin\theta.\cos D.\cos L.}{\cos A.} \qquad (6)$$

But, we have seen that, when the star is on the prime vertical,

$$\sin\theta = \frac{\cos A}{\cos D}$$

whence we have in such case

$$d\,A = -d\,\theta.\cos L. \qquad (7)$$

which will enable us to deduce with sufficient accuracy, for several minutes before and after the passage of the star across the prime vertical, the variation in the hour angle corresponding to any given change in altitude within those limits. And which, if thought desirable, may with a little additional trouble, be extended to full half an hour on each side of the prime vertical.

I have hitherto supposed the declination of the star to be constant: and if the observations be confined to a short period of the year, no sensible error would arise from such assumption. But, if the tables be intended for frequent use, as opportunities may occur, it will be necessary to determine the corrections to be made to each star (both in the altitude and in the hour angle) on account of this assumption. And, first, with respect to the altitude.

By differencing the equation (3), making A and D variable, we have

$$d\,A.\cos A = d\,D.\frac{\cos D}{\sin L.}$$

$$d\,A = d\,D.\frac{\cos D}{\cos A.\sin L.}$$

$$= d\,D.\cot.D\ \tan A. \qquad (8)$$

And by differencing the equation (5), making θ and D variable, we have, for the hour angle,

$$d\,\theta.\sin\theta = -d\,D.\frac{\cot L}{\cos^2 D}$$

$$d\,\theta = -dD.\frac{\cot L}{\sin\theta.\cos^2 D}$$

$$= -dD.\frac{\cot\theta}{\sin D.\cos D.} \qquad (9)$$

If there be any reason for suspecting an error in the assumed latitude of the place, the corrections for the altitude and hour angle may, in like manner, be deduced from the following equations:

$$d\,A = -d\,L.\tan A.\cot L. \qquad (10)$$

$$d\,\theta = d\,L.\frac{\cot\theta}{\sin L.\cos L.} \qquad (11)$$

but, as far as regards the subject of this paper, I shall presume that the latitude of the place is known with sufficient accuracy.

I shall now proceed to show the application of these formulæ to the subject of this communication: and perhaps this cannot be better effected than by forming into a table a few of the values which I have computed for my own observatory, in N. Lat. 51° 33′ 34″.

The first column of Table I. contains merely the names of the stars. The second contains the assumed declination of the star, from which the values in the subsequent columns are deduced. The third contains the altitude of the star when on the prime vertical, deduced from equation (3): and the fourth, the hour angle, or distance from the meridian, at the same point, deduced from equation (5). The fifth and sixth columns contain respectively the variations in altitude and in the hour angle, arising from an *increase* of 1″ in the declination of the star. In this case it will be obvious that the variation of A will be *plus*, and of θ *minus*: but, if the apparent declination be *less* than that assumed in the table, these signs must be changed. These two columns are deduced from equations (8) and (9).

Table II. contains the variation in the hour angle corresponding to a given variation in the altitude, deduced from equation (7). It will give the value sufficiently accurate for a quarter of an hour on each side of the prime vertical, which may be considered as a sufficient period for such observations: but I have extended it to double that distance, by the help of the general equation (1). In proportion however as we extend these limits, the table will be less correct, since it will not in such cases apply equally to every star: whereas *on* the prime vertical, and contiguous thereto, the value depends wholly on the latitude of the place of observation.

This being premised, I shall attempt to show the use of the tables, by means of the following

Rule.

Find, in the Nautical Almanac, or any other ephemeris, the apparent declination of the star, and note down the number of seconds between the value there given and the assumed value in the second column of Table I. Multiply with this number (considered always as *plus*) the numbers which express the variation of A and the variation of θ, in the fifth and sixth columns, affixing the proper signs thereto: and apply those results to the values of A and θ respectively, as given in the third and fourth columns; which will give A and θ *corrected* for the true declination. Correct the observed altitude for refraction;

and take the difference between such corrected value, and the value of A corrected as above. With this difference of altitude find the corresponding variation in the hour angle, by Table II. Then if the observed altitude (corrected for refraction) be *less* than A corrected, *add* it to θ corrected as above; but, if *greater, subtract* it therefrom. This result *added* to the apparent right ascension of the star on the day of observation, will give the correct sidereal time, if the observation be made towards the *west*; but if made towards the *east*, the result must be *subtracted* from the right ascension of the star (increased by 24^h, if necessary).

Example I.

On March 23, 1822, at $9^h\ 24^m\ 44^s$, sidereal time by the *clock*, in N. Lat. $51°\ 33'\ 34''$, I observed the altitude of *Aldebaran*, near the prime vertical westerly, to be $21°\ 57'\ 39''$, when corrected for refraction. Required the *correct* sidereal time?

Apparent $D = 16°\ 8'\ 44''$

Assumed $D = 16\ \ 8\ \ 0$

$$\text{difference} = 44 \times \begin{cases} +1''.3090 = +57''.60 = \text{correction of A} \\ -0\ .0589 = -\ \ 2.59 = \text{correction of } \theta \end{cases}$$

	ALTITUDE.	TIME.
	° ′ ″	h m s
By Table I	20 46 45.2	5 6 54.33
correction	+ 57.6	− 2.59
tabular value corrected	20 47 42.8	5 6 51.74
observed value corrected	21 57 39.0	
	1 9 56.2	= −7 30.03
		4 59 21.71
	Æ =	4 25 43.81
correct time =		9 25 5.52
observed time =		9 24 44.00
clock too slow =		21.52

EXAMPLE II.

On August 21, 1822, I took six double sets of the altitude of *Arcturus*, in the latitude above mentioned, when near the prime vertical westerly, with the altitude and azimuth instrument, first with the face left, and then with the face right: the mean of the whole, when corrected for refraction, was 24° 58′ 9″, at $19^h\ 6^m\ 45^s.2$ sidereal time by the clock. Required the error of the clock?

Apparent D = 20° 6′ 42″
Assumed D = 20 7 0

$$18 \times \begin{cases} -\ 1''.3310 = -\ 24''.00 \text{ for A.} \\ +\ 0\ .0627 = +\ \ 1.13 \text{ for } \theta. \end{cases}$$

	ALTITUDE.		TIME.
By Table I =	26° 2′ 49″.1		$4^h\ 52^m\ 23^s.60$
correction =	− 24.0		+ 1.13
corrected =	26 2 25.1		4 52 24.73
observed =	24 58 9.0		
	1 4 16.1	=	+ 6 53.57
			4 59 18.30
	Æ	=	14 7 34.90
	correct time	=	19 6 53.20
	observed time	=	19 6 45.20
	clock too slow	=	8.00

The advantage attending this method is the great facility with which the computations are made, compared with those which are required by the general formula (1). The time and labour employed in calculating the tables are not great: and when once effected they will serve for a considerable period, and prevent a recurrence of error. To persons travelling for any astronomical purpose, this method may prove highly useful. The stars which they can conveniently observe (for the purpose of deducing their time) during their stay in any one place, can be but few in number: and by the formation of a small table of this kind (which will not cost them more time or labour than the computation of one day's work by means of the general formula) they may multiply

their observations and diminish the chance of error, without any increase of trouble. And it may be useful to remark that if a table be formed for such occasional and temporary purposes, it will not be necessary to compute the values contained in the fifth and sixth columns: since we have only to calculate the equations (3), (5) and (7); which are extremely simple in their formation.

In the preceding examples I have supposed the observed time to be *sidereal* time: but if the clock be adjusted to *mean solar* time, we must convert the correct sidereal time into mean solar time, by any of the known rules, prior to its comparison with the time given by the clock. This operation however, *affects only the final result.*

Although I have, in the preceding investigations, confined the subject to the case of the *stars* only, yet it will be evident that the same principles may be occasionally applied to the *sun*. There are two objections however to the general application of them in this case: 1°. the sun cannot, for nearly two thirds of the year, be observed at a convenient altitude, on the prime vertical, in these latitudes: 2°. the constant change in the declination of the sun would render it necessary in most cases to extend the table to an inconvenient size. Nevertheless in some instances, near the summer solstice, when the change of declination is small and slow, this method may be adopted with success.

The mode of deducing the time from *single* altitudes of the sun or stars, possesses many advantages that will not apply to other methods in use for that purpose. M. Biot even prefers it to the transit instrument: but, setting this aside, it is evidently superior, in many respects, to the method of *equal* altitudes. The principles, on which this latter method is formed, may be more simple, and the practical application of them may be more easy: but there are many serious objections to the adoption of them. 1°. The observer is obliged to wait six, eight or ten hours between each series of observations. 2°. Flying clouds or an unfavourable change of the weather, may render his first set of observations (particularly in this country) totally useless as to this method. 3°. A change in the temperature and density of the atmosphere at the respective periods of observations may cause a variation in the apparent altitude, and thus lead to an erroneous result. 4°. The time ultimately deduced by this method, is not the time at which either the first series of observations or the second was taken; but some imaginary point between the two, the correct determination of which depends on the going of the clock during the interval. 5°. The *impossibility* of detecting, by this method, whether the clock has

stopped *, or deviated from its usual rate, during the time elapsed between the observations.

The method of *single* altitudes is free from all these objections. The observer, in a few minutes, completes his observations, and deduces his time with the greatest accuracy, from principles which are decided at the moment. Nothing is left to future contingency: and by the help of the formulæ which I have detailed in this paper, the computations are rendered as easy as by the method of equal altitudes.

To those persons, however, who still prefer the method of equal altitudes, and who observe the sun for this purpose, it may be useful to know that the following formula will give the results more correctly than the usual tables on this subject. Make

$D =$ the declination of the sun at noon.

$\mu =$ the double daily variation of the same †.

$L =$ the latitude of the place.

$h =$ half the time elapsed between the observations.

$$A = \frac{h}{720^{\text{hours}} \times \sin 15\, h.}$$

$$B = \frac{h}{720^{\text{hours}} \times \tan 15\, h.}$$

Then will the correction for equal altitudes of the sun be

$$\begin{aligned} &- A.\ \mu.\ \tan L. \\ &+ B.\ \mu.\ \tan D. \end{aligned}$$

This formula was first given in 1811, by M. GAUSS, in the *Monatliche Correspondenz*, vol. xxiii. page 402, accompanied by tables of the value of the logarithms of A and B, for *every minute* of the half interval from 0^h to 6^h, computed by M. GERLING: which will be found highly useful to those who are partial to this mode of deducing the time. These tables have since been reprinted by M. Schumacher in his *Sammlung von Hülfstafeln*, (the first part of which appeared during the course of the last year,) and are a valuable acquisition to the practical astronomer.

---

* In winding up a clock, when nearly down, the second hand will sometimes stop, and sometimes even move *backwards*, owing to the *going-spring* of the clock not being sufficiently strong.

† That is, the difference between the declination at the preceding and following noon. M. Schumacher, in his *Astronomische Hülfstafeln* for 1821, 1822, 1823 and 1824, has given these values for every day in the year: an example worthy of imitation in other ephemerides.

TABLE I.

Computed for Latitude 51° 33′ 34″.

Names of the Stars.	Assumed Declination.	Altitude. = A.	Hour Angle. = θ.	Variation for every 1″ *increase* in Declination. in A. +	in θ. −
	° ′	° ′ ″	h m s	″	s
α Orionis	7 22	9 25 18.2	5 36 26.37	1.283	0.0541
Regulus	12 50	16 28 26.5	5 18 19.80	1.298	.0566
Aldebaran	16 8	20 46 45.2	5 6 54.33	1.309	.0589
Arcturus	20 7	26 2 49.1	4 52 23.60	1.331	.0627
α Andromedæ	28 6	36 58 0.8	4 19 41.77	1.410	.0751
Castor	32 16	42 58 5.9	3 59 41.93	1.475	.0855

TABLE II.

Showing the variation in θ corresponding to the given variation in A: computed for Latitude 51° 33′ 34″.

Variation in Altitude.	Variation in the Hour Angle.	Variation in Altitude.	Variation in the Hour Angle.	Variation in Altitude.	Variation in the Hour Angle.
Seconds.	s	Minutes.	s	Degrees.	m s
1	0.11	1	6.43	1	6 26.10
2	0.21	2	12.87	2	12 52.30
3	0.32	3	19.30	3	19 19.10
4	0.43	4	25.74	4	25 46.20
5	0.54	5	32.17	5	32 14.10
6	0.64	6	38.61		
7	0.75	7	45.05		
8	0.86	8	51.48		
9	0.96	9	57.91		
10	1.07	10	m. 1 4.35		
20	2.14	20	2 8.70		
30	3.22	30	3 13.05		
40	4.29	40	4 17.40		
50	5.36	50	5 21.75		
60	6.43	60	6 26.10		

XXIX. *On a new Method of computing Occultations of the Fixed Stars. By J. F. W. Herschel, Esq., F.R.S. Foreign Secretary to the Astronomical Society of London, &c. &c. Communicated in a Letter to Charles Babbage, Esq. F.R.S., &c. &c.*

Read 10th January, 1823.

Dear Sir,

THE following method of computing an occultation will, I believe, be found simple in theory and exact in practice, and has the advantage of being stated in a manner which no one who understands the use of algebraic signs can misinterpret.

I have the honour to be,
Dear Sir,
Yours very sincerely,
John F. Wm. Herschel.

i. From the moon's right ascensions as given in the Nautical Almanac, and the apparent right ascension of the star on the day of occultation, compute the moment of true conjunction in right ascension to the nearest minute.

ii. Compute the real apparent zenith distances of the moon's centre and of the star, at the instant of conjunction in RA.—applying in the case of the moon the correction for parallax on the supposition of the earth's ellipticity, according to the usual well-known formula for that correction *.

iii. Compute the apparent azimuths of the moon's centre, and of the star for the moment of conjunction, correcting that of the moon for the earth's ellipticity *. The azimuths are to be reckoned from the north, eastward, and if the object be west of the meridian, its azimuth is to be regarded as greater than 180°.

iv. Determine from the azimuths and zenith distances so found whether the occultation is likely to be accelerated or retarded by the effect of parallax. If doubtful, leave it undecided.

* See T. Mayer, *Tabulæ Motuum Solis et Lunæ*, &c., p. cii.

v. Repeat the computations of zenith distances and azimuths for an hour *before* the moment of conjunction, if parallax be likely to *accelerate* the occultation; but for an hour after that moment, if it be likely to retard it, or if it be dubious which will be the case.

vi. The zenith distances and azimuths of both luminaries being thus computed for two instants of time at an interval of an hour from each other, let the *earliest* of these instants be assumed for an epoch. Let Z and z denote the respective zenith distances (apparent) of the moon and star, and A, a, their azimuths at this epoch; and let the same letters accented denote their values at the end of the subsequent hour.

vii. Compute α and β in seconds of space by the following formulæ:

$$\alpha = (Z'-z')-(Z-z)$$
$$\beta = (A'-a').\ \sin z'-(A-a).\ \sin z.$$

viii. Compute P and Q as follows:

$$P = \frac{(Z-z).\ (A'-a').\ \sin z' - (Z'-z').\ (A-a).\ \sin z}{\sqrt{\alpha^2+\beta^2}}$$

$$Q = \frac{\alpha\,(Z-z)+\beta(A-a).\ \sin z}{\alpha^2+\beta^2}$$

then will P be an arc, expressed in seconds, and equal to the least distance of the star from the moon's centre, and Q will be the time (in fractions of an hour) from the given epoch when the minimum distance is attained; and this, added to or subtracted from the epoch, according to its sign, will give the moment of the nearest approach, which in ordinary cases is the same with the middle of the occultation.

ix. Compute from the Nautical Almanac the moon's semidiameter at the instant last determined. Correct it for the augmentation, and when so corrected call it ϱ. If ϱ be less than P, there will be no occultation.

x. The semiduration of the occultation in all ordinary cases will then be expressed (in fractions of an hour) by

$$\sqrt{\frac{\varrho^2-P^2}{\alpha^2+\beta^2}}$$

and this added to, and subtracted from the time of the middle of the occultation will give the moments of emersion and immersion respectively.

If extreme precision be required, the computation must be repeated, assuming for the epoch the moment of nearest appulse already found, and instead of an hour's interval between the first and second instants, allowing only ten minutes.

In this re-computation, Q, instead of being expressed in fractions of an hour, will be in fractions of ten minutes; so that if the value given by the formula be multiplied by 10, Q will now be expressed in minutes, and must be applied with its proper sign as a correction to the first or approximate time of nearest appulse.

Demonstration.

Assuming the place of the moon's centre to be determined by two co-ordinates originating in the star, one of which x runs westward in a direction parallel to the horizon, and the other y, perpendicular to it, downwards, we shall have

$$x=(A-a).\sin z\text{; and } y=Z-z$$

Assuming then x and y to increase uniformly during one hour, we shall have (if t be the time elapsed since the epoch)

$$x=f+g\,t$$
$$y=f'+g't$$

and eliminating t, we shall arrive at an equation of the form

$$y=p+qx$$

To determine p and q, let x_0 and y_0 be the values of x, y, corresponding to the assumed epoch, and $x_{,}$ and $y_{,}$ those corresponding to the subsequent hour, then we have

$$y_{,}=p+qx_{,}$$
$$y_0=p+qx_0$$

whence

$$p=\frac{x_{,}y_0-y_{,}x_0}{x_{,}-x_0}\text{;} \qquad q=\frac{y_{,}-y_0}{x_{,}-x_0}$$

Now

$$x_0=(A-a)\sin z\text{;} \qquad x_{,}=(A'-a').\sin z'$$
$$y_0=Z-z \quad\text{;} \qquad y_{,}=Z'-z'$$

Consequently

$$y_{,}-y_0=(Z'-z')-(Z-z)=\alpha$$
$$x_{,}-x_0=(A'-a').\sin z'-(A-a).\sin z=\beta$$

Whence

$$p=\frac{(A'-a')\,(Z-z).\sin z'-(Z'-z')\,(A-a).\sin z}{\beta}$$
$$q=\frac{\alpha}{\beta}$$

Now since the distance of the star from the moon's centre is $\sqrt{x^2+y^2}$, its nearest appulse will be determined by the equation

$$x\,dx+y\,dy=o$$

that is

$$x+q\,(p+q\,x)=o$$

whence we find

$$x = -\frac{pq}{1+q^2}; \quad y = \frac{p}{1+q^2}$$

$$\sqrt{x^2+y^2} = \frac{p}{\sqrt{1+q^2}}$$

$$= \frac{(Z-z)(A'-a').\sin z' - (Z'-z')(A-a).\sin z}{\sqrt{\alpha^2+\beta^2}}$$

Moreover the apparent path actually described by the moon's centre (referred to the star) in an hour being

$$\sqrt{(x_{\prime}-x_{0})^2 + (y_{\prime}-y_{0})^2} = \sqrt{\alpha^2+\beta^2}$$

while that described from the epoch to the moment of nearest appulse is

$$\sqrt{(x-x_0)^2 + (y-y_0)^2}$$

the time of describing the latter space will be

$$\pm 1^{\text{hour}} \times \sqrt{\frac{(x-x_0)^2 + (y-y_0)^2}{(x_{\prime}-x_0)^2 + (y_{\prime}-y_0)^2}}$$

which, by making the necessary substitutions and reductions, becomes

$$-\frac{x_0(x_{\prime}-x_0) + y_0(y_{\prime}-y_0)}{(x_{\prime}-x_0)^2 + (y_{\prime}-y_0)^2}$$

or,

$$-\frac{\alpha(Z-z) + \beta(A-a).\sin z}{\alpha^2+\beta^2} = Q$$

But the numerator of this fraction represents, obviously, the value of $\frac{1}{2}\alpha$ (x^2+y^2) at the epoch, taking 1 hour for the differential of the time, and is therefore + or − according as $\sqrt{x^2+y^2}$ is increasing or diminishing. Now if the distance be increasing at the epoch, the moment of nearest appulse must be then already past, and the time Q must be subtracted from the epoch to give that moment. This is the reason why the sign − is prefixed to the whole fraction when cleared of the radical sign.

XXX. *The Results of Computations relative to the Parallax of α Lyræ, from Observations made with the Greenwich Mural Circle.* By the Rev. Dr. Brinkley, *Andrews Professor of Astronomy in the University of Dublin, F.R.S.* &c.

Read March 14, 1823.

THE opposite results deduced from the observations of the Greenwich mural circle and of the circle at the observatory of Trinity College, Dublin, relative to the parallax of certain fixed stars, have induced me to examine more minutely the Greenwich observations. The apparent confirmation of the observations made here, which I have recently deduced from the determination of the solar nutation, seemed to render the discordance of the instruments still more remarkable.

I have been much surprised to find that one of the simplest methods of examining the observations of the mural circle appears to point out a parallax in α Lyræ not sensibly differing in quantity from that which I had deduced by the circle of the College observatory.

From the nature of the mural circle, the arch intercepted between the stars ought to be found the same at all seasons of the year, provided the proper corrections be made for annual variation, aberration, nutation and refraction, and provided there be no sensible parallax. If the arch be not found the same, some cause must exist, either arising from the instrument or some actual change in the relative position of the stars.

With a reference to the question of parallax, α Lyræ has been chosen for examination, because it appears to have the greatest parallax of any star *near* the zenith. The pole-star, when above the pole, from the great number of observations that are to be found of it, from its having no sensible parallax according to my observations, and from some other circumstances, is an eligible star for comparison with α Lyræ. Using one star only as a point of comparison for other stars, reduces in some measure the mural circle to the simpli-

city of reversing instruments, in which the zenith point is determined by a plumb-line. The pole star besides is particularly pointed out as being the ultimate object of reference for all stars in the use of the mural circle.

If the divisions of the circle were perfectly exact, it would be immaterial whether the position of the telescope remained the same at the opposite seasons in which the intercepted arches are compared. But, admitting the divisions of the Greenwich circle to be as accurate as human ability could execute them, it is not to be expected the effects of different positions of the telescope will not be visible in such small quantities as are the objects of our inquiry. Therefore the arches only have been compared, when the position of the telescope was the same in both summer and winter.

The results of three years are,

	Summer Intercepted Arch between α Lyræ & Polaris.	Winter Intercepted Arch.	Diff.
	° ′ ″	° ′ ″	″
1813. Position Tel. 0°. 6 Micr.	49 42 10.49	49 42 12.48	1.99
1814. Position Tel. 0. 2 Micr.	49 42 12.10	49 42 13.92	1.82
1815. Position Tel. 10. 2 Micr.	49 42 13.61	49 42 15.70	2.09

These intercepted arches are reduced to Jan. 1. 1815.

Instead of using the intercepted arches, we may proceed another way, which must necessarily lead us to the same conclusions, but which will enable us to examine more satisfactorily the results given by the instrument during a considerable period.

If to the observations of the pole-star, reduced by the common equations, we apply the polar distance of the pole-star as given in the standard catalogue, we obtain the corrections for index error. Applying these to the observations of α Lyræ, each being reduced to a given time (1 Jan. 1815), we shall have a numerous series of observations of α Lyræ for our consideration. Then comparing the summer observations with those in the winter, we deduce the effects of parallax. It is obvious this conclusion will not be affected by any error in the standard catalogue. The only cause of applying the standard catalogue is to be able to refer each observation of α Lyræ to a single point in the manner we refer by our circle a star to the zenith point.

In this manner, with the Greenwich circle and by BRADLEY'S refractions, α Lyræ

1813 6 micr.	46 summer observations / 22 winter ditto	give $p = 1''.41$.
1814 2 micr.	32 summer observations / 33 winter ditto	give $p = 1''.18$.
1815	42 summer observations / 46 winter ditto	give $p = 1''.46$.

When the reductions are made by my own Table of refractions (the same in quantity as the French), the results become

For 1813 $p = 1''.12$
1814 $p = 1\ .01$
1815 $p = 1\ .16$

Mean $p = 1\ .10$ by the Greenwich circle.

The result of all my observations by the Dublin circle, as given in my paper on solar nutation, is $p = 1''.12$.

The manner in which these results have been deduced from the Greenwich observations will be easily understood from the annexed series of places of the pole-star and α Lyræ.

But it may be proper to make some remarks on the computations for each year.

In 1813 the observations of Polaris in May and June have been used for the index correction to be applied to the observations of α Lyræ in July and August. This was necessary, as no observations of Polaris when above the pole were made in July and August; and it is evident, from an examination of the index errors as given by Mr. Pond for 1813, that no material change occurred till November in that year.

The index corrections by Polaris above pole are,

19 obs. in June	$-1''.53$
13 obs. in October	$-1\ .31$
13 obs. in November	$-1\ .78$

The mean is $-1''.53$, precisely the same as in June.

It is to be noticed that in this year the mean of the summer and winter observations (with 6 microscopes) give the mean Z. dist. 1815 = 51° 22′ 54″.16, differing but little from the standard catalogue (6 microscopes), 51° 22′ 54″.57,

the result of 170 observations; which shows that this use of the pole-star only for the index correction gives an exact result.

In 1814, it appears that no change in the index correction took place after April 17, till the end of the year. Mr. POND himself considers the correction to have remained permanent during that time, and the observations of Polaris tend to prove this. Indeed the index correction by that star is $-14''.83$ from the observations in April, May and June, which is somewhat greater than $-3''.96$, the correction given by the observations in November, December and January. But the former being more in favour of the parallax of α Lyræ, I have preferred deducing the index correction from a mean of the observations of Polaris between April 1814 and February 1815. This mean is $-14''.31$.

In 1815 no change in the index correction appears to have taken place between May 24 and the end of the year. The observations of Polaris also in this year show a remarkable steadiness. By it, the correction in May and June appears to have been $+ 6''.04$, and in November and December $+ 6''.46$. In January it appears to have increased. As $+ 6''.04$ is more in favour of the parallax of α Lyræ than $+ 6''.46$, I have assumed the mean of these quantities for the index correction in July and August.

The near agreement of the quantities of parallax deduced in each of these years by the mural circle, and of that deduced by the Dublin circle, is very remarkable.

If there were no other stars to refer to than α Lyræ and the pole-star, the results from the observations that have been considered would seem sufficient to place beyond all doubt the conclusions relative to the parallax of α Lyræ. But if we consider the observations, sufficiently numerous, of γ Draconis, we shall also find that we must admit a parallax in this star nearly equal to that of α Lyræ*, which in itself is highly improbable, and is in direct opposition to the observations of the Dublin circle. Here then is the great point of difficulty.

* Index errors, using the standard catalogue.

	Summer 1814.	Winter 1814—15.	Summer 1815.	Winter 1815—16.
Polaris	$-14''.83$	$-13''.96$	$+6''.04$	$+6''.84$
γ Draconis	-15.08	-16.23	$+4.18$	$+2.83$
α Lyræ	-14.53	-16.36	$+4.60$	$+3.01$

The result from the pole-star and α Lyræ is in direct opposition to the result from γ Draconis and α Lyræ. The places of the pole-star on the instrument in summer and winter, when contrasted with those of γ Draconis and α Lyræ, is a new source of difficulty, and is not easily explained whether we admit or deny the existence of parallax. But the argument deduced from the apparent constancy of the arch between γ Draconis and α Lyræ is much weakened. Had the places of the *three* stars remained the same or been changed by nearly the same quantities and *in the same direction,* the argument would have been a powerful one.

A close examination of the Tables of index errors, as given by Mr. Pond, will point out some notable discordances in the arches between stars much nearer together than α Lyræ and γ Draconis. Thus in September 1816, when the observations were conceived to be more exact than at other times, the arch of polar distance, less than 2° between γ Draconis and η Ursæ Majoris, differed from the same arch in November by 2″.1.

Notwithstanding the great number of observations that have been made by the Greenwich instrument, a reference to the published Tables of index errors will show that, besides Polaris and γ Draconis, there are no other stars to be found properly circumstanced for making a satisfactory comparison with α Lyræ in the manner that has been done with Polaris. But it may be remarked, that an examination of the Tables of index errors for several years, published with the observations for 1816, will by no means be unfavourable to the conclusions that have been deduced from the observations of Polaris and α Lyræ.

In the three years which have been considered, the mural circle may be supposed to have been in a more perfect state than it was afterwards. Besides the three above mentioned, there were only two years that could have been used in this comparison from the want of corresponding observations, viz. 1812 and 1817. In 1812 the position of the telescope was so often changed, that it has not been thought right to use the observations for parallax, although in fact they gave it greater than in the subsequent years. As affording, however, an addition to the series of results of the polar distance of α Lyræ at the two seasons, the deductions are annexed.

In 1817 there is no information given in the observations, as in former years, of the changes in the index correction. When a change of some seconds took place in November of that year, it is not noted; and there may have been other

changes more minute not noticed. If, however, the index error remained the same through June, July and August, the results of this year would be against parallax.

In instituting the above examination of the observations of the mural circle, my only object has been to endeavour to throw light on a very difficult subject. There cannot be a doubt that Mr. Pond's method of deducing the index correction is most proper for observations in general, such as the polar distances of the sun, moon and planets, &c. But when used for parallax, there appear too many sources of error mixed up together, to ascertain their precise effect on the mean index correction.

The only method of examining the capability of the instrument for this purpose seems to be, that of comparing the differences of the polar distances of two stars in the opposite seasons, in the manner that I have done for the pole-star and α Lyræ. There are however many obstacles that will circumscribe both the choice of stars and number of observations.

In the volume containing the observations for 1816, Mr. Pond has given a correction for the results of several years when two microscopes only were used. Now it appears that this correction is merely empirical, and in adopting it the non-existence of parallax seems to have been assumed.

It is difficult to say how this correction may be affected by parallax. The very circumstance of the correction being necessary appears to increase the difficulty of deducing exact conclusions relative to parallax from the Greenwich circle, unless by the simple method of comparing two stars.

This becomes more evident if we consider the discordances between the north polar distances of α Lyræ deduced by help of Polaris in 1814 and 1815, and that in the standard catalogue.

	January 1, 1815.		
By the observations of 1814 (2 micr. tel. 0)	51°	22′	55″.71
By the observations of 1815 (2 micr. tel. 10)	51	22	57 .35
Standard Catalogue	51	22	54 .57

The comparison of single observations is still more remarkable.

The single results for α Lyræ, July 17, 1812, and December 10, 1815, differ by 10″, and several others differ by above 8 seconds. It appears necessary for conciliating these observations, to use all possible allowance by adopting parallax, as well as change of figure, inequalities of division, &c.

It would not be right to conclude these remarks without noticing one circumstance that shows in a striking manner the skill with which this instrument has been constructed and supported. There appears nothing by which we can state with certainty that the index error of this iustrument changed from April 16, 1814, to the end of the year, and also from May 24, 1815, to the end of the year. This permanency of the mural circle in summer and winter surpasses considerably that of the fixed telescope used for α Cygni. In that instrument a change, as has been remarked elsewhere, took place, amounting to several seconds between January and June.

α LYRÆ.

Telescope 0°. Six Microscopes.

1813.		Mn. N.P.D. Jan. 1, 1815.	Coeff. of p.
June 22	Ind. cor. } −1″.53	51° 22′ 52″.11	+0″.88
23	An^l var. −6.00	53.96	88
24	−7.53	53.34	88
25		53.25	88
26		53.60	88
27		53.81	88
28		53.01	88
July 5		53.20	87
6		54.14	87
9		53.77	86
10		53.63	86
11		52.73	86
12		53.58	85
13		52.64	85
16		53.40	84
17		54.33	83
18		54.27	83
19		53.72	82
23		51.75	80
24		52.41	79
25		52.14	79
27		53.19	77
28		52.54	77
29		52.27	76

1813.		Mn. N.P.D. Jan. 1, 1815.	Coeff. of p.
July 30	Corr. −7″.53	51° 22′ 51″.87	+0″.76
Aug. 1		52.00	74
3		52.65	73
5		53.71	71
7		53.97	69
9		53.13	66
10		53.12	65
11		53.09	64
12		52.80	63
13		53.31	62
15		53.16	60
16		52.98	59
17		53.19	58
19		53.90	55
20		53.51	54
21		52.67	53
22		53.28	52
23		53.51	51
24		53.72	50
26		52.93	47
30		53.37	43
31		54.18	42

46 observ. mean 51° 22′ 53″.19+0″.72 p.

1813, 1814.		Mn. N.P.D. Jan. 1, 1815.	Coeff. of p.
Nov. 1		51° 22′ 54″.30	−0″.48
3	Ind. cor. } −1″.78	54.49	50
4		54.22	51
6	An^l var. −6.00	53.53	54
8	−7.78	54.51	56
11		54.72	60
18		56.07	67
20		54.55	70
27		55.12	76
30		55.71	78
Dec. 15	Ind. cor. } −2″.80	55.85	86
21		54.56	88
23	An^l var. −6.00	54.44	88
26	−8.80	55.77	88
30		55.51	88
31		56.32	88
Jan. 11		54.88	85
12	Ind. cor. } −3″.22	55.81	84
29	An^l var. −6.00	55.68	74
30	−9.22	55.36	73
31		54.65	72
Feb. 2		54.92	70
3		54.59	69
6		54.71	66
14		54.51	57
17		56.05	53
19		56.48	51
20		55.89	50
22		55.16	47
24		55.36	44
25		55.68	43
26		55.38	42

32 observ. mean 51° 22′ 55″.14−0″.66 p.

From the above 53″.19+.72 p=55″.14−66 p. ap=1″.41

The mean Thermometer

Summer { α Lyræ 59°, Polaris 56 }
Winter { α Lyræ 35, Polaris 37½ }
Hence is deduced by French infinitives p=1″.12

The under corrections are deduced from the polar distance of α Polaris, as given in the standard catalogue published with the observations of 1815, the observed places reduced to the mean being taken from the same volume.

The mean zenith distances of α Lyræ here given are deduced from those observed. The reductions of which to the mean are given in the volume of observations for 1814. To which, instead of the correction there applied, the correction deduced from Polaris is applied.

Index Corrections deduced from Observations of POLARIS above Pole.—Two Microscopes.

Summer 1814.	Obs. N.P.D. reduced to Jan. 1, 1814.
April 16	1° 41′ 17″.51
20	17.41
29	18.33
30	17.89
May 2	17.46
6	17.32
10	16.72
15	16.59
16	17.11
17	15.85
18	16.78
19	16.55
20	16.93
26	15.43
28	17.06
29	16.80
June 8	16.75
10	15.86
15	17.68
16	16.98
19	17.61
Mean of 21 observ.	1° 41′ 16″.98
Stand. cat.	1 41 2.15
Index corr.	— 14.83

Winter 1814-15.	Obs. N.P.D. reduced to Jan. 1, 1814.
Nov. 6	1° 41′ 16″.00
7	15.21
8	14.72
10	16.15
12	15.96
14	16.39
16	17.05
19	16.66
21	16.81
22	16.96
29	15.51
30	15.88
Dec. 1	14.71
3	14.15
4	14.88
9	16.87
15	15.95
16	17.32
20	18.94
21	18.21
22	16.91
1815. 31	16.15
Jan. 1	15.12
8	16.75
12	16.39
17	14.42
18	16.26
19	17.36
28	15.34
Feb. 2	16.21
5	14.39
Mean of 31 observ.	1° 41′ 16″.11
Stand. cat.	1 41 2.15
Index corr.	— 13.96

Summer 1815.	Obs. N.P.D. reduced to Jan. 1, 1815.
May 24	1° 40′ 37″.58
26	37.81
27	36.21
29	35.37
30	38.35
June 7	35.87
8	39.94
9	35.76
15	35.08
18	35.43
19	36.16
29	36.41
Mean of 12 observ.	1° 40′ 36″.66
Stand. cat.	1 40 42.70
Index corr.	+ 6.04

Winter 1815-16.	
Nov. 2	1 40 36.95
3	37.19
6	34.57
7	36.76
9	36.15
14	34.48
15	36.40
16	36.73
17	37.59
18	35.14
22	36.30
23	36.10
25	35.37
26	36.30
27	36.27
29	36.84
Dec. 3	36.16
4	38.02

Winter 1815-16.	Obs. N.P.D. reduced to Jan. 1, 1815.
Dec. 7	1° 40′ 36″.13
8	36.36
10	36.53
11	36.30
12	36.68
13	35.87
17	36.31
20	36.45
22	35.30
24	35.78
25	34.06
27	37.83
30	36.53
Mean of 31 observ.	1° 40′ 36″.24
Stand. cat.	1 40 42.70
Index corr.	+ 6.46

1816.	
Jan. 2	1 40 35.13
3	35.53
4	35.02
11	35.98
13	35.64
17	34.57
20	34.27
26	33.78
Feb. 1	34.79
4	36.54
8	37.07
9	35.22
12	35.65
13	32.80
14	34.26
21	35.97
24	35.82
Mean of 17 observ.	1° 40′ 35″.18
Stand. cat.	1 40 42.70
Index corr.	+ 7.52

Supposing the correction for index error to have remained constant from April 1814 to the February following, which is evidently unfavourable for parallax, the index correction during that time will be = −14″.31, which is applied to the observations of α Lyræ, and the results are as in the following page. Mr. Pond supposes no alteration to have taken place between April and the end of the year, and the index corr. by Polaris between January 1 and February 5, = −13″.65.

There does not appear to have been any change of consequence between May 1815 and the end of the year, from a comparison of the index errors of the first 12 observations and of 31 at the end of the year. Therefore supposing the index corr. in July and August to be that resulting from a correct mean between those, it will be = +6″.34.

The correct mean from the observations between Nov. 2 and Feb. 24 = +6″.84.

The preceding observations of Polaris and the following of α Lyræ are calculated from the observations themselves

α LYRÆ.

Telescope 0°.—Two Microscopes.

Summer 1814.		Mn. N.P.D. Jan. 1, 1815.	Coeff. of p.	Winter. 1814-15.		Mn. N.P.D. Jan. 1, 1815.	Coeff. of p.
June 30	Ind. cor. } −14″.31	51° 22′ 55″.99	+ 0″.88	Nov. 6	Ind. cor. } −14″.31	51° 22′ 57″.49	− 0″.54
July 1	An^l var. − 3.00	54.87	88	8	An^l var. − 3.00	55.31	56
2	−17.31	51.66	88	10	−17.31	56.50	58
5		54.17	87	12		55.09	61
6		54.69	87	13		55.96	62
10		52.73	86	16		55.77	65
11		55.08	86	19		55.68	68
12		56.19	85	22		56.18	72
15		54.83	84	28		56.11	77
16		54.35	84	Dec. 1		56.76	79
17		54.45	83	4		56.55	81
18		54.56	83	6		56.22	82
21		55.41	81	20		57.52	87
22		52.23	80	21		56.66	88
23		55.04	80	22		56.50	88
24		55.50	79	1815. 30		57.68	88
25		55.35	79	Jan. 2		57.53	88
26		55.31	78	6		56.18	87
27		56.65	77	7		56.45	86
29		57.12	76	8		57.88	86
30		55.58	76	9		57.14	86
31		54.75	75	10		55.89	85
Aug. 1		55.11	74	11		57.28	85
2		54.70	74	12		57.12	84
4		54.01	72	17		56.78	82
5		54.88	71	18		56.96	81
6		54.82	70	23		56.30	78
8		54.06	68	27		57.05	75
9		54.50	66	28		55.58	74
11		54.25	64	29		56.79	74
12		55.23	63	Feb. 3		56.69	69
14		55.44	61	4		57.48	67
				8		57.28	63
Mean of 32 observ. 51° 22′ 54″.80+0″.78 p.				Mean of 33 observ. 51° 22′ 56″.62 −0″.76 p.			

The equation is

$$54''.80 + .78\,p = 56''.62 - .76\,p.$$

$$\text{or } p = 1''.18.$$

Mean Thermometer

Summer	α Lyræ	61°	Hence is deduced, using French refractions,
	Polaris	53	$54''.80 + .75 + .78\,p = 56''.62 + .48 - .76\,p.$
Winter	α Lyræ	39	or $p = 1''.01$.
	Polaris	40	

α LYRÆ.

Telescope 10°.—Two Microscopes.

Summer 1815.		Mn. N.P.D. Jan. 1, 1815.	Coeff. of p.
July 2	Ind. cor. +6".3	51° 22' 54".59	+0".88
3		53.58	88
4		56.13	87
5		56.14	87
7		54.43	87
9		56.80	86
11		54.99	86
12		56.87	85
13		56.92	85
16		58.95	84
18		55.90	83
20		53.62	81
21		55.29	81
23		56.32	80
25		54.07	79
26		55.83	78
27		56.16	77
28		55.78	77
29		56.35	76
30		57.27	76
31		56.67	75
August 1		56.39	74
2		55.91	74
3		56.85	73
4		57.53	72
6		57.10	70
7		57.87	69
8		57.34	67
9		56.33	66
11		56.27	65
14		56.79	61
16		57.83	59
17		58.12	57
19		57.16	55
21		56.92	53
22		56.10	52
24		55.27	50
25		57.10	49
26		56.81	48
27		56.41	47
29		56.57	44
31		55.83	41

Mean of 42 observ. 51° 22' 56".31+0".71 p.

Winter. 1815-16.		Mn. N.P.D. Jan. 1, 1815.	Coeff. of p.
Nov. 3	Ind. cor. +6".84	51° 22' 58".71	−0".50
4		58.50	51
5		56.81	52
13		56.96	62
14		57.81	63
18		58.03	67
20		57.82	69
23		58.57	73
26		58.55	75
27		58.04	76
28		58.96	77
Dec. 4		59.10	81
8		59.34	83
9		59.30	83
10		60.85	84
12		59.34	85
13		59.26	85
14		59.37	86
18		59.01	87
22		59.03	88
23		58.40	88
24		58.85	88
25		58.91	88
29		58.99	88
1816. Jan. 1	Ind. cor. +6".84	59.40	88
2		58.04	88
3	An¹ var. +3.00	57.88	87
6	+9.84	58.71	86
9		57.32	85
10		58.95	85
15		57.20	83
17		57.51	81
29		57.76	74
30		57.16	73
31		57.78	72
Feb. 7		58.80	64
8		54.90	63
9		57.89	62
11		57.76	60
13		59.17	58
16		58.29	54
20		59.09	50
23		56.13	46
25		59.31	43
27		59.76	41
28		59.26	39

Mean of 46 observ. 51° 22' 58".40−0".72 p.

The equation is

$$56''.31 + .71\,p = 58''.40 - .72\,p.$$

$$\text{or } p = 1''.46$$

Mean Thermometer

Summer { α Lyræ 60°, Polaris 58 }
Winter { α Lyræ 36, Polaris 37 }

Hence is deduced, using French refractions,

$$56''.31 + .84 + .71\,p = 58''.40 + .41 - .72\,p.$$

$$\text{or } p = 1''.16$$

α LYRÆ.

Summer. 1812.	By pole-star.	Mn. N.P.D. Jan. 1, 1815.	Coeff. of p.
July 6	Ind. cor. } −1″.19	51° 22′ 54″.76	+0″.87
Telescope 7		54.95	87
(10°) 9	An^{l} var. −6.00	55.71	86
10	−7.19	55.60	86
14	Ind. cor. } −7″.26	51.98	85
17	An^{l} var. −6.00	50.54	83
(20°) 21		50.64	81
22	−13.26	52.17	80
23		52.72	80
26	Ind. cor. } +14″.98	53.32	78
28		54.75	77
(0°) 30	An^{l} var. −6.00	55.45	76
31		54.75	75
Aug. 1	+8.98	54.98	74
6	Ind. cor. } +14″.66	52.92	70
12		53.15	63
13	An^{l} var. −6.00	53.36	62
(0°) 15		51.44	60
16	+8.66	52.72	59
17		51.82	58
18		51.70	57
19		52.17	55
20		52.75	54
21		53.42	53

Mean of 24 observ. 51° 22′ 53″.24+0″.72 p.

Winter. 1812.		Mn. N.P.D. Jan. 1, 1815.	Coeff. of p.
Nov. 2	Ind. cor. } −1″.21	51° 22′ 56″.05	−0″.49
3		56.80	50
Telescope 7	An^{l} var. −6.00	55.31	55
(10°) 8		55.34	56
13	7.21	56.95	62
15		55.90	64
19	Ind. cor. } −7″.26	55.35	68
(20°) 20		55.91	70
21	An^{l} var. −6.00	54.70	71
22		55.29	72
Dec. 2	+1.26	55.77	80
6		55.16	82
8		55.78	83
9		55.61	83
10		55.51	84
12		54.54	85
13		55.05	85

Mean of 17 observ. 51′ 22′ 55″.59−0″.71 p.

The equation is

$$53''.24+0.72\,p=55''.59-0.71\,p.$$

$$\text{or } p=1''.64.$$

The above are collected from N. P. D. published in the volume containing the observations for 1814. The index errors there given are applied with a contrary sign and also two years annual variation, together with the index error by the pole-star. By this process the column containing the N. P. D. Jan. 1, 1815, is obtained.

XXXI. *On the Differences of Declination of certain Stars, according to different Astronomers; and on Refraction, &c. Extracted from a Letter of M. J. J. LITTROW, Director of the Imperial Observatory at Vienna, to the Foreign Secretary.—Dated Vienna, January* 3, 1823.

Read 11th April, 1823.

I WISH to give you a few words relative to the declinations of Mr. POND you were so good as to send me; but first I send you here a statement of the declinations as observed and calculated by BESSEL with great care. The column A* is from his recent observations with the three-foot meridian circle of REICHENBACH; B from his old observations with CARY's small circle; C is the annual variation in declination arising from precession, and proper motion; the following columns exhibit the differences between the result A, and those of other astronomers and instruments, viz. D the difference of A and B both reduced to the same epoch; the others E, F, G, speak for themselves; H is the well-known standard catalogue of Mr. POND, and I is from the Nautical Almanac of 1821.

Among these determinations, and those you sent me in your last letter, we may observe many stars which nearly agree in their differences; but discordancies are not wanting. The result of both is, *that BESSEL's declinations collectively* are more southern than those of POND, PIAZZI, ORIANI, and BRINKLEY. This is the chief point on which elucidation is wanted: but to obtain it appears very difficult. I cannot approve, however, of calling the differences between BESSEL's and the other results, without further observation, "error of BESSEL's catalogue," as is done in the catalogue you communicated to me†. They differ, it is true; but which is right is yet undecided.

In a former letter I have told you the grounds of my belief that BESSEL has gone to work with all possible care, and that he is, both in a theoretical and practical point of view, one of the first astronomers of our time. But this is

---

* See the Table at pages 347 and 348.

† [This catalogue was communicated by the Astronomer Royal to Mr. LITTROW through the foreign secretary, and transmitted by him without comment or addition.

no proof of the correctness of his declinations. Let us then wait a little longer for the decision.

After all, I see no reason why each party should be at present so very strenuous in defending his own results. Let us keep on observing with all possible circumspection ; do all we can, and leave the decision to time, which must come at last if we will only accumulate data sufficient. Mr. Pond seems to me somewhat needlessly anxious for the reputation of his observations at Greenwich. After all, has he not all along had great advantages over all others? Is not his instrument, made by the first living artist, one of the best on earth? Do not his observations agree as well as possible with themselves? Do they not agree too with those of Oriani, Piazzi, and Brinkley ? If we were all as strongly disinclined as we are inclined to him, what could justify us in imputing errors to him or his colleagues? Nor can the "*anguis in herbâ*" be sought in the circumstance of the one instrument being Reichenbach's and the other Troughton's ; for Bessel's first circle was of English make, and Oriani's was by Reichenbach. Mr. Pond is therefore in very good company with Brinkley, Oriani, and Piazzi; and, from all we know, these astronomers have convinced themselves by every means in their power of the exactness of their declinations. Here then we have authority against authority, and, to be candid, the greatest weight of it is on Mr. Pond's side. But authority is no proof; and, as aforesaid, instead of insisting, we ought to wait.

Meanwhile one remark presses itself on our attention, which I will mention for your consideration. Most astronomers seem to me to rely too much on their instruments, as free from errors. In my opinion each instrument, as an *individuum,* has its own peculiarities, its marks and blemishes, and we should before all things get acquainted with those before we confide wholly in it, just with the caution with which we deal with those living instruments of our own species—with mankind. To use even chef-d'œuvres of the greatest artists just as they come from their maker's hands, with no other care than keeping all in adjustment, often reversing, &c. &c., seems to me not enough where (as in this case) the question is about the utmost accuracy.

Whether other astronomers do this, or what they do, or what leave undone, I know not. But this I know, that Bessel does it. Only look, for example, into the introduction to his first, sixth, and seventh volume of the Königsberg Observations. I know of no such investigations entered into by other astronomers. I do not say that others do nothing similar—perhaps much better; I only say that nothing of the kind has come to my knowledge, and therefore

I confess it appears to me BESSEL is right when he says that the disagreement "will never cease till every observer shall succeed in eliminating out of his results the *individuality* of his instrument *(sit venia verbo).*"

The English instruments appear to me far preferable in their construction to our repeating circles. In the former the two ends of the axis are solid; in the latter they are not fastened at all. In the former the circle and telescope are supported firmly in the middle; in the latter all hangs on *one* side, which can never be good. In the former there is but one axis on its solid bed; in the latter are three which move one within another, &c. On the other hand, I believe our division to be much better than yours, nor do I doubt that the English circles would give much better results were they *so* divided. Could I only show you our great dividing engine made by REICHENBACH in Vienna, the mere aspect of it would convince you. It is (partially at least) drawn and described in "*Gilbert's Annalen der Physik,*" 1821, on occasion of a dispute between REICHENBACH and LIEBHEN, ... Look too, at the account of the apparatus of four microscopes which BESSEL has given in the introduction to his Observations, and which he had made for examining the divisions of his circle. In the circle of CARY he found notable errors. In the 3-feet meridian circle of REICHENBACH, according to his researches, the probable error of each division is only $0''.325$. This is saying a great deal.

I know not whether your English artists bestow so much care as ours on the ends of the steel axes or pivots *(zapfen)*: with us they are not cylinders but cones, and the part between the cones is quite free. But the essential thing is the lever of contact *(fühlhebel)*: when the pivot is finished, it is brought on its own support, and the extremity of the short arm of a lever brought in contact with it by a peculiar adjustment, and the axis then turned round, when every minute deviation from the circular form is indicated on a divided arc by the extremity of the other arm of the lever, which is twenty times longer, and read off by a lens, being magnified 130 times. REICHENBACH states that he dates the perfection of his instruments from the invention of the lever of contact

* * * * * * * * * *

I must take this opportunity to observe that I have read with particular pleasure the demonstration given by your excellent analyst IVORY, in the *Philosophical Magazine* for 1821, of the elegant formula of MAYER, which has often been sought among us, though BESSEL had already known and acknowledged that MAYER's correction of the refraction for the thermometer is the true one, though differing from that assumed by all other astronomers. But, with-

out here speaking of this thermometrical correction, the *form* itself of MAYER's expression for the mean refraction is very remarkable. Calling z the apparent zenith distance, MAYER's formula is

$$r = \frac{\text{AR.} \sin z}{\cos z + \sqrt{\text{A}^2 + \cos^2 z}} \qquad \text{I.}$$

$$\left.\begin{aligned} \text{or } \tan x &= \frac{\text{A}}{\cos z} \\ r &= \text{R.} \sin z . \tan \frac{x}{2} \end{aligned}\right\} \qquad \text{II.}$$

Where R is the mean refraction, and A a constant (this magnitude A, according to MAYER, is not entirely constant but depends on the thermometer, as is also generally acknowledged). If we abstract at present from the correction of temperature and make A strictly constant, we are enabled to represent our best refraction tables (those of LAPLACE, BESSEL, &c.) only from $z=0$ to $z=80°$, within which limits the well-known formulæ of BRADLEY, SIMPSON, and many others, suffice, so that MAYER's has no advantage. If, however, we allow A to be in some slight degree dependent on the zenith distances, the coincidence may be carried much further. It is thus that I have tried to represent LAPLACE's tables, as given by CARLINI in the Milan Ephemeris.—Here is what I have found.

Put R = 1845″.7, the horizontal refraction according to CARLINI, and A = 0.06265 − 0.00002. tan z, then we find

z	r by formula I.	r by CARLINI.	Difference.
20	21.1	21.1	0″.0
40	48.5	48.6	− 0 .1
60	99.8	100.0	− 0 .2
70	157.4	157.9	− 0 .5
80	317.3	317.9	− 0 .6
81	350.8	351.3	− 0 .5
82	391.6	392.0	− 0 .4
83	442.2	442.6	− 0 .4
84	506.6	506.7	− 0 .1
85	590.5	590.2	+ 0 .3
86	703.0	702.6	+ 0 .4
87	858.6	858.8	− 0 .2

All being calculated for the barometer 28 Paris inches, and the thermometer

of REAUMUR + 10°.—See the ephemeris of Milan, 1817, page 100.—The formula of LAPLACE from which this table is constructed, is

$$\tau = 28 \cos z \qquad a = 0.7175935$$

$$r = 1624 \sin z \left\{ (2 - a - 2a\tau^2)\psi + a\tau \right\}$$

$$\text{where } \psi = \frac{1}{2\tau}\left\{1 - \frac{1}{2\tau^2} + \frac{1.3}{2^2.\tau^4} - \frac{1.3.5}{2^3.\tau^6} + \&c.\right\}$$

or, when $z > 82°$

$$\psi = \frac{\sqrt{\pi}}{2}\left\{1 + \tau^2 + \frac{\tau^4}{1.2} + \frac{\tau^6}{1.2.3} + \ldots\right\} - \tau\left\{1 + \frac{2\tau^2}{1.3} + \frac{2^2\tau^4}{1.3.5} + \&c.\right\}$$

for z under 82° we obtain from the foregoing formulæ immediately this series,

$$r = 58'' \tan z \left\{1 - \frac{1+a}{2\tau^2} + \frac{3(1+2a)}{2^2\tau^4} - \frac{3.5}{2^3\tau^6}(1+3a) + \frac{3.5.7}{2^4.\tau^8}(1+4a) - \&c.\right\}$$

It is, perhaps, not uninteresting to see that one may represent all these complicated expressions by the simple little formula

$$r = \frac{\text{A R.} \sin z}{\cos z + \sqrt{\text{A}^2 + \cos z^2}}$$

as far as $z = 87°$. I have not leisure at present, being desirous to dispatch this letter, to examine whether the approximation cannot be carried on nearer the horizon by taking $\text{A} = a - b. \tan z + c. \tan z^3 - d. \tan z^5 +$, &c.; but I have no doubt it may. A few days ago I made with this view an examination of BESSEL's refractions as given by him in the *Fundamenta Astronomiæ*. His formulæ are frightfully complicated and wearisome, as you may easily convince yourself; yet the whole of his table as far as $z = 86° 30'$ is represented very well by the following formula;

$$\begin{cases} \tan x = \dfrac{0.0535506 - 0.000234 \tan z}{\cos z}; \\ r = \left\{ 2173.7939 - 0.03. \tan z^2 \right\}. \sin z. \tan \dfrac{x}{2} \end{cases}$$

Thus we find

z	BESSEL.	Formula.	Difference.	z	BESSEL.	Formula.	Difference.
10°	10.1	10.2	−0″.1	75°	211.4	211.5	−0″.1
20	20.9	21.1	−0 .2	80	315.1	314 9	+0 .2
30	33.2	33.5	−0 .3	81	348.1	347.8	+0 .3
40	48.3	48.6	−0 .3	82	388 4	387.9	+0 .5
45	57.5	57.7	−0. 2	83	438.3	437.9	+0 .4
50	68.5	68.9	−0 .4	84	501.7	501.5	+0 .2
55	82.0	82.4	−0 .4	85	584.6	584.7	−0 .1
60	99.3	99.7	−0 .4	85½	636.6	636.5	+0 .1
65	122.8	123.2	−0 .4	86	696.9	697.2	−0 .3
70	156.7	157.0	−0 .3	86½	769.1	769.0	+0 .1

This coincidence seems to me too near not to call for a closer theoretical examination, for which however I have hitherto wanted time.

I have already mentioned to you in a former letter, Frauenhofer's circular micrometer, if I mistake not. It consists of a steel ring fastened in the eye-tube on a plane glass, and *after* being so fastened turned exactly round on both sides. This I believe is the only way to get perfect circles, (viz.) to turn them *in situ*, since they are sure to change their figure when detached from the piece which holds them on the lathe. I have now procured such a circle, and notwithstanding the extreme cold ($-12°$ Reaumur) determined its diameter by the passage of two stars, whose declinations differ by nearly the whole diameter to be measured, and for greater security have employed several pairs of stars, and between the observations often turned the eye piece round, to examine all points of the periphery. Frauenhofer wrote me word that only the inner circle would be found exactly round, but I find both equally good. It is very pleasing to see such a ring hanging freely as it were in the heavens without perceiving how it is suspended, and moreover very advantageous, as we see the stars come which we would observe. I send you some of the results of these observations, which I think agree very well.

Inner Circle.

α and ε Aquilæ.

I. obs. $\frac{1}{2}$ diam. =	779″.4 in arc.
II.	780 .2
III.	778 .6
IV.	779 .7
V.	779 .2
VI.	780 .1

Maia and Merope.

I.	778 .5
II.	779 .0
III.	778 .8

Outer Circle.

α and γ Cor. Bor.

I. semidiam.	905″.2
II.	906 .1
III.	907 .5
IV.	906 .4
V.	907 .6
VI.	908 .0
VII.	907 .7

	A Declination. 1820.	B Declination. 1815.	C Ann[l] Var. 1820.	D Bessel. 1815.	E Piazzi. 1800.
α Aurigæ	+45° 48′ 9″.12	+45° 47′ 44″.72	+ 4″.478	−1″.93	−0″.81
α Cygni	44 38 28.47	44 37 26.21	+12.563	+0.53	+2.14
α Lyræ	38 37 17.77	38 37 4.10	+ 2.962	+1.01	+1.69
α Geminor.	32 16 21.05	32 16 54.66	− 7.190	−2.27	+1.20
β ———	28 27 5.54	28 27 44.14	− 8.087	−1.77	+0.50
β Tauri	28 26 40.40	28 26 20.09	+ 3.712	−1.68	+0.42
α Andromedæ	28 5 46.59	28 4 3.28	+19.906	−3.78	+0.52
α Coronæ	27 19 34.44	27 20 34.61	−12.483	−2.28	+3.31
α Arietis	22 36 22.32	22 34 56.11	+17.350	+0.57	+1.67
α Bootis	20 7 25.43	20 9 0.75	−19.009	+0.25	+2.26
α Tauri	16 8 17.16	16 7 37.29	+ 7.855	−0.54	+2.86
β Leonis	15 34 40.04	15 36 20.92	−20.083	+0.47	+3.07
α Herculis	14 36 10.45	14 36 32.52	− 4.614	−1.05	+4.20
α Pegasi	14 14 19.05	14 12 41.94	+19.258	−0.83	+2.98
γ ——	14 10 56.22	14 9 13.02	+20.028	−3.06	+0.97
α Leonis	12 50 33.58	12 51 59.71	−17.310	−0.39	+2.69
α Ophiuchi	12 41 55.66	12 42 10.37	− 3.125	−0.97	+4.04
γ Aquilæ	10 10 53.97	10 10 12.28	+ 8.286	−0.31	+2.40
α ——	8 24 0.69	8 23 14.89	+ 9.002	−0.84	+3.78
α Orionis	7 21 50.69	7 21 45.21	+ 1.267	+0.91	+0.60
α Serpentis	6 59 54.84	7 0 53.37	−11.791	−0.47	+2.34
β Aquilæ	5 57 50.84	5 57 9.29	+ 8.488	+0.84	+3.38
α Canis min.	5 40 40.32	5 41 23.13	− 8.797	−0.82	+4.28
α Ceti	3 22 37.67	3 21 23.46	+14.491	−1.72	+1.59
β Virginis	2 46 42.81	2 48 23.20	−20.289	−1.05	+1.48
α Aquarii	− 1 11 25.48	− 1 12 48.98	+17.195	+2.45	+2.93
α Hydræ	7 53 1.68	7 51 44.39	−15.273	+0.96	+2.27
β Orionis	8 25 4.22	8 25 27.36	+ 4.661	+0.22	+1.86
α Virginis	10 13 7.69	10 11 33.88	−19.027	−1.34	+2.84
1. α Capricor.	13 3 25.59	13 4 20.91	+10.581	−2.47	+4.89
2. α ———	13 5 43.49	13 6 40.64	+10 609	−4.16	+4.65
1. α Libræ	15 14 33.27	15 13 14.64	−15.405	+1.57	+2.54
2. α ——	15 17 15.05	15 15 58.17	−15.374	−0.03	+2.94
α Canis maj.	16 28 37.15	16 28 14.68	− 4.483	+0.10	+2.05
α Scorpii	26 1 23.00	26 0 39.17	− 8.649	+0.52	+3.05
α Pisc. aust.	30 34 26.68	30 36 2.81	+18.836	+0.03	+3.80

	F Oriani. 1811.	G Brinkley. 1813.	H Pond. 1813.	I Pond. 1820.
α Aurigæ	″....	+1″.67	+1″.88	−0″.12
α Cygni	+1.02	+1.08	+2.42	+1.53
α Lyræ	+1.36	+2.05	+2.39	+2.23
α Geminorum	....	+1.21	+2.05	−0.05
β ———	....	+1.92	+1.57	−0.54
β Tauri	+1.69	+1.44	+2.02	+0.60
α Andromedæ	....	+2.43	+3.15	+0.41
α Coronæ	+2.15	+2.60	+2.71	+2.56
α Arietis	....	+2.43	+2.69	+0.68
α Bootis	+1.35	+2.07	+2.45	+1.57
α Tauri	+2.79	+1.96	+2.54	−0.16
β Leonis	....	+2.95	+2.09	+1.96
α Herculis	+2.35	+2.54	+3.18	+2.55
α Pegasi	+2.51	+2.93	+4.13	+1.95
γ ——	....	+2.80	+2.98	+3.78
α Leonis	+2.60	+2.25	+2.61	+2.42
α Ophiuchi	+2.47	+1.88	+3.27	+2.34
γ Aquilæ	....	+2.60	+3.34	+4.03
α ——	+2.51	+2.38	+3.43	+2.31
α Orionis	+2.76	+2.36	+3.60	+1.31
α Serpentis	+2.12	+3.13	+3.24	+2.16
β Aquilæ	....	+3.27	+4.39	+5.16
α Canis min.	+3.04	+3.29	+4.22	+0.68
α Ceti	....	+1.81	+3.15	+4.33
β Virginis	....	....	....	+2.19
α Aquarii	....	+4.04	+4.19	+4.48
α Hydræ	....	+3.85	+3.54	+4.68
β Orionis	+2.78	+2.68	+3 15	+4.22
α Virginis	+3.00	+3.13	+3.16	+4.69
1. α Capricorni	....	+3.47	+4.16	+4.59
2. α ———	+3.68	+5.62	+5.35	+6.49
1. α Libræ	....	....	+6.66	+7.27
2. α ——	....	+4 76	+4.65	+5.05
α Canis maj.	+5.36	+1.59	+5.16	+1.15
α Scorpii	+2.65	+5.57	+5.74	+4.00
α Pisc. aust.	+3.71	....	....	+2.68

XXXII. *On the Theory of Astronomical Instruments.*

By BENJAMIN GOMPERTZ, *Esq. F.R.S.*

PART THE FIRST.

Read January 11, 1822.

ASTRONOMY has been divided into two branches; one called practical astronomy and the other denominated theoretical astronomy; and under the latter head the theory of astronomical instruments should be contemplated: but the connection of the two with each other is so intimate, that a competent knowledge of the one cannot be acquired without an adequate acquaintance with the other. The instruments supply data to the theory; but it is theory which invents the instruments; theory which points out the cause of the defects; theory which directs the practitioner to the requisite improvements. It is theory which teaches the proper use, and extends the originally intended application of the complete instrument: and theory, if I mistake not, will likewise offer the means of obtaining from well-invented, but what may be termed ill-constructed instruments, very accurate results. In this, however, it seems necessary that their construction should not be so ill as to allow of an improper laxity in the motions or warping of their parts: but theory will supply a remedy for bad division, improper direction of motion, (provided it be determinate,) defects of perpendicularities, and of parallelisms, and of innumerable other defects.

If accurate instruments were to be obtained with the same ease as those are acquired which are less perfect, it would be a vain use of the power of knowledge to search through the intricate precepts of theory to attain an object which might be immediately furnished by an accurate instrument: but it is to be regretted, that the expense of good instruments is often far beyond the means of those who could use them to the best advantage for science; and this fact renders the inquiry of the best use to be made of a bad instrument an interesting subject to the philosopher.

Astronomical instruments may be formed either by direct construction or by

inverse construction. By direct construction, I mean a formation after the proper rules of practical mechanics as to the shape, situation, and motion of the different parts and divisions of the limb, &c. In this mode of construction, for the instrument to be good, the greatest accuracy and attention will be required in the workman.

By inverse construction of an instrument, I mean a formation of an instrument sufficiently strong, whose intended motions are perfectly determinate, whose form may not be as accurately the required form as art can approach; but whose parts, which are not intended for motion, are perfectly stationary; and whose limbs, which are to indicate the angles to be taken, are to have the value of the different portions pointed out from observed angles, which may be taken with the instruments by the assistance of theory, indicating the defects of the construction, from observation and properly valuing the influence they may have. If, for instance, with the intention of forming and fixing a transit instrument, we form an instrument of which the axis of motion is not scrupulously fixed perpendicularly to the meridian, nor even parallel to the horizon, nor the line of collimation of the telescope scrupulously perpendicular to the axis of motion, nor the altitude limb perpendicular to the axis of motion, the value of the portions of the altitude-limb may be obtained by inverse construction, and the theory of the instrument laid down such, that, with certain provisos, the instrument with all its defects may answer the purpose both of a well-constructed transit instrument and of a mural quadrant. Though the terms direct and inverse construction are expressions hitherto, I believe, never used, I venture to introduce them, as I believe that a proper distinction of our different modes of operation will often lead to great acquirements.

Astronomical instruments are generally formed chiefly by direct construction; but, for the more accurate adjustment of some of the parts, I believe the indirect construction is always used. The little leisure I can command has only at present permitted me to attempt to offer a partial theory of Hadley's sextant or of other instruments of analogous construction; a subject which I prefer to commence with, in consequence of the peculiar elegance and extensive application of those instruments. And in considering the theory, I have not confined myself to those instruments in their most perfect form and adjustments; but I have had in view to show that, with all probable defects, except weakness of parts, improper difficulty of motion, and certain other defects which it will be necessary to avoid, theory will direct the mode of obtaining from them very accurate results.

Though the intent of Hadley's sextant requires the plane of the horizon reflector and the plane of the index reflector to be perpendicular to a certain plane, in which the whole limb or graduated arc of the instrument is required to be situate; and also requires the line of collimation of the telescope to be parallel to this plane; and requires the limb to be divided into accurately equal parts, zero to be at a particular point, and corresponding to which the situations of the two reflectors are to become parallel to each other; and also stipulates for the performance of various other difficulties: Theory and inverse construction will point out remedies for inaccuracies in all these requisites. In constructing the instrument, if, of the two reflectors to be used, one has its two surfaces nearer to parallelism than the other, it may be preferable to use the best for the index-glass; because, whilst the line of collimation of the telescope is fixed in respect to the horizon-glass, the error in the horizon-glass becomes a constant quantity. Instead of darkening-glasses, I think opaque screens may often be used with great advantage; for we avoid by that means the bad effect of improperly ground darkening-glasses; and at the same time, within certain limits, we can preserve the two images of the same colour and light, if we choose. For these effects I propose two opaque screens to be adapted, one for the horizon-glass or for the direct view, which is to be moveable behind the horizon-glass by means of a lever properly applied; and the other for the index-glass or reflected view, to be moveable in front of the index-glass by means of a lever; and by properly moving the screens, any proportion and diminution of light, within extensive limits, may be obtained at pleasure.

When a telescope is adapted to the instrument which is furnished with these opaque screens, it will not be necessary to diminish the aperture; and it will be far from a defect, if I judge rightly, for the aperture of the telescope to be larger than the horizon-glass, provided it be not an improper aperture for the telescope. In using a great magnifying power to these instruments in terrestrial observations, the phenomenon, which I term the commixture of lights, would become extremely embarrassing, were it not for the use either of opaque screens or darkening-glasses, or at least some contrivance for the same purpose. The use of opaque screens will enable us to perform an interesting experiment on the effect of the commixture of lights; for having supposed the two screens so adjusted, that the view by reflection and the direct view shall be equally bright, we may darken one of the views by its proper screen without intercepting the whole of the rays coming from that view, so that it shall totally disappear; and then, by using the other screen to darken the

other view, the rays of the first, though not greater in quantity than before this last screen was used, will again produce a distinct view; and, with care, we may proceed to a second disappearance and a subsequent re-appearance, and probably we may proceed further. The light of the sky may be brought down by reflection, so as totally to cause a previously distinct view of a terrestrial object to vanish; but the screen will again make it appear. And it may be an amusement, having obtained the proper proportion of light by means of the screens, in observing a distant shut-window, to give the appearance of opening and shutting the window by merely moving the index. These screens may be serviceable in measuring the proportion of light in different objects, and in showing the quantity of day-light necessary to destroy the impression of the light of a star. It is well known that a telescope for a near object must have the eye-glass drawn more out than for a distant object; hence it will happen, in constructing the instrument in an economical mode, if instead of procuring a reflector of a glass ground to a plane, we should have met with a concave or a convex glass for one of our reflectors, as these would make distant objects appear nearer, it follows that, in measuring with our instrument the angles of distant objects, if we adjust the telescope for distinct vision for one of the views, it would give indistinct vision for the other view; if this should be intolerable, a lens may be put before the reflector to correct the effect. If the glass reflector be prismatic, on looking at the sun &c. in it, we may see several images of different brightness: it will be necessary in that case to fix on one for the subject of observation, and the image from the quicksilvered part will be situate, not as if it were reflected by either the anterior or posterior surface, but as if reflected by a surface which, if it be the brightest image which we contemplate, I shall call the first apparent posterior plane of reflection. When a ray comes from a point at a distance, on a glass whose surfaces are ground parallel to each other, an eye situate on the same side of the surface as the radiant point, will receive the images from the reflection of the two surfaces together as one image; but if the radiant point be near the glass, it may receive them as two objects or more, from the internal reflections, as it would from a prismatic glass reflector; and therefore, to discover if the two surfaces are parallel by means of the coincidence of the reflected images, the radiant point should be at a distance, otherwise we might be induced to consider them not parallel, when in reality they are so. And it likewise follows, from what has been said above, that if of the two reflectors one should be concave or convex, though it should be metal, and the other glass, whose surfaces are

truly parallel, that in virtue of the first bringing the distant object near, we may see a repetition of images.

If a represent the principal focal distance of an object-glass of a telescope, and the eye perceives distinctly, from the rays coming from an infinite distance, and the eye-glass be adjusted to see perfectly at an infinite distance, then, if to see a near object distinctly through the telescope the eye-glass be required to be pulled out the small quantity k; then will the distance of that object be $\frac{a^2}{k} + a$ from the object-glass; for if r be this distance, as the distance of the conjugate focus from the object-glass will be $a+k$, we have by the focal theorem $r+a+k : r :: \overline{a+k} : a ; \therefore a+k : k :: r : a ; \therefore r = \frac{a^2}{k} + a$. Hence, if a mirror should happen to be a portion of a sphere of radius b, concave towards the object; and the object-glass of a telescope regarding the reflection of a distant object, should be distant c from the face of the mirror, then, since a distant object reflected by the mirror will appear from the object-glass of the telescope at the distance $c - \frac{1}{2}b$: consequently a and k being as before, we have $c - \frac{1}{2}b = \frac{a^2}{k} + a$; and therefore $b = 2c - \frac{2a^2}{k} - 2a$ for the radius of the mirror. If b comes out negative, the mirror will be convex; if k is equal to o, the mirror will be a plane.

Since the invention of these doubly-reflecting goniometers, the chief deviation of form has been in the portion of the graduated limb; and the instrument has accordingly received the name of the octant, sextant and reflecting circle. Besides which, it may be proper to mention that Atwood had considered a doubly-reflecting goniometer to be applicable to the accurate measure of small angles, by having the two reflectors much inclined to the plane of the graduated limb.—(See his paper in the Philosophical Transactions, in which he has considered the theory of this instrument.) And I observe, that the instrument, in the common construction, inasmuch as having the reflectors perpendicular to the plane of the graduated limb, may, I think, be applied with great advantage as a micrometer; in which case the graduated limb may be very small, say one degree, more or less, according to the purpose intended. If this is to be fixed to any telescope in particular, the construction may be more simple than if meant to be used occasionally as a micrometer to different telescopes. In the former mode, without being very minute in the description of the construction of it, suppose ADBC (Plate V. fig. 1.) a telescope, AD the object-end, and BC the eye-end; AH a strong arm to the telescope-tube

at A; on it fix the horizon-reflector GH; on the opposite side of the axis of the telescope at D, fix another arm. Let the index-glass FK be moveable about a centre somewhere in this arm: let CE be the small graduated limb fixed to the eye-end of the telescope, and let KE be an arm or index, one end moving on CE, with a micrometer-screw, if necessary, the other end being fixed to the back of the index-glass, which it serves to move about a centre. The position of HG must be such, that some one line FG drawn from one glass or reflector to the other, shall make the same angle with the perpendicular to GH, that this last makes with the axis of the telescope, or at least with some line in the telescope, otherwise the light reflected by FK to GH will not be reflected by GH down the telescope. The apparatus should likewise be provided with opaque screens, above described, or with dark glasses. And the longer the telescope is, the longer will be the radius KE of the arc CE, which of course is an advantage. This instrument will serve to measure distances on land, by one station, arising from what I have called in the Second Part, the parallax of the instrument, within degrees of accuracy, depending on the distance of the two reflectors from each other; and, in consequence, it may be convenient to place the reflectors at a greater distance than when the instrument is only intended to measure angles in the heavens. The same figure will show the form which may be used for a micrometer which is to be applied to different telescopes occasionally, if the part ADBC be conceived to represent, instead of the telescope, a tube in which the telescope is to be fixed. This apparatus, with but a small alteration, will serve for another purpose, which I prefer not treating of at present, not being anxious to anticipate the subject of a paper which I have it in contemplation to write.

XXXIII. *On the Theory of Astronomical Instruments.*

By BENJAMIN GOMPERTZ, Esq. F.R.S.

PART THE SECOND, WITH A PLATE.

Read June 14, 1822.

Lemma. IF there be two parallel straight lines, so terminated that a line perpendicular to them, which bisects the one, shall likewise bisect the other, I say the right lines joining the contrary extremes of those lines will be equal to each other, and that the angles made by the said parallels with the lines joining those extremes, are equal to each other.

Fig. 2 and 3. Plate V. Let AB, CD be the two parallel lines, EF the bisecting perpendicular cutting AB, DC in E and F: draw CG, DH parallel to EF, cutting AB in G and H; therefore, because $AE = EB$, $CF = ED$; and therefore $GE = EH$, it follows that $AG = BH$; consequently, in the right-angled triangles ACG, BDH right-angled at G and H, AG being equal to BH; $CG = HD$, AC and BD will be equal to each other, and also the angle A will be equal to the angle B.—Q.E.D.

Corollary. If in a right line, perpendicular to the plane of a plane mirror, there be taken two points equidistant from the mirror, but on contrary sides, then will a right line, joining one of the points with an object, be equal to the right line, joining its reflection by the mirror and the other point; for let EF be the reflector, C and D the points, A the object, B its reflection; therefore $AE = EB$; and therefore EF, which is perpendicular to AB and DC, bisects them; and therefore by the Lemma $AC = DB$.—Q.E.D.

Problem I. Fig. 4. Plate V. The position of the eye, of the horizon-glass, and index-glass of a HADLEY'S sextant, or other goniometer constructed on similar principles, being given; and also the positions of a line passing through the eye and the reflected object in the horizon-glass; and the distance of the real object from the eye being given; it is required to find the angle subtended at the eye by the real object, and the reflection of it by the object-glass; the ray passing to the eye from the said reflection being parallel to the plane which is perpendicular to the two glasses?

Suppose it done DC the index-glass, EC the horizon-glass, H the eye, A the object, B its reflection in the index-glass, R the second reflection, which is in the horizon-glass: draw Hg perpendicular to CD, cutting it in g, produce Hg to e, making $eg = g\mathrm{H}$; also make $e\mathrm{E}$ perpendicular to CKE, cutting it in E; produce $e\mathrm{E}$ to f, making $f\mathrm{E} = e\mathrm{E}$; join f, R; e, B; H, A; R, B; and A, B; and let RB cut CE in M; therefore (corollary to the Lemma*) $f\mathrm{R} = e\mathrm{B} = \mathrm{HA}$, which last is a given magnitude by hypothesis, but f and RH are given in position; therefore R is given in position; therefore R is given; consequently RM is given in position and magnitude; because CE is given in position, and the angle M is given in magnitude; therefore RB its double is given; and consequently B is given; and because CD is given in position, A will in consequence be given; but R is given; consequently the angle RHA is given.—Q.E.D.

Corollary 1. Since $\angle \mathrm{AH}e = \angle \mathrm{HAB} =$ (by the Lemma) $\angle \mathrm{AB}e =$ (if CD cut RB in n) $\angle \mathrm{DB}n + \angle n\mathrm{B}e =$ (by the Lemma) $\angle \mathrm{DB}n + \angle f\mathrm{RB} = \angle \mathrm{DB}n + \angle \mathrm{BRH} + \angle f\mathrm{RH}$; it follows, by adding $\angle e\mathrm{HR}$, that the angle $\mathrm{AHR} = \angle \mathrm{DB}n + \angle \mathrm{BRH} + \angle f\mathrm{RH} + \angle e\mathrm{HR}$; but $\angle \mathrm{DB}n = \angle \mathrm{MC}n$ by reason of the right $\angle\angle$ M and D; and by reason of the right-angles M and g, $\angle \mathrm{BRH} + \angle e\mathrm{HR} = \angle \mathrm{MC}n$; therefore the angle $\mathrm{AHR} = 2\angle \mathrm{MC}n + \angle f\mathrm{RH}$.

Corollary 2. Draw fs perpendicularly to RH, cutting it in S; then it is evident, if the instrument be small in comparison to the distance of the object, that fs will be small in comparison with $f\mathrm{R}$; and therefore that the angle $f\mathrm{R}s$, which may be termed the correction necessary from the parallax of the instrument, will be small; and if the distance may be considered as infinite, the correction will vanish; and then also HR may be considered $=$ AH, as it will differ from it but by a finite quantity.

Corollary 3. If the real image and the image in the horizon-glass appear in one line, since the angle AHR does then vanish, it follows, in that case, that the angle $f\mathrm{R}s$, the correction arising from the parallax of the instrument, will be equal to $-2\angle \mathrm{MCD}$; and consequently, from the given dimensions of the instrument, the distance may be determined.

Corollary 4. L being the centre about which the index-glass turns, draw $\mathrm{LQ} \perp \mathrm{CE}$, cutting it in Q, which produce to P, making $\mathrm{PQ} = \mathrm{QL}$, and join P, f;

* A known law of reflection in plane mirrors being here assumed, namely, that a right line joining a point and its reflection in the mirror, is bisected by the mirror and is perpendicular to it.

and L, e; consequently, since fe is $\perp$ QC, and $fe = e$E (according to the Lemma) Pf = Le; but because $eg = g$H, and Lg perpendicular to eH, Le is = LH a given magnitude; and L and P are given; consequently e and f are in the circumferences of given circles about the respective given points L and P as centres.

Corollary 5. Hence, and because the sine of the angular error fRs arising from the parallax of the instrument is equal to fs divided by fR, considering radius unity; and because fR is equal to AH, the distance of the real object from the eye, a given magnitude; it follows, that as fs is the greatest, when if produced, if necessary, it will pass through the centre P, the error arising from the parallax of the instrument will be the greatest in that case. And if the circle about P cut sH, there will be a case or limit in which the error is constantly positive, and another limit in which it is constantly negative, a particular case of greatest positive error, and a particular case of greatest negative error. And the sum of the sines of these greatest errors, taken independently of sign, will be 2 LH divided by the distance of the real object from the eye. There will likewise be two cases where the error vanishes, namely, when the point f falls in the right line s H; for then fs evidently vanishes. But if the said circle be wholly without the line RsH, the error will either always be positive or always negative, and will admit of one case of *maximum* only and one case of *minimum* only, and the corrections will respectively be to be added or subtracted, accordingly as the circle falls on a different side, or on the same side of the right line RsH; and the difference of the sines of the greatest and least errors will be equal to 2 LH divided by the distance of the real object from the eye. But, lastly, should the circle in question touch the said right line, there will be one case where the error is a *maximum,* and one case where the error vanishes; and the sine of the greatest error will be equal to 2LH divided by the distance of the real object from the eye; and it will be to be added or subtracted as in the last case. Whence, of all instruments, LH remaining the same, those will be subject to the greatest error, arising from the parallax of instrument, whose circle, described with the radius equal to LH about P, does not meet the right line RH; and those whose said circle is cut by RH, are subject to the least *maximum* error arising from parallax.

Corollary 6. Draw LU a perpendicular to RH, cutting it in U; let LP produced cut RH in t, draw fr a parallel to RH, cutting LP in r; and let PV be parallel to RH, cutting fs in V; also let LC cut RH in W. Then since $\angle$ LWH = $\angle$ CWK, it follows that $\angle$ CKW + $\angle$ KCW = $\angle$ WLH + $\angle$ LHW; and there-

fore $\angle$ WLH = $\angle$ CKW + $\angle$ KCW − $\angle$ LHW; and therefore $\angle e$LH = 2 $\angle$ CKW + 2 $\angle$ KCW − 2 $\angle$ LHW. Also $\angle$ QLH being equal to two right angles − $\angle$ LtH − $\angle$ LHW it is = one right angle + $\angle$ CKW − LHW. Hence $\angle e$LQ, or by the Lemma $\angle f$PQ, is equal to one right angle + $\angle$ CKW − $\angle$ LHW − (2 $\angle$ CKW + 2 $\angle$ KCW − 2 $\angle$ LHW) = one right angle − angle CKW − 2 $\angle$ KCW + $\angle$ LHW; but the angle frL is equal to the complement of $\angle$ CKW to a right angle, because it is equal to $\angle$ QtK; therefore the angle rfP, or its equal $\angle f$PV being equal to $\angle fr$L − $\angle f$PQ, is equal to one right $\angle$ − $\angle$ CKW − (one right angle − $\angle$ CKW − 2 $\angle$ KCW + $\angle$ LHW) = 2 $\angle$ KCW − $\angle$ LHW. Whence fV, which is equal to Pf × sine of $\angle f$PV to radius unity, or LH × sine of $\angle f$PV, is equal to LH × sine of (2 KCW − LHW). And PY or Vs being equal to Pt × sine of $\angle$ PtY = (2 LQ − Lt) × sine of PtY = − LH sine of LHW + 2 LQ. sine of PtY. Hence because $\angle$ PtY is the complement of the angle CKW to a right angle, we have fs = − LH. $\{$ sine of LHW + sine of (2 KCW − LHW) $\}$ + 2 LQ. cos of CKW. Hence, No. 1, in a given instrument to find when the error vanishes, we have by making $fs = o$, sine of (2 KCW − LHW) = $2\,\frac{\text{LQ}}{\text{LH}}$. cos of CKW − sine of LHW; and therofore as LQ, LH, $\angle$ CKW are known, 2 KCW is immediately known, which will be the angle marked on the limb of the instrument, and will be the true angular distance AHR, causing the error to vanish. No. 2: If m be the distance of the real object from the eye, the sine of the error of the angle shown on the limb is $-\frac{\text{LH}}{m}\Big\{$ sine of LHW + sine of (2 KCW − LHW)$\Big\}$ + $2\,\frac{\text{LQ}}{m}$ cos of CKW. No. 3, And therefore if by any means the error or parallax is known on a given instrument corresponding to a given angle C, the distance of the object from the eye is had.

Corollary 7. Because sine of $\overline{\text{2 KCW} - \text{LHW}}$ = sine of 2 KCW. cos of LHW − sine of LHW. cos of 2 KCW; and LH sine of LHW = LU, and LH cosine of LHW = UH, it immediately follows from the value of fs in Cor. 6, that fs = 2 LQ. cos of CKW + LU. cos of 2 KCW − UH. sine of 2 KCW − LU; and therefore it appears that if in the right line RH there be taken a point O, such that LU + UO. sine of 2 KCW, be equal to LU. cos of 2 KCW + 2 LQ. cos of CKW; and the place of the eye or H coincide with the point O, fs, and consequently the error arising from the parallax of the instrument will vanish; that is, if UO be equal to $\frac{\text{LU. cos of 2 KCW} - \text{LU} + \text{2 LQ. cos of K}}{\text{sine of 2 KCW}}$, and the eye

be placed at O, there will not be any error arising from the parallax of the instrument; that is to say, $\angle$ AOR will be equal to $2 \angle$KCW. But the place of this point O will vary with the angle C, other things remaining the same.

Corollary 8. But H remaining the place of the eye, draw $H\mu \perp AO$ cutting it in μ, and because the $\angle$ HAO is equal to the difference of the $\angle$ AHR and $\angle$ AOR; that is, the difference of the angle AHR and $2 \angle$KCW; it follows that the angle HAO is equal to the angle fRs, which has been proved to be the value of the same difference, and $H\mu = fs$: and this would also be evident by considering that $H\mu = HO \times$ sine of AOR $= HO \times$ sine of 2 KCW. And with regard to our figure it may be observed that $\angle$ AHR being according to the scheme greater than $2 \angle$ KCW, that is $\angle$ AOR, we have placed the order of the letters R, H, O properly.

Corollary 9. When the real object and the reflection of it in the horizon glass are in one line, if m represent the distance of the real object from U, the distance from eye or AH will be $= m +$ UH (since in this case A is in the line HR); then since by corollary 3 we shall have the error $- 2 \angle$KCD, and by corollary 5 the sine of this error is $= \frac{fs}{AH}$, we have $fs = -$ AH. sine of $2 \angle$KCD, or $(-m +$ UH). sine of $2 \angle$ KCD; but fs by corollary 7 is $= -$ LU $-$UH.sine of $2 \angle$KCD $+$ LU. cos $2 \angle$ KCW $+ 2$ LQ. cos of CKW; it thence immediately follows that m the distance of the real object from the fixed point U is $=$ (LU $-$ LU.cos of $2 \angle$ KCW $- 2$ LQ. cos of KCW) $\div$ sine of $2 \angle$ KCW; and this affords a very easy method when the parallax of the instrument is sufficiently large in proportion of the distance of the object, to find that distance by one observation. And I also observe that if $2 \angle$KCW be small, which will be the case when the object is distant, provided however that 2 LQ .cos of CKW be large in proportion of LU.versed sine of $2 \angle$ KCW; for our theorem we may use an approximation, namely the value $\frac{\text{LQ. cos of CKW}}{-\text{arc KCW to radius unity}}$ for the distance from the fixed point U, where I remark that LQ.cos of CKW is constant for all distances.

Corollary 10. Join AU and draw $U\nu \perp AO$ cutting it in ν, then because the angle subtended at an eye at U by the object A and its reflection R, is the angle AUR; and because the angle AOR $= 2 \angle$ KCW, marked on the limb, the angle UAO the difference of these two is the correction to be affixed to the nominal angle on the limb for the real angle subtended at U, and because $U\nu =$ UO. sine of AOU $=$ UO. sine of $2 \angle$KCW; and sine of UA$\nu = \frac{U\nu}{UA}$, it

follows that the sine of the said correction is = (by corollary 7) (LU . sine of 2 KCW − LU + 2 LQ . cos of K) ÷ AU.

Corollary 11. Draw WQ′⊥CK cutting it in Q′, then since, after the index glass CLD is fixed in any particular position, it cannot signify about what point it may have moved to arrive at that position, fs will not be altered by supposing W to have been the centre of motion instead of L; that is, fs will not be altered by supposing L to coincide with W; in which case ∠LHW will vanish and Q will coincide with Q′; and therefore by cor. 6 we shall have fs = 2 WQ′. cos of CKW − WH. sine of 2 C = 2 WK. sine of CKW . cos of CKW − WH . sine of 2 C = WK . sine of 2 CKW − WH . sine of 2 C.

Problem 2. If a goniometer be formed after the manner of a HADLEY's sextant, except that neither of the reflectors is of necessity perpendicular to the plane of the divided limb, which I shall call the plane of the instrument, nor the visual ray or line of collimation of the telescope parallel to the plane of the instrument; then if there be given the inclination of each reflector to the plane of the instrument, the inclination of the visual ray or line of collimation of the telescope both to the horizon-glass, and to the plane of the instrument, and likewise the angle intercepted by the intersections of the reflectors with the plane of the instrument. It is required to determine the angle made by the visual ray with the plane which is perpendicular to the two reflectors; and the angle made by the intersections of the said reflectors with the aforesaid plane perpendicular to them, or, which is the same thing, the inclination of the two reflectors with each other?

Suppose it done about any point C [Fig. 5. Pl. V.] given at pleasure, and any radius: let a sphere be described; draw the great circle ABE parallel to the plane of the instrument, the great circle GKL parallel to the horizon-glass or reflector, FDL a great circle parallel to the index-glass or reflector, cutting GKL in L. Let CM be parallel to the visual ray or line of collimation, through which draw the great circle KMH perpendicularly to GBL, cutting it in K, and draw the great circle AGFE perpendicularly both to GBL and LDF, cutting ABDE in A and E, GBL in G, FDL in F, and KMH in H. Let the great circle M$\tau\varrho$ be perpendicular to the plane of the instrument, and therefore to ABE cutting this in τ. And let MN be a great circle perpendicular to AGFE cutting it in N; then, since all lines and planes parallel to lines and planes which make given angles with each other, make with themselves angles respectively equal to the given angles of the original lines and planes to which they are drawn parallel, we have the inclination of

the horizon-glass to the plane of the instrument equal to the angle ABG, the angle of the index-glass to the plane of the instrument equal to the angle FDE, the angle made by the intersections of the two reflectors with the plane of the instrument, (which in HADLEY's quadrant is half the nominal angle of the limb,) measured by the arc BD; the inclination of the visual ray to the horizon-glass measured by the arc KM, and the inclination of the visual ray to the plane of the instrument measured by the arc $M\tau$: all of which angles are given. Moreover, the arc GF, which is to be found, is the measure of the angles of the intersections of the two reflectors with the plane, which is perpendicular to them, and is measured by the angle L; for L will be the pole of GF; and the arc MN measures the angle which the visual ray or line of collimation makes with the plane perpendicular to the two reflectors, which is also required. And because the great circles GFH, KMH are perpendicular to the great circle GKB, therefore H is the pole of GKB. Now in the great circle spherical quadrilateral $MKB\tau$ right-angled at K and τ, having MK, $M\tau$, and the angle $KB\tau$, the supplement of ABG, KB will be given; for we shall have sine of $KB\tau$: tan of MK : : $\frac{\text{sine of } M\tau}{\text{sine of MK}}$ + cos of $KB\tau$: sine of KB*; and in the great circle spherical triangle BLD we have the angles B and D, and BD, to find both L one of the requisites, and BL; ∴ we have KL, and therefore KG its complement to a right ∠, GL being a quadrant, as L is the pole of GF; and because H is the pole of GKB, GK is the measure of the angle NHM. Hence we have given MH the complement of KM, H the complement of KL, and the ∠N of the spherical triangle MNH a right angle to find MN, the inclination of the visual ray to the plane which is perpendicular to the two reflectors.

* In the great circle spherical quadrilateral (fig. 5.) $KM\tau B$, if K and τ are right ∠ ∠, I say sine of $KB\tau$: tan of MK : : $\frac{\text{sine of } M\tau}{\text{sine of MK}}$ + cos of $KB\tau$: sine of KB. For let the great circle MB be drawn, then because sine of MB$=\frac{\text{sine of MK}}{\text{sine of KBM}}$ and also $=\frac{\text{sine of } M\tau}{\text{sine of } MB\tau}$, it follows that sine of MK : sine of $M\tau$: : sine of KBM : sine of ($KB\tau-$KBM) or sine of $KB\tau$. cos of KBM $-$ cos of $KB\tau$. sine of KBM; that is, sine of MK : sine of $M\tau$: : 1 : sine $KB\tau$. cot of KBM $-$ cos of $KB\tau$; but cot of KBM $=$ cot of MK. sine of KB; therefore sine of MK : sine of $M\tau$: : 1 : sine of $KB\tau$. cot of MK. sine of KB $-$ cos of $KB\tau$; therefore sine of KB $=$ $\frac{\frac{\text{sine of } M\tau}{\text{sine of MK}} + \text{cos of } KB\tau}{\text{cot of MK. sine of } KB\tau}$.

Corollary 1. Hence we see that when the planes of the reflectors are not perpendicular to the plane of the instrument, the nominal angle indicated by the index, which is double the angle of the limb, is not double the angle made by the two reflectors with each other; and we have the $\angle$ BLD made by the two reflectors with each other, from the property cos of BLD $=$ sine of LBD. sine of LDB. cos of DB$-$cos of LBD. cos of LDB, considering radius unity; and it is double this angle, which should be the angle named on the limb.

Corollary 2. If the index-glass be perpendicular to the plane of the instrument, then we have cos of BLD $=$ sine of LBD $\times$ cos of DB. And if the horizon-glass be perpendicular to the plane of the instrument, we have cos of BLD $=$ sine of LDB. cos of DB.

Corollary 3. If $a=$sine of LDB. sine of LBD, or its equal $\frac{1}{2}.\left\{\text{cos of (LBD} - \text{LDB)} - \text{cos of (LBD} + \text{LDB)}\right\}$; and $b=$ cos of LBD $\times$ cos of LDB, or its equal $\frac{1}{2}.\left\{\text{cos of (LBD}-\text{LDB)} + \text{cos of (LBD} + \text{LDB)}\right\}$, $z=$ BD the real angle measured on the instrument; that is, not its double, which is the nominal angle, $z+w$ the angle made by the reflectors with each other, we shall have cos of z. cos of w $-$ sine of z. sine of $w = a$ cos of $z - b$. Put sine of $w=e$; $\because$ cos of $w=\sqrt{1-e^2} = 1-\frac{e^2}{2}-\frac{e^4}{2.4}$&c., and our equation will become e. sine of $z + (\frac{e^2}{2}+\frac{e^4}{2.4}$&c.). cos of $z=\overline{1-a}$. cos of $z+b$; hence it is evident if $1-a$ and b be both small, which is the case when the reflectors are nearly perpendicular to the plane of the instrument, that e will be small, and if z be not so near o that $\frac{(1-a).\text{ cos of } z+b}{\text{sine of } z}$ cannot be considered sufficiently small, then will e or w itself be very nearly $=\frac{(1-a).\text{ cos of } z+b}{\text{sine of } z}$. The criterion of the justice of the assumption is the value of $\frac{e^2}{2}$ cos of z; for if this be much smaller than e sine of z with that assumed value of e, then is e approximated by it. If e sine of z and $\frac{e^2}{2}$ cos of z are of the same degree of smallness, which may be easily judged from the equation, then for an approximation we may still neglect e^4 &c., and our equation for determining e will be e sine of z $+\frac{e^2}{2}$ cos of $z = (1-a)$ cos of $z+b$. But lastly, if e sine of z be of an insignificant value with regard to $\frac{e^2}{2}$ cos of z, then we shall have for e, the approxi-

mation $\sqrt{\frac{2.\overline{1-a}.\text{ cos of }z+2b}{\text{cos of }z}}$, and in all these cases for e we may take w, as in consequence of w being small it will not differ much from it.

Corollary 4. If when the reflectors are nearly $\perp$ plane of the instrument $z=o$, $e=\sqrt{2.\overline{1-a+b}}$ nearly; and when $z=90°$ then $e=b$ accurately, because in this case the part neglected multiplying the powers of e^3 &c. vanishes. The accurate value of e when $z=o$ is immediately obtained by making $z=o$, in the equation cos of $\overline{z+w}=a.$ cos of $z-b$, as we shall thence have cos of w or $\sqrt{1-e^2}=a-b$; if in this for a and b we put their values in Cor. 3 we have cos of $w=-$cos of (LBD+LDB); and $w=180-$LBD$-$LDB when $z=o$, and this may be immediately discovered from the contemplation of the figure. If $z=180°$ we have cos of $w=a+b=$cos of (LBD$-$LDB), and in that case w = the difference between LBD and LDB.

Corollary 5. As sine of KB $=\frac{\text{tan of MK}}{\text{sine of KB}\tau}\times(\frac{\text{sine of M}\tau}{\text{sine of MK}}+\text{cos of KB}\tau)$. If the line of collimation of the telescope be nearly parallel to the plane of the instrument, and the reflectors are nearly perpendicular to the plane of the instrument, or, which is the same thing, if Mτ be small, the angles B and D be nearly right angles, KB will be small, unless cos of MK be small, that is, unless the line of collimation of the telescope be nearly perpendicular to the horizon-reflector; consequently supposing cos of MK not small, Mτ small, and B and D nearly right $\angle\angle$, we may write KB, for sine of KB, in the above value; and it is, by the bye, a constant quantity, whilst the position of the horizon-reflector and line of collimation of the telescope remains fixed with respect to the plane of the instrument; moreover as cot of BL is generally $=\frac{\text{sine of B. cot of LDB}}{\text{sine of BD}}+$ cos of B. cot of BD, we shall have, in case B and D are nearly right angles, and sine of BD not very small, nearly $90°-\text{BL}=\frac{90°-\text{BDL}}{\text{sine of BD}}+(90°-\text{LBD})$ .cot of BD; consequently supposing also that the line of collimation of the telescope is nearly parallel to the plane of the instrument, but not nearly perpendicular to the horizon-reflector, we shall have nearly $\text{KL}=90°+\text{KB}-\frac{90°-\text{BDL}}{\text{sine of BD}}-(90°-\text{LBD})$. cot of BD; and therefore because sine of MN (or MN itself nearly) is equal to sine of MH. sine of MHN = sine of MH .cos of KL, it will be nearly $=\left\{\frac{90°-\text{BDL}}{\text{sine of BD}}+(90-\text{LBD}).\text{ cot of BD}-\text{KB}\right\}$ $\times$ cos of MK, that is, this will be nearly the inclination of the line of collima-

tion to the plane perpendicular to the two reflectors: these things however are only true when sine of BD is great in proportion of $90°-$ BDL and $90°-$ LBD; if, on the contrary, it should be extremely small in comparison of those quantities, the cot of BL would be very great, or BL very small; and if BD should be nothing, BL would likewise vanish or be equal to $180°$, which would also be the case if BD were $= 180°$. And in these cases the sine of the inclination of the line of collimation of the telescope to the plane perpendicular to the two reflectors $= \pm$ cos of MK $\times$ cos of KB, upper or under sign to be taken according as LB is taken o, or $180°$.

Corollary 6. In the spherical triangle AGB right-angled at G we have $\angle$B and BG, which is the complement of BL, to find AB or the $\angle$ACB; and $\because$ ACD is also known. We have also AG or $\angle$ACG; and in the spherical triangle MNH right-angled at N having MH and MN, we have NH, and consequently we have its complement GN or $\angle$GCN. This corollary is added for the requisite of Problem 4.

Problem 3. In the goniometer described in Problem 2, having the data of that Problem, but the distance of the real object from the eye infinite, it is required to determine the angle, the true object, and the reflection of it in the horizon-reflector, makes at the eye?

Fig. 6. Find by Problem 2 the inclination of the visual ray with the plane which is perpendicular to the two reflectors; and also the angle made by the two intersections with each other, which is made by the reflectors with the said plane perpendicular to them; and conceiving this plane to pass through the object, it will also pass through the reflection in the horizon-glass. Then if a point be taken in this plane any where, not at an infinite distance from the intersections of the said planes with the plane of the reflectors, it follows from Prob. I. Cor. 2. that the angle subtended at that point by the real and reflected object is equal to twice the angle made by the two intersections; and that the said point is equally distant from the real and reflected object, and it will therefore be given. Let H be the eye, KC, LC the intersections of the horizon and index reflectors respectively, with the plane passing through the real object A, and perpendicular to the said reflectors. Let R in the said plane AKCL be the object in the horizon-glass; draw HO perpendicular to the plane AKCL, cutting it in O; join A,H; R,H; A,O; and R,O; then, from what we have said, we have $\angle$KCL given, and the angle AOR$=2\angle$KCL, also AO$=$OR, and consequently AH$=$HR; and the $\angle$HRO is given, it being the angle made by the line of collimation or visual ray with the plane perpendicular to the two

reflectors, and therefore the ratio of RH to RO is given; but in the isosceles triangle AHR, having the vertical ∠AHR, we have the ratio of AH or HR to AR; but we have also the ratio of RH to RO or AO, we have consequently the ratio of AO or OR to AR; and consequently the vertical angle AOR of the isosceles triangle of AOR the required angle is had.—Q.E.I.

Corollary 1. Because AR=2 RO. sine of $\frac{1}{2}$∠AOR, and also equal to 2HR. sine of $\frac{1}{2}$∠AHR, it follows that sine of $\frac{1}{2}$∠AHR= $\frac{\text{RO}}{\text{HR}}$ sine of $\frac{1}{2}$∠AOR =sine of ($\frac{1}{2}$∠AOR)×cos of ORH.

Corollary 2. If ∠AHR= ∠AOR− w, we shall have sine of $\frac{1}{2}$ AOR. cos of $\frac{1}{2}$ w−cos of $\frac{1}{2}$ AOR×sine of $\frac{1}{2}$ w=sine of $\frac{1}{2}$∠AOR×cos of ORH; take each side from sine of $\frac{1}{2}$∠AOR, and we have sine of $\frac{1}{2}$ AOR.versed sine of $\frac{1}{2}$ w×cos of $\frac{1}{2}$ AOR. sine of $\frac{1}{2}$ w=sine of $\frac{1}{2}$∠AOR×versed sine of ORH; ∴ if ORH is very small $\frac{1}{2}$ w=tan of $\frac{1}{2}$∠AOR×versed sine of ORH nearly, or $w=\frac{1}{2}\cdot\overline{\text{ORH}}^2$×tangent of $\frac{1}{2}$∠AOR nearly, if ORH here stands for the arc to radius one corresponding to ∠ORH; this, however, will cease to be an approximation when cot of $\frac{1}{2}$ AOR is not large in comparison of versed sine of $\frac{1}{2}$ w. If it were small in comparison of versed sine of $\frac{1}{2}$ w, we should have nearly versed sine of $\frac{1}{2}$ w=versed sine of ORH, or w=2 ORH nearly; and this would be its exact value when ∠AOR=180°. And 2 ORH is evidently the greatest value w will admit of; for otherwise cos of $\frac{1}{2}$ AOR.sine of $\frac{1}{2}$ w, consisting of a product of two terms, of which both are assumed affirmative, would be negative, which is absurd. In which remark I observe, that I consider the ∠AOR on that side which is not greater than two right-angles, and in which case AHR is less than AOR, and consequently w positive.

Problem 4. In the goniometer of Problem 2, given the inclination of the horizon and index-glasses to the plane of the instrument, their intersections with the plane of the instrument, and from which plane the distance of the eye is given, and also the point where a perpendicular from the eye to it, cuts it, being given; the inclination of the line of collimation of the telescope with the horizon-glass; and also with the plane of the instrument; and the distance of the eye from the object all given, to find the angle subtended at the eye by the real and reflected object?

Fig. 7. Suppose it done; let A be the real object, R the object in the horizon glass, and AR will be in the plane, which is perpendicular to the two reflectors, and passing through the object. Let O be the eye; from O draw OH perpendicularly to the plane AQR, which is perpendicular to the two re-

flectors, cutting it in H; let GNMZ be the horizon reflector, cutting the plane ARQ in GZ; GNLY the index reflector, cutting the plane ARQ in GY; also through O draw the planes CMLO, NMLO; the first parallel to the plane ARHQ, the second parallel to the plane of the instrument; the horizon glass cutting these planes in CM, NM respectively; and the index glass cutting them in CL, NL. Join AH, RH; draw MZ, LY, parallel to HO, cutting GZ, GY in Z and G. Let GZ produced cut RH in K, and GY produced cut RH in W; and by problem 2 we have the $\angle$ORH, made by the line of collimation, with the plane perpendicular to the two reflectors, and also the $\angle$ZGW, or $\angle$MCL; and from cor. 6, prob. 2, we have the $\angle$NML, which is equal to the angle made by the line MN, which is in the section of the horizon reflector with the plane of the instrument, with the line ML, which is in the section of the plane, perpendicular to the two reflectors, and the plane of the instrument (that is, $\angle$BCA of fig. 5); but OM is equal to the distance of the point, where the perpendicular to the plane of the instrument, passing through the eye, cuts the said plane, from the section of the horizon glass and that plane; when after being diminished by the rectangle of the distance of the eye from the plane of the instrument, with the cot of the angle made by the horizon glass with the plane of the instrument, it is divided by the sine of OMN*, consequently MO is given, and OL is found by a similar operation with respect to the index glass; and MLO being the common section of CML and NML, is a right line; and therefore ZYH is a right line. Moreover $\angle$ GKH is the angle made by the line GK, which is the section of the horizon-glass, and the plane perpendicular to the reflectors, with the line which is the

* For NMO being as before, let OM, fig. 8, be $\perp$MN, and cut it in M'; let N'M''O' be the plane of the instrument, cutting the horizon-glass in N'M'', and cutting the right line OO', which is perpendicular to O'M''N' in O'; also let M'M'' be the section of NM'M''N' and M'OO', and join O'M''; then is OO' the distance of the eye from the plane of the instrument; but the planes NM'O, N'M''O', parallel to each other, being cut by a third plane, their sections OM', O'M'', with the said plane are parallel to each other; and, for a similar reason, is NM' parallel to N'M''; but OM' is $\perp$NM; $\therefore$ O'M'' is $\perp$N'M'', and therefore O'M'' is the given distance from the point of intersection of the line drawn from the eye $\perp$ the plane of the instrument to the intersection of the horizon-glass, and the plane of the instrument; draw Os $\parallel$ M'M'', cutting OM'', in s, then will OsO' be the inclination of the horizon-glass with the plane of the instrument, and by reason of the parallels, M'O=M''E; $\therefore$ M'O=M''O'−OO'. cot of OsO', and $\text{OM} = \dfrac{\text{OM}'}{\text{sine of OMN.}}$

section of the last-mentioned plane, and the plane perpendicular to it, passing through the visual ray (in the figure of problem 2, this is represented by the angle GCN). And $\angle$GZY, or CML, is what is represented in figure to problem 2, by ACG. Therefore, because OL=YH, and ZH=MO, we have in the triangle KZH, given ZH, $\angle$KZH and $\angle$GKH, we therefore have KH and ZK; and because we have ZY, $\angle$KGY, by problem 2, and the angle GZY we have GZ, and therefore we have GK; and consequently having GK and the angles of the triangle KGW we have KW. Draw H*ge* $\perp$ GWg, making $eg=gh$, draw e F$f\perp$GZE, making Ef=Ee, and join fR; therefore since A is an object, which after being reflected in the index-glass GY, is reflected from thence to the virtual focus R by the horizon-glass GK, by problem 1, Rf is equal to AH, and the angle AHR=2$\angle$KGW+ $\angle f$RH. Draw $fs\perp$RH, cutting it in s, and as f is given with respect to H, and the line HR, and OA and the angle ORH are given, put AO=a, HS=b, $sf=c$, and cotangent of $\angle$ ORH=n, also HO=x; $\therefore$ RH=nx, AH or R$f=\sqrt{a^2-x^2}$; $\therefore$ RS= $\sqrt{a^2-c^2-x^2}$; $\therefore$ RH=$\sqrt{a^2-c^2-x^2}+b$=(from above)nx; $\therefore$ $a^2-c^2-x^2=n^2x^2-2nbx+b^2$; $\therefore$ $x^2-\frac{2nb}{1+n^2}\cdot x=\frac{a^2-c^2-b^2}{1+n^2}$; and $x=\frac{nb+\sqrt{\overline{a^2-c^2-b^2}.\overline{1+n^2}+n^2b^2}}{1+n^2}$; therefore RS=$(nx-b=)\frac{n\sqrt{\overline{a^2-c^2-b^2}.\overline{1+n^2}+n^2b^2}-b}{n^2+1}=\frac{\sqrt{\overline{a^2-c^2-b^2}.\overline{\frac{1}{n^2}+1}+b^2}-\frac{1}{n^2}b}{1+\frac{1}{n^2}}$ and the tangent of the angle fRS $=\frac{(1+\frac{1}{n^2})c.}{\sqrt{(a^2-c^2-b^2)\overline{\frac{1}{n^2}+1}+b^2}-\frac{1}{n^2}b}$. And therefore $\angle f$RS; and consequently $\angle$AHR is had; and because AH and RH are now known, AR is had; but AO and OR is had, therefore the required $\angle$OR is known.—Q.E.I.

Problem 5. To explain the passage of a ray of light falling on the anterior surface of a prismatic glass reflector, and refracted in its passage to the posterior surface, thence reflected to the anterior surface, and from thence refracted into the air: the anterior and posterior surfaces, and the incident ray being all given in position.

Fig. 9. Let DC be the incident ray falling on the anterior surface of the prism at C, about which conceive a sphere to be described with a given radius; let the great circle ARBH of the sphere be the section of the anterior surface of the prism and sphere; RXWHY the section of the sphere, with

the plane parallel to the posterior surface of the prism, passing through C; ADKPBW a great circle, perpendicular to the plane ARBH of the anterior surface of the prism, passing through the incident ray DC; and it will pass through the first refraction, which let be in the direction KC: let π be the pole of RXWHY; and suppose P the pole of ARBH; in the circle YKπX take $\pi E = \pi K$, then will the ray KC falling on the posterior reflecting surface of the prism, which is parallel to RXWH, be reflected in the direction CE, in which direction it will fall in the inside of the prism on the anterior surface ARBH, through the pole of which draw the great circle PE, and in it take I, so that sine of PI is to the sine of PE as the sine of PD is to the sine of PK; that is, in the ratio of the sine of incidence to the sine of refraction, from the air into the prism, then will PI be parallel to the returning ray. Draw the great circle Pπx, cutting ARB in x, draw PR and πR; then as P is the pole of AR, PR is a quadrant; and because π is the pole of RW, πR is a quadrant; $\therefore$ because PR and πR are quadrants, R is the pole of Pπx; $\therefore$ Rx is a quadrant; $\therefore$ $\angle$APπ, which is equal to the $\angle$ACx, exceeds ACR by a right $\angle$, and is therefore given, and Pπ measuring the inclination of the anterior and posterior surfaces ARB, RXW is also given; and KP is given; hence in the spherical triangle KPE, Pπ bisecting KE there is given KP, $\angle$KPπ and Pπ to find PE, and the angle KPE. For this purpose, in the great circle Pπ continued, make $\pi\varrho = P\pi$, and draw the great circle Eϱ; then because $\pi\varrho = \pi P$, $\pi E = K\pi$, and the angle Pπ K$= \angle\varrho\pi$E, $\therefore$ in the spherical triangles PπK, $\varrho\pi$E, we have $\varrho E = PK$, and the $\angle\varrho = \angle KP\pi$; $\therefore$ in the spherical triangle ϱPE we have given Pϱ, which is equal to twice the inclination of the surfaces of the prism to each other, side ϱE, which is equal to PK, and the $\angle$PϱE, which is equal KPϱ, which itself has been proved to be equal to $\angle$ACR, increased by a right $\angle$, to find PE and $\angle$EPϱ. Hence putting the sine of incidence to the sine of refraction from the air into the medium of the prism, as $1:n$; $AD=p$, $AK=q$, $\angle ACR = a$, b, the arc which measures the inclination of the surfaces, we have cos of $q = n$ cos of p; ϱE $(=PK)$ the complement of q to a quadrant; $P\varrho = 2\,b$, $\therefore$ because cos of PE$=$sine of Pϱ. sine of ϱE. cos of PϱE$+$ cos of Pϱ. cos of ϱE, we have cos of PE$= -$ sine of $2b$. cos of q. sine of $a +$ cos of $2b$. cos of q. Hence PI is had, and the angle KPE will be had; and since cot of $\varrho PE = \frac{\text{sine of } P\varrho}{\text{sine of } \varrho}$. cot of ϱE $-$ cos of Pϱ. cot of $\varrho = \frac{\text{sine of } 2\,b.\ \text{tan of } q}{\text{cos of } a}$. cos of $2\,b$. tan of a; and cotan of KP$\pi = -$ tan of a; $\therefore$ tan of KPE $=$

$$\frac{\text{cos of } KP\pi + \text{cot of } \varrho PE}{\text{cot of } KP\pi.\ \text{cot of } \varrho PE - 1} = \left(-\text{tan of } a + \frac{\text{sine of } 2\,b.\ \text{tan of } q}{\text{cos of } a} + \text{cos of } 2\,b.\ \text{tan of } a\right)$$

divided by $-\frac{\text{sine of } 2b.\ \tan \text{ of } q.\ \text{sine of } a}{\cos \text{ of } a^2} - \frac{\cos \text{ of } 2b.\ \text{sine of } a^2}{\overline{\cos \text{ of } a}|^2} - 1 =$ (cos of a. sine of $2b$. tan of $q - \frac{1}{2}$ versed sine of $2b$. sine of $2a$) divided by $1+$(sine of $2b$. tan of $q-$ versed sine of $2b$) . sine of a.

Corollary 1. If b be small, tan of KPE, or KPE itself, is nearly$=-2b$. tan of q. cos of a.

Corollary 2. If for the sake of brevity we put sine of $2b=e$, sine of $a=g$, we have sine of PE$=\sqrt{1-\text{sine of } q^2.\ \overline{1-e}+2e\sqrt{1-e^2}.\ g.\ \text{sine of } q.\ \cos \text{ of } q - e^2g^2.\ \cos \text{ of } q^2}$ $=\sqrt{(1-e^2g^2-e^2).\ \overline{\cos \text{ of } q}|^2 + e^2+2e^2.\sqrt{1-e^2}.\ g.\ \text{sine of } q.\ \cos \text{ of } q}$; sine of PI$=$ $\sqrt{\frac{1-e^2g^2-e^2.\ \overline{\cos \text{ of } p^2} + e^2+2e\sqrt{1-e^2}\ .\ g.\ n.\ \text{sine of } q.\ \cos \text{ of } p}{n^2}}$; put it equal to cos of $(p-w)$, that is $=$ cos of p. cos of w $+$ sine of p. sine of w; therefore $(1-e^2g^2-e^2 - \overline{\cos \text{ of } w}|^2).(\cos \text{ of } p)^2 =$ sine of p. cos of p. sine of $2w + \overline{\text{sine of } p}|^2 \times$ (sine of $w)^2 - \frac{e^2+2e\sqrt{1-e^2}.\ g.\ n.\ \text{sine of } q.\ \cos \text{ of } p}{n^2}$: therefore when e or $\angle$ of the prism is small, $w=\frac{2bg.\ \text{sine of } q}{n.\ \text{sine of } p}$ nearly; unless sine of p is small, of the same degree as e, or smaller. If sine of $p=o$, we shall have cos of $p=1$, cos of $q=n$, and we should have $1-e^2g^2-e^2-\cos \text{ of } w^2=-\frac{e^2+2e.\ \sqrt{1-e^2}.\ gn.\ \sqrt{1-n^2}}{n^2}$ and sine of $w=$nearly$\sqrt{\frac{-2eg.\ \sqrt{1-n^2}}{n}}$

Corollary 3. Drawing the great circle DI, let the arc DI be bisected in Q, and if about Q as a pole the plane of a great circle be conceived to be formed as a simple reflector, then will incident rays parallel to DC, falling on this reflector, pass in the direction parallel to CI, that is parallel to the same line, as they do when at their incidence they are acted on by the compound prismatic reflector in question: and the said plane, or imaginary reflector, may be called the *first apparent posterior plane of reflection:* and if PQ be joined, the arc PQ will measure its inclination to the plane AHB; and the angle QPB will represent the complement to a right angle of the angle which AC makes with section of the said apparent posterior plane of reflection, and the plane ARB; for let CZ be the said intersection, cutting the circumference AHB in Z, and let the great circle PQ cut AHB in V, and draw the great circles QZ, PZ; then as Q is the pole of the said apparent posterior plane, of which Z is a point in the circumference, QZ is a quadrant, and because P is the pole of AZBH ∴ PZ is a quadrant, and therefore Z is the pole of the great circle PQV; ∴ ZV is a quadrant, and AZ is ∴ the complement of BV, or of the $\angle$ BPQ.

And since DQ=QI, we have sine of DP. sine of DPQ=(sine of DQ. sine of Q=sine of IQ. sine of Q=) sine of PI. sine of QPI=sine of PI×(sine of DPI. cos of DPQ − cos of DPI. sine of DPQ); whence cot of DPQ = $\frac{\text{sine of DP}}{\text{sine of PI. sine of DPI}}$ + cot of DPI; and sine of PQ = $\frac{\text{sine of }\frac{1}{2}\text{ DI. sine of D}}{\text{sine of DPQ}} = \frac{\text{sine of }\frac{1}{2}\text{ DI. sine of D}}{\text{sine of DPQ}} = \frac{\text{sine of }\frac{1}{2}\text{ DI. sine of IP. sine of IPB}}{\text{sine of DPQ. sine of DI.}}$

Corollary 4. Adopting the notation of corollary 2, we have DP=PI−w, and consequently from cor. 3, cotangent of ∠ DPQ = $\frac{\text{cos of } w + \text{cos of DPI}}{\text{sine of DPI}}$ − $\frac{\text{cot of PI}}{\text{sine of DPI}}$. sine of w; and if the angle of the prism or b be small, we shall have by cor. 1, taking small arcs for their sines, ∠IPB measured by its arc to radius one, = $2\,b$. tan of q . cos of a nearly; and from cor. 2, $w = \frac{2\,bg.\text{ sine of } q}{n.\text{ sine of } p}$ nearly; we therefore have, by neglecting all powers of b but the first, cot of ∠DPQ = − $\frac{\text{cot of PI}}{\text{sine of DPI}}$. sine of w = −cot of PI. $\frac{2\,bg.\text{ sine of } q}{n.\text{ sine of } p}$ divided by $2\,b$. tan of q. cos of a, all powers of b but the first having been neglected, and this is evidently = − $\frac{\text{tan of } a.\text{ cos of } q}{n.\text{ sine of } p}$. cot of PI = − $\frac{\text{tan of } a.}{\text{tan of } p}$. cot of PI = − $\frac{\text{tan of ACR}}{\text{cotan of PD}}$. cot of PI; or writing PD instead of PI its near value, we have cot of ∠DPQ = −tan of ACR nearly; that is, R and Z will coincide nearly. And we are taught that if we wish to have the angle DPQ more accurately, we must take in quantities beyond those which are small of the first order in the corollaries 1 and 2.

Corollary 5. The angle of the prism being small, we have the angle PDI small, and ∴ DP nearly=DQ, and PI nearly equal to QI; and as DQ is= QI, we therefore have nearly DP=½DI=PI; whence from corollary 3, sine of PQ, or PQ itself, is nearly equal to $\frac{\overline{\text{sine of DP}}\rceil^2.\text{ sine of EPB}}{\text{sine of DPQ. 2. sine of DP. cos of DP}} = \frac{\text{tan of DP. sine of EPB}}{2\text{ sine of DPQ}}$ = cor. 3, nearly $\frac{\text{tan of DP. sine of EPB}}{2\text{ cos of } a}$ = by cor. 1, nearly $\frac{\text{tan of DP}}{2\text{ cos of } a}$. $2\,b$. tan of q . cos of a = b . cot of p . tan of q = $\frac{b.\text{ cos of } p.\text{ sine of } q}{\text{sine of } p.\text{ cos of } q} = \frac{b.\text{ sine of } q}{n.\text{ sine of } p}$, that is, this is nearly the value of PQ when the ∠ of the prism is small.

Corollary 6. Let MN be the plane of the instrument, cutting the anterior

surface of the reflector in M, and the apparent posterior plane of reflection ZN in N; then considering the angle MCR given, which is the angle measured on the anterior surface of the prismatic reflector by the section of the instrument with it, and the common section of the two surfaces of the reflector: and the angle ZCR being determinable from proper data from cor. 3, MZ will be given; and the $\angle$ MZN, also determinable from proper data from cor. 3, being considered given; and the angle M, the inclination of the anterior surface of the prism, with the plane of the instrument, being supposed given; in the spherical triangle MZN, having the $\angle$ Z, $\angle$ M, and MZ, we shall have MN and $\angle$ N; that is, the angular distance of the line where the apparent posterior plane of projection cuts the plane of the instrument from the line where the anterior surface of the reflector cuts the plane of the instrument, and the angle the said posterior plane of reflection makes with the plane of the instrument. And if the $\angle$ of the two surfaces of the reflector be small; that is, if the two surfaces of the reflector be nearly parallel, then from cor. 5, provided p be not extremely small, the angle $MZN = \frac{b.\ \text{sine of } q}{n.\ \text{sine of } p}$ nearly: and considering M nearly a right angle, we have $MN = \text{sine of } MZ \cdot \frac{b.\ \text{sine of } q}{n.\ \text{sine of } p}$ nearly $=$ from cor. 4, sine of $MR \times \frac{b.\ \text{sine of } q}{n.\ \text{sine of } p}$ nearly; in consequence of the near coincidence of R and Z. Also cos of MNZ=sine of NMZ. sine of MZN $\times$ cos of MZ $-$ cos of M $\times$ cos of MZN generally, will, if $NMZ = 90^\circ - \epsilon$, and ϵ and the $\angle$ MZN be both small, give $90^\circ - MNZ =$ nearly MZN. cos of $MZ - \epsilon$, or $MNZ = 90^\circ - MZN$. cos of $MZ + \epsilon$; or putting $\frac{b.\ \text{sine of } q}{n.\ \text{sine of } p}$ for MZN, we have the angle MNZ; or from cor. 4, in consequence of the near coincidence of Z and R, the angle MNR, which the apparent posterior plane makes with the plane of the instrument $= 90^\circ - \frac{b.\ \text{sine of } q}{n.\ \text{sine of } p} \cdot \text{cos of } MZ + \epsilon = 90 - \frac{b.\ \text{sine of } q}{n.\ \text{sine of } p} \cdot \text{cos of } MR + \epsilon$ nearly. In the application of this problem and its corollaries to the horizon and index glasses when prismatic, tracing the ray backwards from the eye to the object; with respect to the horizon-reflector, p will represent the angle made by the line of collimation of the telescope with the anterior surface of the said reflector, and will be constant, as will also be b, q, n, MR and ϵ. With respect to the index-reflector, if the prismatic $\angle$ of it be supposed small, if the two reflectors be nearly perpendicular to the plane of the instrument, and the line of collimation of the telescope

nearly parallel to the same plane; then if K be put for the angle the line of collimation of the telescope makes with the horizon-reflector h, half the angle marked on the limb, that is half the nominal angle, which is the angle made by the two reflectors with each other; then is $p=K-h$ nearly: and b and n will be constant; also, if the axis, about which the index reflector moves, be nearly perpendicular to the plane of the instrument, MR and ε will also be constant.

(To be continued.)

XXXIV. *A Supplement to the Theory of Astronomical Instruments; being the Equation of the Reflecting Instrument.* By BENJAMIN GOMPERTZ, Esq. F.R.S

Read December 12, 1823.

LET A be the distance of the object from the eye; Z the angle marked on the instrument, which is double the real angle; K the angle the line of collimation makes with the horizon-glass, which is supposed to be a point in its surface; B the distance of the centre about which the index-glass moves from the eye; the angle formed by the line of collimation of the telescope with the line joining the centre about which the index-glass moves, and the point of the horizon-glass cut by the line of collimation of the telescope $=$ H; the distance of the centre about which the index-glass moves from the horizon-glass measured by a line drawn from the said centre perpendicular to the horizon-glass $=$ D: then if all things are correct as to position, the angle X subtended by the real and reflected object at the eye $=$ Z $+$ the angle whose sine is $(\frac{B}{A}\times \left\{\text{sine of } H + \text{sine of } (Z - H)\right\} + 2\frac{D}{A}.\ \cos \text{ of } K)$; or putting $M = B.$ sine of $H + 2$ D. cos of K, which will be constant in the same instrument, we shall have the true angle $X = Z +$ the angle whose sine is $\frac{M + B.\ \text{sine of } \overline{Z - H}}{A}$; for the reason of which see Problem 1, Cor. 6, page 357. But if the reflectors are not perpendicular to the plane of the instrument and the line of collimation of the telescope not parallel to the said plane, then of the four angles about B made by the planes LBG and ABD (Supplement to fig. 5. Plate V.) let $ABG = 90^\circ - h$ represent the angle of the horizon-reflector $d' b' c'$ makes with the plane of the instrument, on the same side of the said plane as the eye is situate, which I shall call the upper side, and reckoned to the aspect of the front of the reflector; and let the angle $BDF = 90^\circ - I$, which will be the angle which the index-reflector makes with the plane of the instrument on the same side of the plane of the instrument as the eye is situate, and having the

aspect of the front of the reflector; l the $\angle$MCπ the angle which the line of collimation of the telescope makes with the plane of the instrument reckoned as tending upwards with respect to the instrument in its course from the eye to the horizon-reflector, I, h, and l being supposed extremely small. Hence we have the angle LBD $= 90° - h$, angle LDB$=90°+$I, and Mπ being $\parallel$KC, $\angle$MCK$=\angle$CMπ=K; Z$'$=2$\angle c'Oc$ which is the nominal angle of the limb, O being the intersection of cb and $c'b'$, and Z double the inclination of the planes of the reflectors to each other; the inclination being reckoned of the same aspect as the angle $c'Oc$; then will BD or angle BCD $= 180 - \frac{1}{2}$Z$'$, and GF $= 180° - \frac{1}{2}$Z. Put $g = \frac{\text{tangent of K}}{\text{sine of }(90°+h)} \times \left\{\frac{\text{sine of } l}{\text{sine of K}} + \text{cos of}(90°+h)\right\}$ $= \frac{\text{tan of K}}{\text{cos of } h} \times \left(\frac{\text{sine of } l}{\text{sine of K}} - \text{sine of } h\right)$, the sine of the constant arc represented by KB, of the great circle passing through the line of collimation of the telescope which is perpendicular to the horizon-reflector, and the great circle which is parallel to the plane of the instrument, and which I take for the arc itself in consequence of its being very small, unless indeed K is near 90°. Then we have a of Cor. 3, Prob. 2, or cos of h. cos of I nearly $= (1 - \frac{h^2}{2}) \times (1 - \frac{I^2}{2}) =$ $1 - \frac{h^2}{2} - \frac{I^2}{2}$ nearly; also $b =$ cos of $(90° - h) \times$ cos of $(90° + I) = -h.$ I nearly. And from Cor. 3, Prob. 2, we have by writing $180° - \frac{1}{2}$Z$'$ for z, and $180° - \frac{1}{2}$Z $= z + w$; and therefore $180° - \frac{1}{2}$Z $= 180° - \frac{1}{2}$Z$' + w$, and also Z $=$ Z$' - 2w$, $Z = Z' - \frac{-2h.I + \overline{h^2+I^2}.\text{cos of }(180° - \frac{1}{2}Z')}{\text{sine of }(180° - \frac{1}{2}Z')}$ nearly $= Z' +$ $\frac{2h.I + \overline{h^2+I^2}.\text{cos of }\frac{1}{2}Z'}{\text{sine of }\frac{1}{2}Z'}$, failing when z equal to o or 180° or nearly so; but when $z = 180°$ then is Z$'$=o, consequently the expression fails when Z$'$=o or nearly so. When Z$'$=o, that is when $z = 180°$, Z or Z$' - 2w$ is equal to twice the difference of LBD and LDB $= \pm\ (2h + 2.$ I), in which that sign of the two $\pm$ is to be taken, which when h and I are interpreted by their proper values and signs, shall cause $\pm\ (2h + 2.$ I) to have the same sign as the vanishing arc Z$'$ has before it vanishes.

Moreover from Cor. 5, Prob. 2, the inclination of the line of collimation of the telescope to the plane perpendicular to the two reflectors $= (\frac{-I}{\text{sine of } z} + h.$ cot of $z - g) \times$ cos of K $= (-\frac{I}{\text{sine of }\frac{1}{2}Z'} + h.$ cot of $\frac{1}{2}$Z$' + g) \times$ cos of K, failing when sine of BD $= o$ or nearly so; in which cases the sine of the said

angle will be $= \pm$ cos of K. cos of g. Therefore from Prob. 3, Cor. 2, supposing the object at an infinite distance, we have X the angle subtended at the eye by the real and reflected object, though Z′ might be extremely small or o, $= Z - (\frac{I}{\text{sine of } \frac{1}{2} Z'} + h.\ \text{cot of } Z' + g).\overline{\text{cos of K}}^2.$ tan of $\frac{1}{2}$ Z′ nearly, but failing when Z′ is 180° or nearly so. Hence X is nearly $= Z' + \frac{2\,h.\,I + (h^2 + I^2).\ \text{cos of } \frac{1}{2} Z'}{\text{sine of } \frac{1}{2} Z'}$ $- (\frac{I}{\text{sine of } \frac{1}{2} Z'} + h.\ \text{cot of } \frac{1}{2} Z' + g) \times \text{cos of } K^2 \times \text{tan of } \frac{1}{2} Z'$, except when $Z' = o$, or 180°, or very near those limits; but when $Z = o$ we shall have X $= \pm\ (2\,h + 2\,I)$ to be taken after h and I are properly interpreted of the same sign as the arc Z′ was previously to its vanishing. To the general equation for the value of X above, the part due to the parallax of the instrument is to be added and we shall have as an approximation $X = Z' + \frac{2.\ h.\ I + (h^2 + I^2)\ \text{cos of } \frac{1}{2} Z'}{\text{sine of } \frac{1}{2} Z'}$ $- (\frac{I}{\text{sine of } \frac{1}{2} Z'} + h.\ \text{cot of } \frac{1}{2}\ Z' + g)^2 \times \text{cos of } K^2 \times \text{tan of } \frac{1}{2} Z' +$ the arc whose sine to radius unity is $(\frac{B}{A}\left\{\text{sine of } \frac{1}{2} H + \text{sine of } (Z' - H)\right\} + \frac{2\,D}{A}.$ cos of K$)$ + the error of the arch. It is to be observed that I have written, in the part due to the parallax, Z instead of Z′ as an approximation: and that g is $= \frac{\text{sine of } l}{\text{cos of } h.\ \text{cos of K.}}$ — tangent of K. tan of h; that is when h is small $g = \frac{\text{sine of } l}{\text{cos of K}}$ $- h.$ tan of K, provided Z is not nearly $= o$ or 180°.

Hitherto we have not considered the reflectors to be defective; but if they be prismatic, as it will be the apparent planes of reflection which are to be considered instead of the anterior surfaces of the reflectors, and therefore if Z″, h' and I′ be with respect to the apparent posterior planes of reflection what Z′, h and I are with respect to the anterior surfaces of the reflectors, we shall have $X = Z'' + \frac{2\,h'.\ I' + (h'^2 + I'^2)\ \text{cos of } \frac{1}{2} Z''}{\text{sine of } \frac{1}{2} Z''} - (\frac{I'}{\text{sine of } Z''} + h'\ \text{cot of } Z'' + g)^2 \times$ cos of $K^2 \times$ tangent of $\frac{1}{2}$ Z″ + arc whose sine is $\left\{\frac{M + B\ \text{sine of } (Z'' - H)}{A}\right\} + G$;

let this be called equation A; where G stands for the index-error and M for B. sine of H + 2 D cos of K, a constant quantity: though indeed g and several of the other quantities which are retained here as they stood in the original equation belonging to the anterior surfaces, stand in need of small corrections; still as it is but in approximations to quantities in themselves small that these quantities are concerned, it would be useless to embarrass the formula with

the alterations; for if absolute accuracy were required there would be other corrections needful.

Let $a' d' b$, $a d b$ (supplement to fig. 5) be planes parallel to the posterior surfaces of the horizon and index reflectors respectively, making the very small angles b', b respectively with the anterior surfaces of the reflectors, which we will call the *angles of the prisms*, cutting the said anterior surfaces in $d' b'$, $d b$, which we will call the *lines of the prisms*; and let the plane passing through the eye and parallel to the plane of the instrument cut the anterior surfaces of the horizon and index reflectors respectively in $b'c'$, $b c$; and the planes $a'd'b'$, $a d b$ respectively in $a'b'$, $a b$: let the angles $d'b'c'$, $d b c$ of this figure (which are the angle RCM of fig. 9), that is the angles which the prismatic lines respectively of the horizon-reflector and index-reflector, make with the plane of the instrument reckoned on the planes of the anterior surfaces of the reflectors, equal to t' and t respectively; let the ratio of sines of incidence to the sine of refraction from the air into the matters of the horizon and index reflectors respectively be $1 : n'$, and $1 : : n$; and considering $90° - h$, $90° - \mathrm{I}$ to have represented the angles which the anterior surfaces of the horizon and index-reflectors make with the plane of the instrument on the side termed the upper side of that plane; and having the aspect of the respective fronts of the reflectors, let $90° - h'$, $90° - \mathrm{I}'$ be the angles which the apparent posterior surfaces of the horizon and index reflectors make with the plane of the instrument reckoned on the same side and aspects as the angles just named were taken; that is let $90° - h$ and $90° - h'$ if we consider the horizon-reflector, or $90° - \mathrm{I}$ and $90° - \mathrm{I}'$ if we consider the index-reflector, represent in fig. 9 the supplement of NMZ and MNZ. Let K be the inclination of the line of collimation of the telescope with the anterior surface of the horizon-reflector, Z double the inclination of the anterior surfaces of the two reflectors with each other. We have therefore from Corollary 6, Problem 5, $90° - \mathrm{I}' = 180° - \left(90° - \frac{b.\ \text{sine of the arc whose cosine is } n.\ \cos \text{ of } (\mathrm{K} - \frac{1}{2}\mathrm{Z})}{n.\ \text{sine of } (\mathrm{K} - \frac{1}{2}\mathrm{Z})} . \cos \text{ of } t + \mathrm{I}\right)$ nearly, and therefore $\mathrm{I}' = \mathrm{I} - \frac{b.\ \text{sine of the arc whose cosine is } n.\ \cos \text{ of } (\mathrm{K} - \frac{1}{2}\mathrm{Z})}{n.\ \text{sine of } (\mathrm{K} - \frac{1}{2}\mathrm{Z})} . \cos \text{ of } t$.

Also $h' = h - b'. \frac{\text{sine of the arc whose cos is } n'.\ \cos \text{ of K}}{n'.\ \text{sine of K}} . \cos \text{ of } t'$; and $\mathrm{Z}'' = \mathrm{Z}' + 2\, b. \frac{\text{sine of the arc whose cos is } n.\ \cos \text{ of } (\mathrm{K} - \frac{1}{2}\mathrm{Z})}{n.\ \text{sine of } (\mathrm{K} - \frac{1}{2}\mathrm{Z})} . \text{sine of } t + 2\, b'. \frac{\text{sine of the arc whose cos is } n' \cos \text{ of K}}{n'.\ \text{sine of K}} . \text{sine of } t'$. In making these substitutions Z′ may be written in the above formula for Z as an approximation.

When Z', which I here suppose not to be subject to index error, is an extremely small quantity U, this will give $= -b.\frac{\text{sine of the arc whose cos is } n.\text{ cos of K}}{n.\text{ sine of K}}$ cos of t,

$$h' = h - \frac{b' \text{ sine of the arc whose cos is } n'.\text{ cos of K.}}{n'.\text{ sine of K}} \text{ cos of } t', \text{ and}$$

$$Z'' = U + 2\,b.\text{ sine of } t.\frac{\text{sine of the arc whose cosine is } n.\text{ cos of K}}{n.\text{ sine of K}}$$

$$+ 2\,b'.\text{ sine of } t'.\frac{\text{sine of the arc whose cosine is } n'.\text{ cos of K}}{n'.\text{ sine of K}};$$ and therefore in this case if X become E we have $E = U \pm (2\,I + 2\,h) -$

$$2\,b.\frac{\text{sine of the arc whose cos is } n.\text{ cos of K}}{\text{sine of K}} \times (\text{cos of } t - \text{sine of } t) -$$

$$2\,b'.\frac{\text{sine of the arc whose cos is } n'.\text{ cos of K}}{n.\text{ sine of K}} \times (\text{cos of } t' - \text{sine of } t') + \text{the arc}$$

whose sine is $(\frac{2\,D}{A}.\text{ cos of K}) + G$. For the sake of brevity put R for the whole of ther ight hand side of the equation except U, G the index error (Z' being now considered subject to index error) and the arc whose sine is $\frac{2\,D}{A}$ .cos of K; and we shall have for extremely distant objects $G = E - U - R$. If the inclination h' to the line perpendicular to the plane of the instrument of the apparent posterior plane of reflection of the horizon-glass which is a constant quantity, be considered the given angle instead of h, the whole effect of the prismatic form of the horizon-glass will appear to be destroyed, as it will be enveloped in the quantity taken for the index error. The expression is subject to particular cases of failure as above pointed out. A prismatic form of an horizon-glass produces an effect from the ray which is transmitted through it from the real object to the eye; but the whole of this effect will also be contained in the quantity used for the index error.

It is to be remarked, with respect to equation A, that as the deductions are from approximations, which did not retain higher powers of b and b' than the first, in making the substitutions we should only retain the first powers of b and b', that we may not be misled by a mixed approximation with regard to the degree of accuracy which we attain; and that if g, I, h, b and b', are all quantities of the same degree of smallness, it would be proper also to leave out g^2, I^2, h^2, b^2, b'^2, I. b, &c., unless we push the approximation to terms containing b^2, &c. inclusively: according to this, if R' be put $= \pm$ $(2\,I + 2\,h) - 2\,b.\frac{\text{sine of the arc whose cos is } n.\text{ cos of K}}{\text{sine of K}} \times (\text{cos of } t - \text{sine of } t) -$

$2\,b'.\dfrac{\text{sine of the arc whose cosine is } n' \cos \text{ of K}}{n' \text{ sine of K}}$. cos of t', we shall have $X = Z' +$ *

$\dfrac{2\,b.\ \text{sine of the arc whose cos is } n.\ \cos \text{ of } (K - \frac{1}{2} Z')}{n.\ \text{sine of } (K - \frac{1}{2} Z')} \times$ sine of $t + E - U - R' +$

arc whose sine is $\left\{\dfrac{M + B.\ \text{sine of } (Z' - H)}{A}\right\}$.

But if b be of the same degree of smallness as I^2, h^2, &c. it will be proper to add to the right hand side of the equation the quantity $\dfrac{2\,h.I + (h^2 + I^2).\cos \text{ of } \frac{1}{2} Z'}{\text{sine of } \frac{1}{2} Z'}$ $-\left(\dfrac{I}{\text{sine of } Z'} + h.\ \cot \text{ of } Z' + g\right)^2 \times$ cos of K^2. tan of $\frac{1}{4} Z^2$; in which I have neglected the introduction of b and b', under the idea that b^2, $b.\,I$, &c. are smaller than the degree of approximation requires us to retain, g being sufficiently nearly $= \dfrac{\text{sine of } l}{\cos \text{ of K}} - h.$ tan of K; h and I being respectively the inclination of the anterior surfaces of the horizon and index reflector, from a perpendicular to the plane of the instrument, which are supposed small, and are both of the same sign when the inclinations are on different sides of a line perpendicular to the plane of the instrument, l also a small angle representing the inclination of the line of collimation of the telescope to the plane of the instrument, and is positive when reckoning from the eye towards the horizon-glass, it tends upwards with respect to the side of the plane of the instrument which we have termed the upper side; and K is put for the inclination of the line of collimation of the telescope to the horizon-glass, reckoned of the aspect of the zero of the division of the limbe.—Note; the angle marked on the limbe called Z', being here supposed subject to the index error G, that is $E - U - R'$; if this should be a large quantity, it ought to be added to the value of Z' in the functions which are added to Z', to get the value of X; should the instrument be subject to eccentricity, the error caused by it should also be taken into consideration.

In tracing the ray backwards from the eye to the object, we have discovered the position of the apparent posterior planes of reflection, approximatively; but should that not be sufficient, we may proceed thus:—

Considering the horizon reflector AM of fig. 9, is what is represented in figure 5 by KB, and having RCM, fig. 9, the angle made by the prismatic line with the section of the anterior surface with the plane of the instrument, we

---

* The part $\dfrac{2\,b.\ \text{sine of the arc whose cosine is } n.\ \cos \text{ of } (K - \frac{1}{2} Z')}{n.\ \text{sine of } (K - \frac{1}{2} Z')}$. sine of t, fails when $K - \frac{1}{2} Z'$ is extremely small.—(See cor. 2, prob. 5.)

have $\angle$ ACR ; $\because$ Cor. 3, prob. 5, we have the angle DPQ ; and therefore as AZ is there proved to be the complement of BPQ to a right angle, and therefore $= 90° - (180° - \text{APQ}) = \text{APQ} - 90°$ it is given; therefore we have MZ, we have also $\angle$ MZN ($=$ PQ) and the angle M ; thence we have MN, and because in the spherical triangle DAZ, (see supplement to fig. 9, the great circle DZ being drawn,) right angled at A, we have DA and AZ, we have DZ and the $\angle$ DZA, and having AZN we have DZN ; hence if the great circle DU be drawn perpendicular to NZU, having DZ and DZU, we have DU, the inclination of the incident ray to the apparent posterior plane, and this is also the inclination of the (really emergent ray but) apparent reflected ray with that plane. Now let every thing be in supplement to figure 5, as originally directed, except that LBK is here to be considered the apparent posterior plane of reflection of the horizon-glass, and CM being the ray passing through the eye, incident on the plane $\parallel$ KBL, if in the great circle KM, we take $\text{KM}' = 180° - \text{KM}$, then CM′ is parallel to ray, apparently reflected from the posterior apparent plane of reflection of the horizon-glass, and incident on the plane parallel to FDL ; draw the great circle M′T perpendicularly to LDT, then will M′T be the inclination of that ray, incident on the anterior surface of the index-glass ; and TD will be the distance of the plane perpendicular to the index-glass, which passes through the portion of the ray lying between the horizon-glass and index-glass, from the intersection of the index-glass with the plane of the instrument. M′ T, D here if referred to the figure 9, in the case of the ray being traced back from the eye to the index-glass, will be represented in that figure by the letters D, A, M. But continuing with the supplement of figure 5, draw the great circle M′L ; then since we have M′K, and KL ($=$ KB $+$ BL), and MKL a right angle, we have M′L and the $\angle$ M′LK ; but we have BLD, therefore we have M′LD ; $\because$ having M′L, and MTL being a right angle, we have M′T, the inclination of the ray to the anterior surface of the index-glass and TL, therefore we have also TD ; hence may the position of the apparent posterior plane of reflection of the index-glass be found.

XXXV. *On the Mercurial Compensation Pendulum.*
By Francis Baily, *Esq. F.R.S. and L.S.*

Read May 9, and June 13, 1823.

1. It is now above a hundred years since Mr. Graham first suggested the method of compensating the expansion and contraction of the pendulum rod, by means of a cylindrical vessel filled with quicksilver, which at the same time served as a bob to the pendulum. The idea was certainly an ingenious one; and has the merit of being the *first* which was proposed to counteract the effect of heat and cold on this invaluable instrument. It appears afterwards to have been laid aside for some time, and to have given way to various other methods which were subsequently proposed and put in practice: but, of late years, it has been revived, and is now in very general use, at least in *this* country. The simplicity of its construction, and its very easy adjustment, are much in favour of its adoption: but I much doubt whether a correct knowledge on these points is generally diffused amongst those persons who are actually employed in the formation of the instrument. And since nothing probably tends so much to the perfection of works of art as a *general* diffusion of the knowledge of the principles on which such works are constructed, I shall proceed to lay before the Society a few remarks which I have drawn up on this subject: avoiding, as much as possible, all allusion to those abstruse parts which will be found so ably treated by other writers, but which would only tend to confuse the practical mechanic.

2. It is well known that the oscillations of a pendulum, in any given period, are so sensibly affected by the occasional variations in the temperature of the atmosphere, that an intimate knowledge of the thermometrical expansion of the substances of which it is composed becomes absolutely necessary to its perfect construction. But, although many philosophers have paid considerable attention to this subject, and made several accurate experiments thereon, there is still a great discordance in many of the results; arising partly from the different modes of conducting the experiments, and partly from the various

quality of the substances under investigation. For, if we take the metals, for instance, we shall find that *bars* differ in their expansion from *wires*: and bars themselves will also differ from each other, according as they have been *wrought*, or *cast*, or subjected to some other process. In fact, the metal from one manufactory will not always agree, as to its expansibility, with a similar metal from another manufactory; although, to outward appearance, there may be no visible distinction of such quality.

3. Under these circumstances it would seem absolutely necessary that we should ascertain, in all cases, the exact expansibility of the very substance employed; and, in truth, this is the only correct mode of proceeding. But, as such a process is, at all times, troublesome, and, in most cases, not easily practicable, we must (I fear) content ourselves with the results that have been obtained by those who have had leisure and a convenient apparatus for such investigations. It fortunately happens, however, that the *differences* in the results are not very great, and that the *ratio* of expansion between two metals, as determined by different operators, does not vary much: so that the artist is seldom at a loss as to the general construction of the pendulum; leaving the minute adjustment of it to a more severe trial after the instrument is completed.

4. The great density and expansibility of *mercury* render it peculiarly fit, not only as a weight for the rod of the pendulum, but also as a means of counteracting the expansion of the rod itself. It therefore becomes an interesting object of inquiry, how far we can depend on the experiments which have, from time to time, been made to ascertain its correct expansion. Unfortunately, the experiments on this substance differ more from each other than those on any of the other metals: a difference probably arising partly from a difference in the quality of the metal itself, and partly from the various modes in which the experiments have been conducted. As those experiments were instituted for various purposes, (some of which have no connexion with the present subject,) and, although correct in principle as applicable thereto, yet may lead to false or erroneous conclusions on *this* subject, I trust I shall not be encroaching too much on the time and attention of the Society if I take a general view of the modes in which they have been conducted.

5. Of all the experiments which have been made on the expansion of mercury, those of MM. Dulong and Petit, inserted in the eleventh volume of the *Journal de l'Ecole Polytechnique*, appear to me to lay claim to the greatest credit. The paper is entitled *Recherches sur la mésure des temperatures, et*

sur les lois de la communication de la chaleur: and it obtained the prize decreed in a public sitting of the Royal Academy of Sciences at Paris, on the 16th of March, 1818. Their method is founded on the well-known principle of hydrostatics; that, when two liquid columns communicate with each other by means of a lateral tube, the vertical heights of their surfaces are exactly in the inverse ratio of their densities. If therefore we can, by any means, measure exactly the heights of two columns of mercury contained in the legs of an inverted syphon of glass, (one of those legs being surrounded with ice, whilst the other is submitted to a known degree of temperature,) we may easily deduce the required expansion. In fact, if h and h' denote the vertical heights of the two columns, producing equal pressure at the temperatures t and t', then, by calling d and d' the corresponding densities, we shall have $hd = h'd'$. But d and d' are in the inverse ratio of the volumes v and v', which are occupied by the same mass of liquid in raising it successively to the temperatures t and t'; whence we have $v' = v\,\frac{h'}{h}$: and lastly $\frac{h'-h}{h\,(t'-t)}$ for the mean co-efficient of the expansion between t and t'. The whole process therefore is reduced to the correct measure of the *temperatures*, and of the *heights* of the columns: and it was in this manner that MM. Dulong and Petit proceeded. The determination of the temperature presented no difficulty; and the *difference* in the heights of the two columns was determined by a peculiar microscopic apparatus, which is minutely detailed in the paper here alluded to. It is almost needless to remark, that, by this method they obtained the *absolute* expansion of the liquid; since neither the form of the vessel, nor *its* expansion, had any influence on the pressure of the liquid contained in it.

6. The *absolute* expansion of mercury being thus obtained, we may easily deduce its *apparent* expansion; since we have only to subtract the *cubical* expansion of the vessel containing the mercury, which may in all cases be assumed equal to three times the *linear* expansion of the same substance. For, the *linear* expansion of a body (represented by unity) at a given temperature, being denoted by e, its total length at that temperature will be $(1+e)$; and its *cubical* expansion will be $(1+e)^3$: which, since e is always very small, may, in all cases, be assumed equal to $(1+3\,e)$. But, in experiments on the mercurial compensation pendulum, it is neither the *absolute* nor the *apparent* expansion of mercury which is required, but an intermediate quantity, which, for the sake of a convenient term of reference, I shall call the *relative vertical expansion;* a quantity depending on the expansibility of the vessel in which it is

contained, and which is found by deducting *twice the linear* expansion of the containing vessel from the *absolute* expansion of the mercury. Thus, if the linear expansion of the containing vessel be denoted by v, and the *absolute* expansion of mercury by m, then will $(m - 2v)$ express the *relative vertical expansion* of the mercury. It is a variation in the *height* only of the mercury in the vessel, which affects its compensating power: and it is evident that the variation in the *height*, caused by an alteration of temperature, is correctly found by this method.

7. I shall next advert to the experiments of MM. Lavoisier and Laplace, which have not yet (as I believe) been detailed in any publication; but the result of them has been given by M. Biot, in his *Traité de Physique*, and in his *Précis elementaire de Physique*. The value given in the former work is erroneous, but is corrected in the latter: and MM. Dulong and Petit state that the error arose from a mistake in the calculations, the true value of the absolute expansion of mercury having been $\frac{1}{5522}$ of its bulk for 180° of Fahrenheit's thermometer, instead of $\frac{1}{5412}$. MM. Dulong and Petit add, that even this value was still considered too great by MM. Lavoisier and Laplace; as they had neglected to boil the mercury. So that the experiments of these four philosophers approximate nearer than the annexed table, to which I shall presently allude, seems to indicate.

8. The Committee of the Royal Society (appointed in 1777) assumed the expansion of mercury to be $\frac{1}{11500}$ of its bulk for every degree of Fahrenheit's thermometer; but no allusion is made to the experiments on which this assumption is founded, neither do they state whether the *absolute* or *apparent* expansion is intended.

9. The experiment alluded to by Mr. Gavin Lowe, in his paper "On the compensation mercurial pendulum," inserted in the *Philosophical Magazine* for August 1819, evidently refers to the *apparent* expansion of mercury; and which, therefore, requires a correction, as I have already mentioned.

10. The experiments of Mr. Cavallo, inserted in the *Philosophical Transactions* for 1781, were made with thermometer tubes, and do not appear to have been conducted with that degree of nicety which is so necessary in all experiments relative to the pendulum.

11. Sir George Shuckburgh, so well known for his philosophical researches, made his experiments with a tube, somewhat like a thermometer, but con-

siderably larger than the common size, and open at one end. It was filled with mercury to a certain height, and then exposed to the temperatures of freezing and boiling repeatedly: the difference of the volume in each instance was afterwards determined by accurately weighing the contents.

12. MM. DELUC, CASBOIS, and General ROY, made their experiments, on the expansion of mercury, with barometers. But this method, however correct it may be for the purposes of barometrical measurements, is totally inapplicable to experiments with the pendulum. It may be proper also to remark that General ROY found that the *rate* of expansion of a column of mercury, in a barometer of the ordinary kind, *varied* with the degree of temperature applied: but, from the experiments of MM. DULONG and PETIT, it does not appear that this circumstance takes place *below* the temperature of boiling water, when the vessel is not hermetically sealed. Consequently, we may, in all cases connected with the pendulum, consider the rate of expansion as uniform. General ROY's experiments are detailed in the *Philosophical Transactions* for 1777, and are divided into three distinct sets, which differ a little from each other. Those of M. Deluc are also to be found in the same volume, corrected by General ROY; they are likewise quoted by MM. DULONG and PETIT. The results, however, as given by these respective authors, appear to differ from each other.

13. The result of the experiments of MM. LALANDE, DELISLE, HALLSTROEM, and DALTON, is given by MM. DULONG and PETIT; and of MM. COTTE, LICHTENBERG, ACHARD, HALLSTROEM, and ROSENTHALL, by Dr. YOUNG in his *Lectures on Natural and Experimental Philosophy*. As I have not seen the detail of those experiments, I do not know what degree of confidence can be placed in their results. There is one point, however, on which all these experimentalists appear to be silent, but which it would nevertheless be important to know: namely, whether any of them performed their experiments on *different* kinds of mercury. And, till this has been decided, we may fairly presume that part of the difference in the results may have arisen (as in other substances) from a diffrence in the quality of the metal.

14. Having thus enumerated the principal experiments that have been made on the expansion of mercury, I shall proceed to an examination of those which have been made, by various persons, on those substances of which the *rod* of the pendulum may be composed. Of all these substances, that of *wood* appears to be the least expansible; but, it is unfortunately so liable to be affected by the moisture of the atmosphere, that, notwithstanding we coat it with

varnish, or sealing wax, or paint, or gilding, or even bake it, and impregnate it with oil, it has seldom been found sufficiently accurate for the refined purposes of the modern astronomer. I would not, however, wish to discourage any attempts to render this substance more fit for general use. Fitted up with a leaden bob (in the manner which I shall hereafter describe), it forms the cheapest pendulum that I know of: and, if placed in a room where there is an uniformity in the atmosphere, it might answer every useful purpose for an economical observatory. At all events, it would form an excellent appendage and improvement to the common household clock, and would be far superior to and much cheaper than the usual and absurd mode of hanging a leaden bob to the end of an iron wire.

15. The next least expansible substance is that of *glass*; which (from the table to which I shall presently allude) will be found to differ in its expansibility, not only according to the materials of which it is composed, but also according to the form into which it is manufactured: viz. as to *rods*, or *tubes*, or *plates*. It is however of so brittle a nature that there is always some risk in using it for the *rod* of the pendulum; although it is the substance now generally adopted for containing the mercury in the bob. An accurate knowledge, therefore, of its true rate of expansion is very desirable in all inquiries of this nature. The authorities on which I have relied, in these cases, are those of General ROY and Mr. SMEATON in England, and of MM. LAVOISIER and LAPLACE, and MM. DULONG and PETIT in France. The experiments of the former were conducted in a manner similar to their experiments on the expansion of metals, to which I shall presently refer: but those of MM. DULONG and PETIT were conducted in a manner peculiarly their own. They, in fact, proposed to obtain the expansion of glass and other bodies by determining the difference between the expansion of mercury and those bodies: for that difference is nothing more than the apparent expansion of the mercury in a vessel made of such substance. Although this species of expansion had already been the object of a great number of experiments, yet MM. DULONG and PETIT thought proper to make it the subject of their own experiments. With this view they made use of a tube of glass about two feet long, and containing about two pounds (troy weight) of mercury. This tube was closed at one end, and terminated at the other in a capillary bore, the capacity of which was an insignificant fraction of the whole tube. The whole apparatus being filled with mercury, and rendered free from air and humidity by repeated boiling, they determined the weight of mercury which escaped when

the temperature of it was raised from the freezing to the boiling point. They remark that the accuracy of this process may be imagined when we recollect that that portion of the tube, which does not partake of the ebullition, is insensible; and that the position of the tube (being horizontal) enables the operator to apply the correction depending on the barometric pressure. The experiments were repeated five times on different tubes, with nearly the same results: and (which is remarkable) they state that they did not find any difference in the tubes of common glass taken from different manufactories, whatever was their diameter or thickness.

16. Of the various *metals,* I shall confine my remarks to those which are likely to be employed on such occasions; commencing with platina the least expansible of the solid metals, and terminating with zinc the most expansible of them all. I am not aware of more than three sets of experiments having been made on the expansibility of platina: those recorded by Mr. Brande, in his *System of Chemistry,* appear to be a mean between those of the French philosophers. The experiments on iron, steel and brass, have been numerous, both in this country and on the continent; and the results have been as various as the experiments. Still, however, those results are confined within certain limits; which may be yet further reduced, or modified, on a more strict examination of the mode in which the respective experiments have been conducted. Much, indeed, depends upon this: for, the method of determining the expansion of bodies may be pursued in various ways, which will be entitled to more or less credit, according to the perfection of the apparatus, and the skill of the operator. The pyrometer of Musschenbroek, the first experimentalist in this way (in the order of time), is not to be compared with that of Mr. Smeaton or of General Roy: nor are the experiments of Cavallo to be placed in competition with those of Sir George Shuckburgh.

17. Mr. Smeaton's experiments were made with a pyrometer which is described in the *Philosophical Transactions* for 1754. The measures taken by this instrument were determined by the *contact* of a piece of metal with the point of a micrometer screw. It was conceived that the accuracy of the observation could be better determined by the *hearing,* than by the *sight* or *feeling*: and Mr. Smeaton found by trial that it was practicable to repeat the same measurement several times without differing $\frac{1}{20000}$ of an inch.

18. General Roy's observations were made with a pyrometer of Mr. Ramsden's construction, as described in the *Philosophical Transactions* for 1785 and also in the *Account of the Trigonometrical Survey*. The measures were

determined by *sight*, with the assistance of a micrometer microscope which could be read off to ·00014 of an inch. It will be seen, by the table inserted at the end of this paper, that the rates of expansion, as determined by General Roy, are generally *less* than those determined by Mr. Smeaton. There is not however much difference in the results: and they agree more nearly than we might previously have supposed from two such opposite modes of obtaining the measures.

19. The experiments of MM. Dulong and Petit on platina, iron and brass were conducted in a manner somewhat similar to that to which I have already alluded. In order to obtain the expansion of these metals, they took a tube of glass about $\frac{7}{10}$ of an inch diameter, and two feet long, closed at one end. They then introduced a small cylindrical rod of the metal under experiment, which was secured in the middle of the tube by means of four transverse pieces, of a length equal to the diameter of the tube. After having joined, to the open end of this tube, another tube with a capillary bore, the whole was filled with mercury; which they repeatedly boiled in order to free it from air and humidity. By exposing it to different temperatures, and determining the weight of the mercury which escaped, they could easily deduce the expansion of the metal: for, the quantity which escaped evidently represented the *sum* of the expansion of the mercury and the metal, diminished by the expansion of the glass. In order to make this calculation, however, it is necessary to know the volume of the bodies at the freezing point: but that of the metal is obtained by its weight and density at that point; and in the same manner may be deduced the requisite deductions for the glass and mercury at the same temperature. Care was taken, in these experiments, to prevent the action of the mercury on the metals. The cubical expansion of the metals being obtained by this process, the linear expansion was deduced by taking $\frac{1}{3}$ of the same, as already alluded to.

20. In the experiments of MM. Lavoisier and Laplace, the metal rod was placed in the usual way in the trough of the pyrometer; but the end of it operated on a lever attached to a telescope, which pointed to a wall at a considerable distance, on which the degrees of expansion were marked on a large scale. A description and drawing of the apparatus are given by M. Biot in his *Traité de Physique*.

21. In the third volume of the *Base du Système Métrique*, page 469, it is stated that M. Borda and the Committee of Weights and Measures made a number of accurate experiments on the expansion of platina, iron and brass;

the result of which I have inserted in the table at the end of this paper: but no account is given of the manner in which those experiments were conducted, nor of the apparatus employed.

22. The experiments of Captain Kater on the expansion of brass were made on bars placed in a trough filled with water, the temperature of which was varied at pleasure. The quantity of expansion was determined by microscopes; in the manner stated in his excellent paper on the Pendulum in the *Philosophical Transactions* for 1818. The expansion assumed by Captain Sabine does not appear to be the result of his own experiments *. There is evidently a typographical error in placing the decimal point in the value given by him in the *Philosophical Transactions* for 1821, which I have corrected in the table subjoined to this paper.

23. In the ninth volume of Mr. Nicholson's Journal, are stated the results of some very accurate experiments made by the celebrated Mr. Troughton. The detail of those experiments is not given; but, whatever comes from so excellent a source, is entitled to the highest credit, and will be duly appreciated.

24. Mr. Hasslar's experiments on iron and brass were conducted on a large scale; the bars being upwards of twenty-six feet long, and intended as measures to be used in a national survey of the coast of the United States of America. A wooden box (of a sufficient size and length to contain the bars freely) was placed vertically against the north wall of a house, but at a little distance from it. The bottoms of the bars rested on a metal plate attached to the wall, and the tops abutted against the end of two levers, each connected with a fine level attached also to the wall. These levels were furnished by Mr. Troughton. The bottom of the box or tube was connected, by means of a lateral tube, with the top of a boiler; and, by passing steam through it, the temperature of the bars, throughout their whole length, could be raised to the boiling point. The index, connected with the levels, being then noted, the whole was left to cool till it came to the freezing point (advantage being taken of the season, in conducting these experiments); when the index was again noticed, and the required rate of expansion deduced. An account of the apparatus employed, and a detail of the experiments, are given in the *Transactions of the American Philosophical Society*, vol. 1. of the new series. In the table annexed to this

* I have since been informed that this intelligent and enterprising officer proposes to deduce it from the variation in the number of vibrations of the very pendulum which was used by him. In fact, the pendulum itself is the best *pyrometer* that can be adopted in *these* cases.

paper, I have taken the mean of *two* sets of experiments. If Mr. Hasslar's apparatus were reduced to a smaller scale, it would form an excellent pyrometer for ordinary purposes.

25. I have inserted the result of M. Musschenbroek's experiments, more to show what degree of accuracy may be obtained with a very clumsy apparatus, than with a view of producing it as an authority. Those who are desirous of examining more in detail the subject of his experiments, may consult Desaguliers's *Course of Experimental Philosophy*, vol. 1, page 439, where they will find a description of the instrument employed: which, I believe, was the *first* that was constructed for such purposes.

26. Notwithstanding all that had been thus done on the subject of expansion, M. Berthoud (so well known for his various publications on horology) made a number of experiments on the expansion of various substances, for the express purpose of determining with what degree of accuracy they might be employed in the construction of pendulums. For this purpose he made use of a pyrometer, still differently formed from those of preceding writers; a description of which is given in his *Histoire de la mésure du temps par les horloges*: vol. 1. The most important part of it consisted of a large slab of marble, 5 feet long, 1 foot wide and 5 inches thick, placed in a vertical position in a case, which contained likewise the materials on which the experiments were to be made: two stout brass pillars were fastened into this slab; one at the top, to which the pendulum, or any other body, was attached; and the other at the bottom, to which a graduated arch of 10 inches diameter was fastened. By means of a lever annexed to the index of this arch, from the bottom of the rod under experiment, the degree of expansion was determined in the following manner. "Rods of various metals were procured, each 461 "lines long, 5 lines broad and 8 lines thick. Each rod was previously put "into a leaden tube, filled with ice; and afterwards placed in the case of the "pyrometer; heat was then applied till Reaumur's thermometer stood steadily "at 27°. The result of the experiments is given in the following table:

Soft steel	69	Tin	160
Steel hammered cold	74	Lead	193
Spring tempered steel	77	Glass	62
Soft iron	75	Silver	119
Hardened iron	78	Softened gold	82
Copper	107	Hard gold wire	94
Brass	121	Mercury	1235

"The above quantities are expressed in 360ths of a line. Thus, soft steel "gave for the absolute quantity of its expansion $\frac{69}{360}$ of a line, in passing from "the freezing point to 27° of Reaumur's thermometer: and thus of the "rest."

27. I have preferred giving M. Berthoud's own explanation of these quantities, because the results differ very materially from those of every preceding writer. He has not stated how he obtained the expansion of mercury. A variation of 27° in Reaumur's thermometer is equal to a variation of 60°·75 in Fahrenheit's thermometer: therefore $\frac{1}{461 \times 360 \times 60.75} = \cdot 000000099186$ will be the constant quantity by which each of the above values must be multiplied in order to reduce them to the rate of expansion according to Fahrenheit's thermometer. The table at the end will show how much these values exceed those of every other experimentalist, except it be perhaps those of Musschenbroek.

28. In order to show at one view the results of the experiments of all the authors alluded to in the preceding pages, as well as of some others which I have collected, on the expansion of various substances used in the construction of pendulums, I have drawn up the table inserted at the end of this paper. The substances are arranged in the order of their expansion; and opposite to each is placed the expansion of the body (considered as unity) for every 1° of Fahrenheit's thermometer. By this method, a constant and easy reference may be at any time obtained, relative to those points: and such comparisons made, as may be required.

29. Having thus determined the expansion of various substances, as accurately as our present means of information will enable us, I shall endeavour to show the practical application of this inquiry in the construction of the mercurial pendulum beating seconds. This pendulum, as usually made by the artist, is composed of a cylindrical glass vessel, of about 2 inches diameter and 7 inches long, filled nearly to the top with mercury, and standing in a steel stirrup, which is annexed to a rod of the same material; to the upper end of which is fitted a spring, on which its vibrations are formed. The parts of this pendulum are minutely described by Mr. Gavin Lowe in the *Philosophical Magazine* for August 1819: and I believe that that description has furnished the model for most of the mercurial pendulums now in use. It is however liable to some objections, and capable of some improvements, to which I shall allude in the sequel. At present I shall confine myself to the principles on

which it is constructed: whereby the practical philosopher may extend the application to other forms of the same pendulum; and to other substances, of which the component parts may consist.

30. It is well known that, if the temperature of the atmosphere remained the same, the oscillations of a pendulum, vibrating in small arcs of equal magnitude, (as all those attached to our modern clocks are made to do,) would also be uniformly the same, and no correction would be required. But, since this is never the case in reality, and the rod of the pendulum becomes longer in warm weather, and shorter in cold, thereby affecting very sensibly its oscillations in any given period, it becomes necessary to counteract this variation in its length, in order that the centre of oscillation may always remain at the same given distance from the point of suspension. The whole theory therefore of the compensation pendulum depends on an accurate knowledge of the ratio of expansion in the different substances of which it is composed. But, before this can be practically applied, it becomes necessary to advert to a few of the well known principles in mechanics, connected with the centre of oscillation.

31. It is obvious to every one, acquainted with those principles, that, if a cylinder AHDE (Plate VI. fig. 1.) be suspended by any diameter AH of its upper circle, on which it is caused to vibrate, the centre of oscillation O will be distant from F two thirds of the length of the cylinder FB, *added to* a small quantity depending on the radius of the cylinder. This minute *additional* quantity may be wholly neglected in the subsequent investigations, as we shall treat only of the *comparative* results of *compound* pendulums, and not of the *absolute length* of the *simple* pendulum.

32. If the same cylinder be suspended from S, by a line SF void of weight, and be made to vibrate on S, the centre of oscillation O will, in this case, be distant from S by the quantity $\frac{(SG)^2 + \frac{1}{3}(FG)^2}{SG}$; where G is the centre of gravity of the cylinder; corresponding with C the centre of the cylinder. It should however be particularly observed that this result is obtained on the supposition that the line SF is void of gravity; which, in practice, can never be the case. Since therefore the rod or line, by which the cylinder is suspended, will always be of a sensible weight, the above formula will not express the true distance of the centre of oscillation from the point of suspension: but it may be determined in the following manner.

33. Let SADFEHB (fig. 2.) be a compound pendulum vibrating about

the axis at S. Let g and o denote the distances, from the point of suspension S, of the centre of gravity and of the centre of oscillation respectively of the rod or line SF, considered as a simple pendulum and vibrating independently of the lower part: and let w be the weight of this rod. Further, let g', o', and w' represent similar quantities in the lower part, or cylinder AHED, considered also as a simple pendulum suspended from S by a line *void of weight*, and vibrating thereon, independently of the upper part, or rod. Lastly, let g'' and o'' denote corresponding quantities in the *compound* pendulum SADFEHB.

Now, since ogw is the sum of the product of each particle of the rod SF, multiplied by the square of its distance from the axis of suspension: and since $o'g'w'$ is in like manner the sum of the product of each particle of the cylinder AHED, multiplied by the square of its distance from the axis of suspension: and since the distance of the centre of gravity of the *compound* body, from the axis of suspension is $g'' = \frac{wg + w'g'}{w + w'}$; it follows that in the *compound pendulum* the distance of the centre of oscillation from the axis of suspension, will be $o'' = \frac{ogw + o'g'w'}{gw + g'w'} = \frac{o + \frac{o'g'w'}{gw}}{1 + \frac{g'w'}{gw}}$: and this is the length of the equivalent isochronous simple pendulum*.

34. But it will be more convenient, in the subsequent investigations, to assume the weight of the bob as a given multiple ($= m$) of the weight of the rod SF: that is, to assume $m w = w'$; whence we have $m = \frac{w'}{w}$. Moreover, if we make the whole length of the rod, or line SF, equal to r, and the length of the bob BF equal to b, we shall have $g = \frac{r}{2}$, and $g' = r - \frac{b}{2}$; consequently $\frac{g'}{g} = \frac{2r - b}{r}$: whence

$$o'' = \frac{o + o' \, m \frac{2r - b}{r}}{1 + m \frac{2r - b}{r}} \qquad \text{(A)}$$

* See Huygens's *Horologium Oscillatorium*, Part iv. Prop. 5; and all the subsequent authors on this subject.

35. Now, we have already seen, by § 31, that $o = \frac{2r}{3}$; and, by § 32, that $o' = \frac{2r(r-b)+\frac{2}{3}b^2}{2r-b}$: consequently we have

$$o'' = \frac{2}{3} \times \frac{[3r(r-b)+b^2]m+r^2}{(2r-b)m+r}$$

$$= \frac{2}{3} \times \frac{\left(3+\frac{1}{m}\right)r^2 - 3br + b^2}{\left(2+\frac{1}{m}\right)r - b}$$

But the length of the simple pendulum beating seconds is always a determinate quantity, and may be denoted by p*: whence we have

$$p = \frac{2}{3} \times \frac{\left(3+\frac{1}{m}\right)r^2 - 3br + b^2}{\left(2+\frac{1}{m}\right)r - b} \qquad \text{(B)}$$

36. Since the rod and the bob (or r and b) are subject to expansion and contraction, according to the temperature of the atmosphere, it is evident that this expression will not, in all cases, denote the true value of p, unless those bodies are of a proper length to compensate each other. The method of determining those lengths, according to the substances employed, will be as follows. Let the linear expansion of the substance, which forms the rod, be denoted by ϱ; and the relative vertical expansion of the substance which forms the bob, by β: and let r' and b' respectively denote what r and b become after expansion. Then, for any given difference, t, in the temperature, (t being equal to the number of degrees of variation in the thermometer, caused by such difference,) we shall have $r' = r(1+t\varrho)$ and $b' = b(1+t\beta)$: and, by omitting the powers of ϱ and β, as too small to be of any value, we shall have $(r')^2 = r^2 + 2r^2t\varrho$, $(b')^2 = b^2 + 2b^2t\beta$, and $r'b' = rb + rb(\varrho+\beta)t$. Whence by substituting these values in the formula above given, we shall have p, after expansion, equal to

* The seconds pendulum is usually adjusted either to *mean solar* time, or to *sidereal* time: the length of the simple synchronous pendulum vibrating, in the open air, 86400 seconds of *mean solar* time in a day, is, in this latitude, 39·133 inches: and the length of a similar pendulum, vibrating 86400 seconds of *sidereal* time in a day, is 38·919 inches. I shall therefore in the subsequent pages assume 39 inches as an approximation that will suit each case.

$$p' = \frac{2}{3} \times \frac{\left(3+\frac{1}{m}\right)(r')^2 - 3\,r'b' + (b')^2}{\left(2+\frac{1}{m}\right) r' - b'}$$

And since p and p' should, under all temperatures, be equal to each other, we have $p - p' = 0$.

If for the sake of brevity and convenience we put

$$A = \left(3+\frac{1}{m}\right) r^2 - 3\,br + b^2$$

$$B = \left(2+\frac{1}{m}\right) r - b$$

$$A' = \left(6+\frac{2}{m}\right) r^2 \varrho - 3\,br\,(\varrho + \beta) + 2\,b^2 \beta$$

$$B' = \left(2+\frac{1}{m}\right) r\,\varrho - b\,\beta$$

we shall have $p = \frac{2}{3} \times \frac{A}{B}$ and $p' = \frac{2}{3} \times \frac{A+A't}{B+B't}$; consequently $p - p' = \frac{A}{B} - \frac{A+A't}{B+B't} = 0$: whence $BA' - B'A = 0$. Therefore

$$+ \left[\left(2+\frac{1}{m}\right) r - b\right] \times \left[\left(6+\frac{2}{m}\right) r^2 \varrho - 3\,br\,(\varrho + \beta) + 2\,b^2 \beta\right]$$

$$- \left[\left(2+\frac{1}{m}\right) r\varrho - b\,\beta\right] \times \left[\left(3+\frac{1}{m}\right) r^2 - 3\,br + b^2\right] = 0.$$

Let us make

$$\frac{\varrho}{\beta} = k$$

then, by assuming $\frac{b}{r} = x$, we shall obtain

$$+ \left[\left(2+\frac{1}{m}\right) - x\right] \times \left[\left(6+\frac{2}{m}\right) k - 3\,(k+1)\,x + 2\,x^2\right]$$

$$- \left[\left(2+\frac{1}{m}\right) k - x\right] \times \left[\left(3+\frac{1}{m}\right) - 3\,x + x^2\right] = 0.$$

which may be ultimately reduced to a cubic equation of the following form:

$$x^3 - \left[\left(4+\frac{2}{m}\right) + \left(1-\frac{1}{m}\right) k\right] x^2 + \left[\left(3+\frac{2}{m}\right) + \left(3+\frac{1}{m}\right) 2k\right] x - \left(2+\frac{1}{m}\right)\cdot\left(3+\frac{1}{m}\right) k = 0.$$

37. This equation for $x \left(=\frac{b}{r}\right)$ depends only on the ratio of the *weight* of the rod to that of the bob, and on the ratio of the *expansion* of the bob to that of the rod. The former may be easily determined by actually weighing the parts: and we may approximate very nearly to the

latter by means of the table inserted at the end of this paper. When x is known, r and b will also be known from the equation (B)

$$p = \frac{2}{3} \times \frac{\left(3+\frac{1}{m}\right) r^2 - 3\,br + b^2}{\left(2+\frac{1}{m}\right) r - b}$$

$$= \frac{2}{3} \times \frac{\left(3+\frac{1}{m}\right) - 3\,x + x^2}{\left(2+\frac{1}{m}\right) - x} \times r$$

whence, since $b = rx$, we have

$$r = \frac{3\,p}{2} \times \frac{\left(2+\frac{1}{m}\right) - x}{\left(3+\frac{1}{m}\right) - 3\,x + x^2} \qquad \text{(C)}$$

and, which being multiplied by x, will give the value of b.

Now, in order to determine x in the preceding cubic equation, we may make use of the method of approximation: and if we assume

$$d = \frac{2}{m} + \left(1 - \frac{1}{m}\right) k$$

$$e = \frac{2}{m} + \left(6 + \frac{2}{m}\right) k$$

$$f = \left(2 + \frac{1}{m}\right) \cdot \left(3 + \frac{1}{m}\right) k$$

the cubic equation may be written $x^3 - 4\,x^2 + 3\,x = dx^2 - ex + f$, or $x\,(3-x).(1-x) = dx^2 - ex + f$: in which k and $\frac{1}{m}$ being, by the nature of the problem, small; d, e, and f, will likewise be small. The three roots of this equation will therefore be nearly 1, 3, and $\frac{f}{3}$: the last of which only can apply to the case of the mercurial compensation pendulum. The first approximation therefore of the value of x will be $\frac{f}{3}$. By substituting this value in the equation $x = \frac{f - ex + dx^2}{(3-x).(1-x)}$, we have for a second approximation, more nearly,

$$x = \frac{3\,f - de}{9 - 4\,f} = \frac{f}{3}\left(1 - \frac{e}{3} + \frac{4\,f}{9}\right) \qquad \text{(D)}$$

and which will be sufficiently near for all ordinary purposes. Having thus deduced x, we may obtain r by means of the equation (C), and thence determine $b = rx$.

38. I shall now bring these several equations under one view, for the convenience of the computer.

$$\left.\begin{aligned}
k &= \frac{\rho}{\beta} = \frac{\text{expan. of rod}}{\text{rel. vert. expan. of bob}} \\
\frac{1}{m} &= \frac{w}{w'} = \frac{\text{weight of rod}}{\text{weight of bob}} \\
e &= 2\left[\frac{1}{m} + \left(3 + \frac{1}{m}\right)\right] k \\
f &= \left(2 + \frac{1}{m}\right) \cdot \left(3 + \frac{1}{m}\right) k \\
x &= \frac{f}{3}\left(1 - \frac{e}{3} + \frac{4f}{9}\right) \\
r &= \frac{3p}{2} \times \frac{\left(2 + \frac{1}{m}\right) - x}{\left(3 + \frac{1}{m}\right) - 3x + x^2} \\
b &= rx
\end{aligned}\right\} \quad (E)$$

39. As an example of the use and application of these formulæ to the case of the mercurial pendulum, let us suppose that the rod SF, fig. 2, is made of steel, whose rate of expansion is $\rho = \cdot 00000636$; and that the bob is composed of a cylindrical column of mercury in a glass vessel*. If we assume the rate of expansion of glass to be $\cdot 00000479$, and the absolute expansion of mercury to be $\cdot 00010010$, then will the relative vertical expansion of the bob be $\beta = (\cdot 00010010 - \cdot 00000958) = \cdot 00009052$.† Let us further suppose that the weight of the rod is half a pound, and the weight of the bob ten pounds; whence $m = 20$, and $\frac{1}{m} = \cdot 05$. In this case we have $k = \frac{\rho}{\beta} = \frac{636}{9052} = \cdot 07026$: whence $e = \cdot 5286$ and $f = \cdot 4393$: and the first approximation of x will be $\cdot 1464$, and the second approximation $\cdot 1464\ (1 - \cdot 1762 + \cdot 1952) = \cdot 1494$.

By substituting this value of x in the equation (C), we have $r = \frac{3 \times 39}{2} \times \frac{2 \cdot 05 - \cdot 1494}{3 \cdot 05 - \cdot 4482 + \cdot 0223} = 42 \cdot 37$; and $b = 42 \cdot 37 \times \cdot 1494 = 6 \cdot 31$. It appears

* In all cases of the mercurial compensation pendulum, the rod SF (Fig. 2) is supposed to pass freely through the cylinder AHED, which rests on a nut at the bottom: so that whilst the expansion of the rod is in the direction SF, that of the bob is in the opposite direction FB. In the pendulum, hereafter described, the effect will be the same, although the cylinder is placed in the stirrup, as in figure 4.

† See § 6.

therefore that a column of mercury about $6\frac{3}{10}$ inches high will compensate a steel rod about $42\frac{4}{10}$ inches long, whose weights are in the ratio of 20 to 1 *; and whose expansions are in the ratio here assumed.

The expansion of steel has been here deduced from General Roy's experiments: but if we assume Mr. Troughton's rate of expansion, we shall have $r = 42{\cdot}47$, and $b = 6{\cdot}58$. Or, if we assume Mr. Smeaton's rate of expansion, we shall have $r = 42{\cdot}59$, and $b = 6{\cdot}81$ inches.

40. In order to prevent the trouble of calculating according to the formula, I have computed the following little table, whereby the value of r and b may in most ordinary cases be discovered by inspection. The artist has only to determine the value of k, or the ratio of the expansion of the rod and the bob, and the corresponding values of r and b will be readily found, according as m is assumed equal to 20, 30, or infinitely great.

$k = \frac{\varrho}{\beta}$	$m = 20$		$m = 30$		$m = \infty$	
	r	b	r	b	r	b
·065	42·11	5·80	42·00	5·75	41·77	5·66
·066	42·15	5·90	42·05	5·85	41·82	5·76
·067	42·20	6 00	42·10	5·95	41·86	5·86
·068	42·25	6·10	42·14	6·05	41·91	5·96
·069	42·30	6·20	42·19	6·15	41·96	6·06
·070	42·35	6·30	42·24	6·25	42·00	6·15
·071	42.40	6·40	42·28	6·36	42·05	6·25
·072	42·45	6·51	42·33	6·46	42·10	6·35
·073	42·50	6·61	42·38	6·57	42·14	6·45
·074	42·55	6·71	42·42	6·68	42·19	6·55
·075	42·60	6·82	42·47	6·79	42·24	6·65

* If the ratio had been 30 to 1; that is, if the weight of the bob had been 30 times the weight of the rod, the height of the mercury would be 6·25 inches.

These are the proper proportions of the lengths of the rod and of the bob, provided we could depend on the rates of expansion as here assumed; but on this point there will always be some uncertainty. It is seldom that the artist has an opportunity of ascertaining, by direct experiment and trial, the correct expansion of the very substances which he employs; neither indeed is it absolutely necessary in the mercurial pendulum, since any subsequent correction may be readily applied (either additive or subtractive) if, upon trial, it should be found not to be accurately compensated: and this forms one of the great advantages of the mercurial pendulum. If a pendulum, constructed agreeably to the principles here laid down, should be found on trial to go *slower* in *warm* weather, and *faster* in *cold*; it is said to be *under*-compensated, and more quicksilver must be poured into the cylinder. On the contrary, if it should go *faster* in *warm* weather, and *slower* in *cold*, it is said to be *over*-compensated, and some of the quicksilver must be taken out of the cylinder. The *quantity* thus added or subtracted, must be the result of *trial and error*. Care must be taken, after either of these operations, to adjust the bob, or the slider, so that the clock may be restored to its true rate.

41. From what has been stated, it will be evident that it is the *height* of the mercury in the cylinder which must be principally attended to; and consequently, that if it be required to make the bob heavier than usual, it must be done by an increase in the *diameter* of the vessel, and not in the height of the column of mercury*.

42. In the investigation of this problem by M. Biot, in his *Traité de Physique*, Vol. I, page 169, he has satisfied himself with determining the height of a column of mercury, which would, under all temperatures,

* In order to determine the diameter of the cylinder ($=d$), corresponding to any given height ($=h$) of the cylindrical column expressed in inches, and to any given weight ($=w$) of the mercury expressed in ounces, avoirdupois, let us assume the specific gravity of mercury to be 13586: whence we have $w = \frac{13586 \times \cdot 7854}{1728} d^2 . h$. Consequently

$$w = 6\cdot 175\, d^2 . h$$

$$d = \sqrt{\frac{w}{6\cdot 175\, h}} = \sqrt{\frac{w}{h}} \times \cdot 161943$$

Therefore, the weight of the bob, expressed in avoirdupois ounces, being (as is usually the case) previously fixed upon, and the proper height of the column of mercury being ascertained, we may readily determine the diameter of the vessel.

render constant the distance between the point of suspension and the centre of *gravity* of the pendulum: the pendulum being supposed to be suspended by a line *void of weight.* "There is," as he justly observes, "but a trifling difference in the distance between the centre of gravity "of the column of mercury, and the centre of oscillation of such a "pendulum. Therefore, by neglecting the effect of the expansion on "this small interval, it will be sufficient, in order to secure the isochronism "of the oscillations, to preserve the same uniform distance between the "centre of *gravity* of the columnn of mercury, and the axis of suspension." This remark applies more forcibly to the simple pendulum than to the compound pendulum. Nevertheless, we may frequently avail ourselves of it, in order to determine an approximate value of the ratio of the length of the bob to the length of the rod: the method of doing which will be as follows. The distance of the centre of gravity of the compound pendulum is, by § 33, equal to $\frac{wg+w'g'}{w+w'}$; which, by the substitutions there alluded to, will become $\frac{\frac{r}{2}+\left(r-\frac{b}{2}\right)m}{1+m}=\frac{r+(2r-b)m}{2(m+1)}$: and since this value must be the same under all temperatures, we shall have $\frac{r+(2r-b)m}{2(m+1)}-\frac{2r'+(2r'-b')m}{2(m+1)}=0$; whence, by substituting $r(1+t\varrho)$ and $b(1+t\beta)$ for r' and b' respectively, we have $r\varrho+2mr\varrho-b\beta=0$: and consequently $\frac{b}{r}=\frac{\varrho}{\beta}\times\frac{2m+1}{m}$: which is a very simple and easy approximation to the value of x in § 37. In the case stated by M. Biot, m is infinite, and therefore the formula becomes $\frac{b}{r}=\frac{2\varrho}{\beta}$. Whence $b=\frac{2\varrho}{\beta}\times r$: which is not strictly correct. In order to determine r, he assumes $p=\left(r-\frac{b}{2}\right)=r-\frac{\varrho}{\beta}r$; whence $r=\frac{\beta}{\beta-\varrho}\times p$.

43. Mr. Lowe, in his investigation of the problem, in the *Philosophical Magazine* above alluded to, appears also to have assumed the distance between the centre of gravity and the centre of oscillation, as a constant quantity under all temperatures. In the simple and *imaginary* pendulum discussed by him and M. Biot, this distance will not amount to the tenth part of an inch: but, in the ordinary pendulum, and indeed in that which Mr. Lowe has afterwards *actually proposed and described* for use, this distance is much greater, and may be sensibly affected by a change of tem-

perature. Mr. Lowe's formula is $b = \left(2+\frac{1}{27}\right)\frac{\rho r}{\beta}$ which appears to be an empirical one.

44. I have already stated, that in the example given by Mr. Lowe, in the *Philosophical Magazine,* the rate of expansion of the mercury is evidently erroneous (see § 6 and 9): and although the result of his investigation comes out nearly the same as in the example stated in this paper, yet this coincidence arises from a *compensation of errors* in the particular case adduced. With respect to the expansions, it is difficult to decide *which* rates ought, in ordinary cases, to be assumed as the foundation of the mercurial pendulum: the artist must choose for himself amongst a variety of well authenticated results. Even the superior accuracy with which some of these experiments have been conducted, may render the results not quite so proper for general use, without a corresponding attention on the part of the artist. Thus, in the experiments of MM. Dulong and Petit, which were conducted with the greatest care and accuracy, the mercury which they employed was carefully purified, and frequently boiled; a circumstance which may probably cause some difference in the rate of expansion when compared with the mercury which forms the common article of commerce in the shops. As this metal however may now be readily obtained *distilled,* at a small additional expense, I should recommend it to be used in this state. The glass vessel should be previously wiped with a clean rag, and well dried: and should any air bubbles remain attached to the glass, (the formation of which it may be difficult to prevent entirely on pouring in the mercury,) they may be removed by passing a feather round the inside of the vessel.

45. I have also stated, in a preceding part of this paper, that the mercurial pendulum, as described by Mr. Lowe, is liable to some objections. In the first place, the frame of the stirrup is composed of *several detached pieces* of steel, fastened together at the four corners by means of screws; which, from the weight and motion of the pendulum, and the variations of temperature, are liable, occasionally, to become loose, and thus cause a working in those parts destructive of the regularity of its motion. The second objection is to the mode of adjusting the pendulum; which, in Mr. Lowe's plan, (and indeed in all those which I have seen,) is by means of a *fine* screw passing through the head of the stirrup. This fine screw, which is made of steel, is not more than one-tenth of an inch in diameter, and

has about 40 turns in an inch; it passes through a brass nut, only one quarter of an inch thick, *which thus supports the whole weight of the bob.* One twenty-eighth part of a turn of this nut will make a difference of one second per day in the rate of the clock* : but it is very doubtful whether a screw of this sort can be depended upon to the twenty-eighth part of a turn; and it may be justly suspected, that the great weight of the bob, constantly pressing the *steel* threads against the *brass* screw, will cause irregularities which may destroy its accuracy; and that when we imagine we have raised the bob by a small quantity, we may, in fact, have lowered it, through the incorrectness of the screw. These, and other circumstances, have probably induced many persons to leave the adjustment of the pendulum of their clock much further from the truth than they would otherwise wish: since they find, by experience, that there is a risk of a greater error in attempting a superior degree of accuracy. Moreover, the screw of the nut is so fine and formed of so soft a substance, pressing against a comparatively hard one, that it is liable soon to wear away, particularly if the rod does not pass *very freely* through the hole made in the upper part of the stirrup. And it was in fact owing to this very circumstance that I once nearly experienced the loss of a vessel of mercury attached to a pendulum in this way. The stirrup dropt from the rod whilst I was in the act of turning the nut; the threads of which, owing to a slight resistance, were cut by the steel screw, and thus the loss of the whole was endangered. To obviate the recurrence of a similar accident, I should propose that the threads of the screw, which unites the rod to the stirrup, should be *much larger* and deeper; that the screw itself should not be less than two-tenths of an inch in diameter, and that it should not have more than about 30 turns in an inch. The pendulum may be brought nearly to its proper rate by means of this *coarse* screw, and the bob should be then firmly secured in its position; and the pendulum may afterwards be accurately adjusted, either by means of a small sliding weight attached to the rod, or by means of a weight or ball attached to a screw below the bob. Both of these methods are the same in principle, and differ only in the magnitude of the weights, and in the space through which they are made to pass; as will evidently appear from the following investigation.

* The variation in the length of a pendulum vibrating seconds, corresponding to a variation of 1″ per day, is very nearly equal to $\frac{2p}{N} = \frac{78}{86400} = \cdot00090$, or nearly $\frac{1}{1000}$ of an inch: p being the length of the pendulum in inches, and N the number of vibrations which it makes in a day. See § 46.

46. Let SB (fig. 3) be considered as a line, void of weight; to the bottom of which, at B, is attached a heavy body, which I shall denote by w', and which, for the purpose of the present inquiry, I shall consider as infinitely small. The length of this pendulum, which I shall denote by p, will be $\frac{(SB)^2 \times w'}{SB \times w'} = SB$. But, if we annex to any part F of this line, the small weight w, the motion of the pendulum will be accelerated; and the length of the simple synchronous pendulum corresponding thereto, which I shall denote by p', will depend on the position of that weight on the line, and will be expressed by $\frac{(SB)^2 \times w' + (SF)^2 \times w}{SB \times w' + SF \times w}$. Whence, the difference between these two values, or the variation in the length of the simple pendulum, caused by the addition of the small weight w, will be

$$p - p' = \frac{SB \times SF - (SF)^2}{SB \times \frac{w'}{w} + SF}$$

But S B is equal to p, and S F (or the variable distance of the small weight w from the point of suspension) may be denoted by d: whence we have

$$p - p' = \frac{p - d}{p \cdot \frac{w'}{w} + d} \times d$$

If N denote the number of seconds which the pendulum, whose length is p, makes in a day; and N' ($=N+n$) the number of seconds which another pendulum, whose length is p', makes in a day; then, since the lengths of two pendulums are to each other in the reciprocal duplicate ratio of the number of their vibrations, we shall have $p : p' :: \frac{1}{N^2} : \frac{1}{(N+n)^2}$; whence $p' = \frac{N^2}{(N+n)^2} p$: and $p - p' = \left(1 - \frac{N^2}{(N-n)^2}\right) p$. But, if p differs very little from p', as is generally the case, n will be very small, and all its powers above the second may be safely neglected. Consequently we shall have $(p - p)' = \frac{2n\,p}{N}$: and the above formula becomes $\frac{2n}{N} \times p = \frac{p - d}{p \cdot \frac{w'}{w} + d} \times d$.

Whence by making $\frac{w'}{w} = m$, as in § 34; and $\left(1 - \frac{2n}{N}\right)\frac{p}{2} = a$, we have

$$n = \frac{(p-d)\,d}{p\,m + d} \times \frac{N}{2\,p}$$

$$d = a \pm \sqrt{a^2 - \frac{2\,p^2\,m\,n}{N}}$$

In the seconds pendulum N is always equal to 86400; and p may be taken equal to 39 inches, without any material error (see the note in § 35): consequently we have $a = (1 - \cdot 000023148 \times n)\ 19.5$; whence we have

$$n = \frac{(39 - d)\, d}{39\, m + d} \times 1107 \cdot 69 \qquad \text{(F)}$$

$$d = a \pm \sqrt{a^2 - \cdot 035208\, m\, n} \qquad \text{(G)}$$

47. In order to show the application of the first of these formulæ to practical purposes, let us suppose that the rod of the pendulum, from the point of suspension to the centre of oscillation, measures, and is divided into 39 inches; and that it is required to know what effect the sliding weight will have on the rate of the clock, when placed successively at each of those divisions; the sliding weight being only one thousandth part of the weight of the bob. In this case $m = 1000$, and d must be successively taken equal to 1, 2, 3, &c., whence we have the following table.

Distance from axis, in inches.	Variation in the rate per day.	Diff.	Distance from axis, in inches.	Variation in the rate per day.	Diff.	Distance from axis, in inches.	Variation in the rate per day.	Diff.
1	+1″·08		14	+ 9″·94		27	+ 9″·20	
		+1·02			+·28			—·46
2	2·10		15	10·22		28	8·74	
		·97			·23			·51
3	3·07		16	10·45		29	8·23	
		·91			·17			·56
4	3·98		17	10·62		30	7·67	
		·85			·11			·63
5	4·83		18	10·73		31	7·04	
		·79			+·06			·68
6	5·62		19	10·79		32	6·36	
		·74			·00			·74
7	6·36		20	10·79		33	5·62	
		·68			—·06			·79
8	7·04		21	10·73		34	4·83	
		·63			·11			·85
9	7·67		22	10·62		35	3·98	
		·56			·17			·91
10	8·23		23	10·45		36	3·07	
		·51			·23			·97
11	8·74		24	10·22		37	2·10	
		·46			·28			1·02
12	9·20		25	9·94		38	1·08	
		·40			·34			—1·08
13	9·60		26	9·60		39	0·00	
		+ ·34			—·40			

48. By this table it will be seen, that the effect of the slider is, in all cases, to accelerate the clock when placed any where between the axis of suspension and the centre of oscillation; which effect is increased the further it is removed from the axis of suspension, till it arrives at the middle of the rod (considering the length of the rod as the length of the simple synchronous pendulum): after which, if the motion be continued downwards, its operation will be the same, but in an inverse order. At equal distances, therefore, from the middle of the rod (so considered), the effect will be the same whether the slider be placed above or below that point: but, it will be more convenient to place it in the *upper part* of the rod, since it may, in this position, be more readily moved without danger of obstructing the pendulum when in motion.

Owing to the crotch of the pendulum, and the rising board of the clock, it is seldom that we can approach the axis of suspension within eight or nine inches. If therefore we place the slider at nine inches from the centre of the spring of the pendulum, we shall have a range of $10\frac{1}{2}$ inches, to produce an acceleration of $3''$ per day.

It will be seen by the table here given, that the effect produced is not the same at each portion of this interval, but varies considerably with its distance from the axis of suspension. Thus, the interval from the 9th to the 10th inch produces a variation in the rate of the clock equal to $+0''\cdot56$; whilst the interval from the 18th to the 19th inch produces a variation of only $+0''\cdot06$. But, it is very easy, by altering the coarse screw, to throw the adjustment on any portion of the rod in which we may think it most advisable to place the slider; and thus extend or diminish the length of our scale at pleasure. It will readily occur to the practitioner, that, in the primary adjustment of the rate of the clock by means of the coarse screw, it will be desirable, when the slider is placed on the upper part of the rod, to cause the rate to be too slow rather than too fast; since it will be more convenient (in order to obtain the final and complete adjustment) to push *down* the slider, than to force it *upwards*, whilst the pendulum is in motion. And this may very easily be done without stopping the clock, or obstructing the motion of the pendulum.

49. If the slider be continued *below the centre of oscillation*, the values will be still nearly the same (at least in most practical cases) as those given in the table, except that they will be *minus* instead of *plus*: or, in other

words, the effect of the application of the slider to this part of the rod will be to retard the clock; and this effect is increased the *lower* the slider is moved*.

In this case, let us suppose the rod to be continued eight inches below the centre of oscillation, and that the slider is attached to this lower part of the rod. In the mercurial pendulum, the bottom of the cylinder is about four inches below the centre of oscillation; so that the slider will have, in the remaining part, a range of about four inches; which, in that portion of the rod, will produce an effect of about $3''$ per day: whereas, in the upper part of the rod, we have a range of $10\frac{1}{2}$ inches to produce the same effect.

50. The maximum effect, produced by a slider of this kind, on the seconds pendulum, when placed at the middle of the rod, may be determined by the formula

$$n = \frac{19.5}{2m+1} \times 1107.65 = \frac{21599 \cdot 6}{2m+1} \qquad \text{(H)}$$

By which it will be seen that the values of n will be nearly in the direct ratio of the weight of the slider. Thus, if the values, in the table above given, be produced by a slider, whose weight is $\frac{1}{1000}$ part of the weight of the bob, double those values nearly will be produced by a slider of double that weight; or whose weight is $\frac{1}{500}$ of the weight of the bob. If, therefore, we wish to produce a greater effect on the rate of the clock, without altering the range of the slider, we have only to increase the weight of the slider in the ratio required.

51. The most convenient form of the slider will be that of a ring or collar, surrounding the rod of the pendulum; and which, by the help of a small spring, may be made to remain in any position on the rod. The size of this ring will depend on its required weight, and on the diameter of the

* A similar result would be obtained by applying the slider *above the axis of suspension*: but this effect is increased the *higher* the slider is moved: as this operation, however, cannot conveniently be carried into effect with pendulums attached to clocks of the ordinary construction, without encountering other difficulties which it is desirable to avoid, I shall not consider this part of the subject.

rod: it will also differ according to the specific gravity of the metal of which it is composed. If the diameter of the rod be $\frac{1}{4}$ of an inch, a ring of platinum, $\frac{1}{10}$ of an inch thick, and $\frac{3}{20}$ of an inch broad, will weigh above $\frac{1}{5}$ of an ounce: which, if the pendulum weigh 12 pounds, will be about $\frac{1}{1000}$ part of its weight. If the ring be made of silver, its breadth must be double, or $\frac{3}{10}$ of an inch; since its specific gravity is only half that of platinum; and if made of brass, its breadth must be $\frac{4}{10}$ of an inch. The actual effect of the ring on the pendulum may be readily ascertained by trying it at different distances on the rod. The form of a ring of this kind, of its real size (when made of brass) is seen in figure 5.

52. The equation (G) will be useful, if we wish to determine the exact distances d, at which the slider ought to be placed from the axis of suspension, in order to increase the rate of the clock n seconds per day. Thus, if it were required to place the slider above mentioned, in such a point that the rate of the clock may be increased $10''$ per day, we shall have $d = 19{\cdot}495 \pm \sqrt{(19{\cdot}495)^2 - 352{\cdot}083} = 14{\cdot}2$ inches, and $24{\cdot}8$ inches. But as these values may be readily found by the table above given, we shall seldom have occasion to advert to this formula.

53. In these investigations I have taken for granted that the centre of oscillation and the centre of gravity of the pendulum are in the same point: which however in fact can never be the case. The values above given are therefore approximations only: but they are approximations exceedingly near the truth, and will answer every practical purpose. The approximation depends on the great weight of the bob compared with the weight of the rod and the slider: and, in the mercurial pendulum, the former exceeds the latter in a considerable degree. Those who are desirous of pursuing the subject more fully may consult the excellent papers of M. Prony in the *Connaissance des Tems* for 1817 and 1820, and in his *Léçons de Méchanique*.

In the first of the works here mentioned, M. Prony has given the following general and correct view of the effect of a small weight attached to an oscillating body. "If we conceive that the weight be placed above the axis of suspension, in such a position that the centre of gravity be exactly in that axis, "the length of the simple synchronous pendulum (or the duration of an oscillation) will be infinite. But if, proceeding from this upper point, we lower

"the weight continually, the length of the simple synchronous pendulum (or "the duration of an oscillation of the system) will diminish more and more, "until the weight reaches a certain point *below* the axis of suspension (which "point is nearly half way between the axis of suspension and the centre of "oscillation), where it produces a minimum in the duration of an oscillation "of the system. After this, the weight being made continually to descend, "the length of the simple synchronous pendulum will augment as the weight "descends: that is, the length and the descent will increase, and become "infinite together."

54. M. Prony states that this method of regulating a clock, by means of a slider attached to the rod, was *discovered* by M. Petremand, one of the artists employed by M. Breguet. But, it seems to have escaped the recollection of this distinguished mathematician that this very method was not only pointed out, but was actually employed by M. Huygens: and, in his *Horologium Oscillatorium,* Part iv. Prop. 23, he has investigated the effects produced by a slider of this kind. In the plate which contains a drawing of Mr. Huygens's clock, the slider is seen attached to the rod: and the same is observed in Mr. Graham's mercurial pendulums. It is also distinctly adverted to in Dr. Derham's *Artificial Clock-maker.* Why it was discontinued, I have not been able to ascertain: unless it were owing to the introduction of gridiron pendulums, whose peculiar construction rendered the application of the slider impracticable, or unavailable. The recent encouragement given to mercurial pendulums may revive and probably continue the use of this ingenious and commodious invention. M. Prony recommends that the slider should be formed of platinum, which, on account of its great specific gravity, is well adapted to this purpose: but, should this metal be considered too expensive, silver, or even brass, will answer as well.

55. There is another mode of regulating the rate of a clock, depending on the same principles as those above mentioned, which may sometimes be conveniently practised. In this case, the small weight (instead of being constantly the same, and variable in position) is *fixed* in one position to the rod, and increased or diminished at pleasure. The point at which any additional weight will have the greatest effect on the rate of the clock, is about the middle of the rod: or, in the seconds pendulum, about 19½ inches from the spring. At this point, a small conical tube may be fixed, as at fig. 6, in which a few small leaden shot may be placed: which, when the clock is adjusted nearly to its proper rate, may be increased or diminished according to the circumstances of the

case. Of the shot, which is called No. 10 by the gunsmiths, about 1600 form an ounce: and one of these shot, if the bob should weigh 12 pounds, will cause an alteration in the daily rate of the clock of 0″,035. Even one of these may be subdivided; so that, by this method, we may regulate the clock to a very great degree of accuracy*.

56. In Plate VI., fig. 4., is a drawing of the mercurial pendulum, as constructed in the manner proposed in this paper. The rod SF is made of steel and perfectly straight; its form may be either cylindrical, of about a quarter of an inch in diameter, or a flat bar $\frac{3}{8}$ of an inch wide and $\frac{1}{8}$ of an inch thick: its length from S to F, that is, from the bottom of the spring to the bottom of the rod at F, should be 34 inches†. The lower part of this rod, which passes through the top of the stirrup, and about half an inch above and below the same, must be formed into a *coarse* and *deep* screw, about two tenths of an inch diameter, and having about 30 turns in an inch. A steel nut with a milled head must be placed at the end of the rod, in order to support the stirrup; and a similar nut must also be placed on the rod *above* the head of the stirrup, in order to screw firmly down on the same, and thus secure it in its position, after it has been adjusted *nearly* to the required rate. These nuts are represented at B and C. A small slit is cut in the rod, where it passes through the head of the stirrup, through which a steel pin E is screwed in order to keep the stirrup from turning round on the rod. The stirrup itself is also made of steel, and the side pieces should be of the same form as the rod, in order that they may readily acquire the same temperature. The top of the stirrup consists of a flat piece of steel, shaped as in the drawing, somewhat more than $\frac{3}{8}$ of an inch thick. Through the middle of the top (which at this part is about I inch deep) a hole must be drilled sufficiently large to enable the screw of the rod to pass *freely*, but without *shaking*. The inside height of the stirrup, from A to D, may be $8\frac{1}{2}$ inches; and the inside width, between the bars, about 3 inches. The bottom piece should be about $\frac{3}{8}$ of an inch thick, and hollowed out nearly $\frac{1}{4}$ of an inch deep so as to admit the glass cylinder freely. This glass cylinder should have a brass or iron cover G, which should fit the mouth

* Since this was written I have seen a mode, somewhat similar, adopted by HENRY BROWNE, Esq. of Portland Place; who has very ingeniously and accurately regulated one of his mercurial pendulum clocks, by means of small pieces of sheet lead, which are loosely placed on the top of the glass cylinder; and thus readily increased or diminished at pleasure.

† The whole of this length is not preserved in the plate, but is represented as *broken* in the middle.

of it freely, with a shoulder projecting on each side, by means of which it should be screwed to the side bars of the stirrup, and thus be secured always in the same position. This cap should not *press* on the glass cylinder so as to prevent its expansion. The measures above given may require a slight modification, according to the weight of the mercury employed, and the magnitude of the cylinder: the final adjustment however may be safely left to the artist. Some persons have recommended that a circular piece of thick plate glass should float on the mercury, in order to preserve its surface uniformly level*. The part at the bottom, marked H, is a piece of brass fastened with screws to the front of the bottom of the stirrup; through a small hole in which a steel wire, or common needle, is passed in order to indicate (on a scale affixed to the case of the clock) the arc of vibration. This wire should merely rest in the hole, whereby it may be easily removed, when it is required to detach the pendulum from the clock: in order that the stirrup might then stand securely on its base. One of the screw holes should be rather larger than the body of the screw, in order to admit of a small adjustment, in case the steel wire should not stand exactly perpendicular to the axis of motion. The scale should be divided into *degrees*, and not *inches*: observing that with a radius of 44 inches (the estimated distance from the bend of the spring to the end of the steel wire) the length of each degree on the scale must be 0.768 inch.

57. A mercurial compensation pendulum may be formed, without the intervention of the *glass* cylinder, by having the cylinder made of *steel* or *iron*, and the top or handle constructed in the same manner as the top of the stirrup, so as to receive the screw of the rod. In this case, the value of m remaining as before, we shall have the value of β (if the cylinder be made of steel) equal to $\cdot 00010010 - \cdot 00001272 = \cdot 00008738$: whence $k = \frac{\rho}{\beta} = \frac{636}{8738} = \cdot 07279$: consequently $e = \cdot 5440$ and $f = \cdot 4551$. The first approximation of x will

* The variation produced in the height of the column of mercury (supposed to be $6\frac{1}{2}$ inches high) by an alteration of $\pm$ 16° in the temperature, will be only $\pm \frac{1}{100}$ of an inch: or in other words $\frac{1}{100}$ of an inch will be the total variation from its *mean* state, by an alteration of 32° in the temperature. It is therefore probable that, in most cases of moderate alteration in the temperature, the *centre* only of the column of mercury is subject to elevation and depression, whilst the exterior parts remain attached to the sides of the glass vessel. It was with a view to obviate this inconvenience that Mr. BROWNE (I believe) first suggested the piece of floating glass.

therefore be $= \cdot 1517$; and the second approximation $= \cdot 1548$: whence $r = 42\cdot 48$, and $b = 6\cdot 59$. The column of mercury therefore must, in this case, be above a quarter of an inch higher than when a glass vessel is used.

58. I have already stated that an economical pendulum for a seconds clock might be constructed by means of a leaden bob attached to a wooden rod: and I shall now show the mode of determining the relative lengths of these two substances. I have preferred lead to zinc on account of its inferior price, and the ease with which it may be formed into the required shape: and, as there is no considerable difference in their rates of expansion, it is equally applicable to our purpose. If we take the rate of expansion of deal to be $\cdot 0000022685$, and that of lead to be $\cdot 0000159200$, as given in the table, we shall have $\frac{\ell}{\beta} = \frac{22685}{159200} = \cdot 1425 = k$. Now, if we assume the weight of the bob to be 100 times the weight of the rod (which will probably be the case) we shall by means of the equation (D) have $x = \cdot 2874\ (1 - \cdot 2926 + \cdot 3832) = \cdot 313$: which, being substituted for x in the equation (C), will give $r = 45\cdot 75$ and $b = 14\cdot 30$ *.

59. This being premised, let us assume $b = 14\frac{1}{3}$ inches, and make the cylinder of any diameter, according to the weight proposed. The construction then of such a pendulum will be as follows. Take a deal rod, of a convenient size, but not less than 46 inches in length, the lower part of which ($14\frac{1}{2}$ inches long) should be made cylindrical, about $\frac{5}{8}$ of an inch in diameter: or the whole rod may be of this size and shape. Procure a leaden weight or cylinder to be cast, with a hole through the centre which will *freely* admit the cylindrical end of the rod, and of such a length that when put in the lathe it may be reduced to the required standard of 14·3 inches †, and to the required weight ‡.

* If the bob were made of zinc, the length of the cylinder would be only 13·88 inches.

† It will be more convenient to have this cylinder too long rather than too short: since we may readily diminish the length of it, if, on trial, it should be found that the pendulum is *over*-compensated.

‡ The following are the respective weights of a leaden cylinder 14·3 inches long, and having a hole, through the centre, equal to $\frac{5}{8}$ of an inch in diameter.

Diameter of cylinder.		Weight.
$1\frac{1}{4}$ inch.	=	6·56 lbs.
$1\frac{1}{2}$	=	9·73
$1\frac{3}{4}$	=	13·47
2	=	17·80
$2\frac{1}{4}$	=	22·70

The lower end of the rod should be formed into a screw, to which a *wooden* nut may be fitted, in order to adjust the pendulum *nearly* to the given rate: and the final adjustment may be made by means of a slider, as above described. A pendulum of this kind will cost but a few shillings, and will answer many useful purposes, as I have found by experience. If the expansion of the rod could be depended on, it would be as accurate as any other. In this construction I have not considered the expansion of that part of the *spring* which is below the axis of motion. The effect of this may be taken into account, at the final adjustment of the pendulum.

60. In all the wooden pendulums which I have seen, the bob has been made of a *lenticular* shape, and oftentimes been fastened to the rod by means of *a pin passed through its centre;* and therefore not constructed with any view to compensation. This shape was, I presume, originally adopted with a view to overcome the *resistance of the air,* and thus reduce the maintaining power of the clock. But it is well known that the air, as a resisting medium, has no sensible effect on the rate of the clock: neither is the rate affected by the shape of the bob; we may therefore choose that shape which will best answer other useful purposes.

61. A variation, however, in the *density of the air* will cause a slight variation in the rate of the clock: and although I am not at present aware that any mode has been proposed for compensating the whole of the errors which arise from that source, yet the amount of them may be determined in the following manner.

Let the specific gravity of water (compared with air considered as unity, when the barometer is 29·27 inches high, and Fahrenheit's thermometer at 53°) be denoted by w. Let us also suppose that air expands $\frac{1}{480}$ of its bulk for every degree of the thermometer*. Let s be the specific gravity of the compound pendulum compared with water considered as unity. Let the height of the barometer be denoted by b; and the height of the thermometer by t. Then will the diminution of gravity, on account of the density of the atmosphere, (the absolute force of gravity *in vacuo* being considered as unity,) be denoted by

* This is the value given by MM. Gay-Lussac and Biot: but other writers are by no means agreed on the correct value. See *Zeitschrift für Astronomie*, vol. ii. p. 6. M. Laplace has recently assumed $\frac{1}{450}$. See his *Système du Monde*, page 89. Edition 1824.

$$D = \frac{b}{29.27\left(1+\frac{t-53^\circ}{480}\right)w.s}$$

$$= \frac{b}{29{\cdot}27\,[1+(t-53^\circ){\cdot}0020833]\,w.s}$$

and the true accelerative force, at that temperature and under that pressure, will be only $(1-D)$. And, at any other temperature $(=t')$ and at any other pressure $(=b')$ we shall have

$$D' = \frac{b'}{29{\cdot}27\,[1+(t'-53^\circ){\cdot}0020833]\,w.s}$$

and the true accelerative force, in such case, equal to $(1-D')$.

62. Now the squares of the number of vibrations, made in the same time by two pendulums of the same length, are to each other as the accelerative forces of gravity by which they are impelled. If therefore we denote the number of daily vibrations, corresponding to $(1-D)$, by N; and the number of daily vibrations, corresponding to $(1-D')$, by $N'=N+n$; we shall have $N^2 : (N+n)^2 :: (1-D) : (1-D')$. Whence, in the seconds pendulum, since N' generally differs very little from N, we obtain for the daily variation in the rate of the pendulum $n = \frac{D-D'}{1-D} \times 43200$: or sufficiently near for all useful purposes

$$n = (D - D')\,43200.$$

If we assume the height of the barometer equal to 29·27 inches, and the thermometer at 53° of Fahrenheit, we shall have $D = \frac{1}{w.s}$; and at any other height of the barometer $(=b')$, and at any other temperature $(=t')$, we shall have

$$n = \frac{1-\frac{b'}{29{\cdot}27}+(t'-53^\circ){\cdot}0020833}{1+(t'-53^\circ){\cdot}0020833} \times \frac{43200}{w.s}$$

63. In order to apply these formulæ to the case of the mercurial pendulum, we shall first ascertain what effect is produced on the daily rate of a clock with a mercurial pendulum, by an alteration in the density of the air occasioned by a change in the *thermometer* only. For this purpose, we shall assume $w.s = 10000$: and let us suppose $t = 32^\circ$ and $t' = 70^\circ$, and that the barometer remains stationary at 29·27 inches. Consequently we have $D = {\cdot}0001045751$, and $D' = {\cdot}0000965795$; whence $n = 0,''3454$.

Thus it appears that an increase of temperature of 38° of Fahrenheit's thermometer (viz. from the freezing point to 70°) will cause an acceleration

(arising from the difference in the *density* of the atmosphere only) of about one third of a second, in the daily rate of a clock with a mercurial pendulum: or about 0″,009 for every degree of the thermometer. In some countries the probable variation of the thermometer may exceed 100° in the course of a year: but the occasional differences arising from this source are corrected by the practical mode of compensation alluded to in page 399; and may therefore be considered as not materially affecting the *mercurial* pendulum.

64. Let us now see the effect produced by an alteration in the state of the *barometer*. For this purpose we shall suppose the thermometer stationary at 53°: and that the barometer stands in the first instance at 31 inches, but afterwards falls to 28 inches: or that $b = 31$, and $b' = 28$. In this case we have $D = \cdot 0001059105$ and $D' = \cdot 0000956611$: consequently $n = 0'',4428$. Whence it appears that a *fall* of one tenth of an inch in the barometer will cause an *increase* in the rate of the clock, equal to 0″,015: and on the contrary, that a *rise* of one tenth of an inch will cause a *diminution* of the same amount.

But when the density of the air diminishes, the *arc of vibration* probably increases: let us therefore endeavour to ascertain what influence this would have on the going of the clock; and whether it may not counteract the effect just mentioned. For this purpose let us assume the semiarc of vibration, in its mean state, equal to x; and let x' denote the semiarc of vibration, in its altered state. Then the variation in the rate of the seconds pendulum will, in consequence of this alteration in the arc of vibration, be denoted by

$$n = 10800 . (\cos x' - \cos x).$$

In most pendulums which I have seen, the semiarc of vibration does not differ much from 2°; consequently the variation in the rate of the clock produced by an increase of 1′ in the semiarc of vibration, would be $n = 10800 \times - \cdot 0000102 = - 0'',11$. Now, if this increase in the semiarc of vibration* were produced by a fall of $\frac{3}{4}$ of an inch in the barometer, it is evident that these two operating causes on the rate of the pendulum would be exactly counterbalanced. Whether the variations in the height of the barometer do produce the effect here alluded to, or in what other degree, I am unable at present to decide, for want of sufficient data; neither am I aware of any experiments that have been conducted with a view to elucidate this point: but from some recent observations which I have made, I am induced to think that

* This increase in the semiarc would not much exceed the $\frac{1}{100}$ part of an inch: a quantity scarcely perceptible by the eye, on the ordinary scale attached to clocks in general.

these effects are in some measure counterbalanced. The subject however is certainly deserving of more particular attention from the practical astronomer.

In these examples I have supposed the specific gravity of the compound mercurial pendulum, compared with air considered as unity, to be denoted by 10000, more for the sake of a practical example in round numbers than as expressing an opinion of its true value. If greater accuracy is required, the practical astronomer will determine these elements for himself, according to the substances made use of in the construction of the pendulum under consideration.

Linear Expansion of various Substances for *one degree* of FAHRENHEIT'S Thermometer.

No.	Substance.		Expansion.	Authors.	Reference.
1	WHITE DEAL		·00000 22685	Captain Kater	Nich. Journ. vol. xx.
2	WHITE DEAL		28444	Dr. Struve	Dorpat Obs. vol. I.
3	GLASS	Bar. tube	43100	General Roy	Phil. Trans. 1785.
4	GLASS	solid rod	44900	Ditto	Ditto.
5	GLASS	Eng. flint	45092	Lavoisier and L.	Biot, Traité de Phys.
6	GLASS		46300	Smeaton	Phil. Trans. 1754.
7	GLASS		47887	Dulong and P.	Journ. de l'Ec. Pol.
8	GLASS	with lead	48444	Lavoisier and L.	Biot, Traité de Phys.
9	GLASS	tubes without lead	48651	Ditto	Ditto.
10	GLASS	tubes without lead	49866	Ditto	Ditto.
11	GLASS	tubes without lead	50973	Ditto	Ditto.
12	GLASS		61495	Berthoud	Hist. de la Mes. du Tems.
13	PLATINA		47583	Borda	Base du Syst. Met.
14	PLATINA		48100	Brande	Syst. of Chem.
15	PLATINA		49121	Dulong and P.	Journ. de l'Ec. Pol.
16	IRON	cast	61700	General Roy	Phil. Trans. 1785.
17	IRON	cast	61800	Lavoisier	Dr. Young's Lect.
18	IRON		63333	Borda	Base du Syst. Met.
19	IRON		65668	Dulong and P.	Journ. de l'Ec. Pol.
20	IRON	soft	67803	Lavoisier and L.	Biot, Traité de Phys.
21	IRON	wire	68613	Ditto	Ditto.
22	IRON	bar	69844	Hasslar	Amer. Phil. Trans.
23	IRON		69907	Smeaton	Phil. Trans. 1754.
24	IRON	nealed	74000	Muschenbroek	Dr. Young's Lect.
25	IRON	hardened	74389	Berthoud	Hist. de la Mes. du Tems.
26	IRON	hammered	77000	Muschenbroek	Dr. Young's Lect.
27	IRON	soft	77365	Berthoud	Hist. de la Mes. du Tems.

Linear Expansion of various Substances for *one degree* of Fahrenheit's Thermometer.

No.	Substance.		Expansion.	Authors.	Reference.
28	Steel	not temp[d].	·0000 059305	Lavoisier and L.	Biot, Traité de Phys.
29		not temp[d].	059975	Ditto	Ditto.
30		blistered	062500	Roy	Phil. Trans. 1795.
31		rod	063596	Ditto	Ditto, 1785.
32		blistered	063900	Smeaton	Ditto, 1754.
33			064200	Lavoisier	Dr. Young's Lect.
34			066100	Troughton	Nich. Journ. vol. ix.
35		nealed	068000	Musschenbroek	Dr. Young's Lect.
36		hard	068056	Smeaton	Phil. Trans. 1754.
37		soft	068438	Berthoud	Hist. de la Mes. du Tems.
38		tempered	068866	Lavoisier and L.	Biot, Traité de Phys.
39		hammered	073398	Berthoud	Hist. de la Mes. du Tems.
40		ditto	076000	Musschenbroek	Dr. Young's Lect.
41		tempered	076373	Berthoud	Hist. de la Mes. du Tems.
42	Gold	soft	081332	Berthoud	Hist. de la Mes. du Tems.
43			086197	Lavoisier and L.	Biot, Traité de Phys.
44		wire	093235	Berthoud	Hist. de la Mes. du Tems.
45	Copper	hammered	094444	Smeaton	Phil. Trans. 1754.
46			106000	Musschenbroek	Dr. Young's Lect.
47			106129	Berthoud	Hist. de la Mes. du Tems.

Linear Expansion of various Substances for *one degree* of FAHRENHEIT'S Thermometer.

No.	Substance.		Expansion.	Authors.	Reference.
48	BRASS		·0000 095456	Dulong and P.	Journ. de l'Ec. Pol.
49			098200	Captain Kater	Phil. Trans. 1819.
50			098888	Borda	Base du Syst. Met.
51			099590	Captain Kater	Phil. Trans. 1818.
52		cast	101850	Captain Sabine	Ditto, 1821.
53		scale	103077	Roy	Ditto, 1785.
54			103706	Lavoisier and L.	Biot, Traité de Phys.
55		cast	104166	Smeaton	Phil. Trans. 1754.
56			104400	Com. on W. and M. May 28, 1821.	Mean of sev. exp.
57		bars	104850	Hasslar	Amer. Phil. Trans.
58			104984	Lavoisier and L.	Biot, Traité de Phys.
59		rod	105155	Roy	Phil. Trans. 1785.
60		trough	105271	Ditto	Ditto
61			106666	Troughton	Nich. Journ. vol. ix.
62		wire	107407	Smeaton	Phil. Trans. 1754.
63			120000	Musschenbroek	Dr. Young's Lect.
64			120015	Berthoud	Hist. de la Mes. du Tems.
65	SILVER		106038	Lavoisier and L.	Biot, Traité de Phys.
66			118031	Berthoud	Hist. de la Mes. du Tems.
67	PEWTER, fine		126852	Smeaton	Phil. Trans. 1754.
68	TIN	grain	137963	Ditto	Ditto
69			158698	Berthoud	Hist. de la Mes. du Tems.
70	LEAD		159259	Smeaton	Phil. Trans. 1754.
71			191429	Berthoud	Hist. de la Mes. du Tems.
72	ZINC		163426	Smeaton	Phil. Trans. 1754.
73		hammered	172685	Ditto	Ditto

Cubical Expansion of *Mercury* for 1° of Fahrenheit's Thermometer.

No.	Cub. Expan.	Authors.	Remarks.	
74	·000 0829180	Dom. Casbois	Quoted by Dulong and Petit.	
75	0847500	Lalande and Delisle		
76	0855000	Cotte	Ditto, by Dr. Young: probably in glass.	
77	0857339	Dulong and P.	*Apparent.*	$\frac{1}{6480}$
78	0860126	Lav. and Laplace	As corrected by Dulong and Petit.	
79	0863778	G. Lowe	Phil. Mag. 1819.	$\frac{1}{6459}$
80	0869565	Com. of Roy. Soc.	Phil. Trans. 1777.	
81	0872144	Schuckburgh	Ditto, without allowing for expan. of glass.	
82	0880572	Lav. and Laplace	Incorrect. See No. 78.	$\frac{1}{6300}$
83	0900000	Cavallo	Phil. Trans. 1781.	
84	0917000	Lichtenberg	As quoted by Dr. Y. mean of several exper.	
85	0920000	Achard	Ditto, in glass.	
86	0941610	Roy	As quoted by Dulong and Petit.	
87	0947593	Ditto	2nd set of Exp. Phil. Trans. 1777.	
88	0970000	Hallstroem	As quoted by Dr. Young.	
89	0973703	Roy	1st set of Exp. Phil. Trans. 1777.	
90	0986805	Deluc	Phil. Trans. 1777. See No. 101.	
91	0992060	Ditto	As quoted by Dulong and Petit.	
92	1001001	Dulong and P.	*Absolute.*	$\frac{1}{5550}$
93	1006077	Lav. and Laplace	As corrected by Dulong and Petit.	$\frac{1}{5522}$
94	1007963	Roy	3rd set of Exp. Phil. Trans. 1777.	
95	1010101	Hallstroem	As quoted by Dulong and Petit.	
96	1011032	Shuckburgh	P. T. 1777, allowing for the expan. of glass.	
97	1026528	Lav. and Laplace	Incorrect. See No. 93.	$\frac{1}{5412}$
98	1028805	Schuck. Lav. & Lap.	As quoted by Dulong and Petit.	
99	1030000	Deluc and Lap.	As quoted by Dr. Young.	
100	1040000	Lord Cavendish	Ditto, mean of Shuckburgh's Exper.	
101	1041500	Deluc	As corrected by Roy.	
102	1048200	Lord Cavendish	As quoted by Dulong and Petit.	
103	1080000	Roy	Probably a typographical error.	
104	1098500	Rosenthall	Quoted by Dr. Young.	
105	1111111	Dalton	Quoted by Dulong and Petit.	
106	1224947	Berthoud	Hist. de la Mesure du Tems.	

Since page 416 was printed off, the following errata have been discovered.

No.	3	*for*	43100	*read*	43119
	4	. .	44900	. . .	44881
	16	. .	61700	. . .	61632
	21	. .	68130	. . .	68613
	23	. .	69900	. . .	69907

XXXVI. *Subsidiary Tables for facilitating the computation of Annual Tables of the apparent places of forty-six principal fixed Stars, computed by order of the Council of this Society: to which is prefixed a statement of the Formulæ employed, and Elements adopted in their Construction. Drawn up by* J. F. W. HERSCHEL, *Esq. M.A. F.R.S. Foreign Secretary.*

Read December 12, 1823.

THE corrections to be applied to the mean places of the stars, to obtain their apparent ones at any given time, arise, as is well known, from their own proper motions in the heavens, and from the effects of precession, nutation, and aberration; and when analytically expressed, consist of a variety of terms, depending on the time elapsed since a given epoch, on the sun's longitude, that of the moon, and of the node of her orbit; and are stated with more or less simplicity by most writers on astronomy. In particular, they have undergone a most elaborate examination by Professor BESSEL, who has brought their theory to that degree of perfection, that little remains to be added to it. For this, and for a history of the successive refinements it has undergone, the reader is referred to the seventh section of the *Fundamenta Astronomiæ* of that eminent astronomer, page 67; to the *Zeitschrift für Astronomie*, vol. vi. page 169; and, more recently, to SCHUMACHER's *Astronomische Nachrichten*, numbers 4 and 34. Our object at present is by no means to add any thing to this theory, but simply to point out the construction of the following tables, their use, and the formulæ from which they have been computed. We shall therefore consider in their order the several corrections, with this view.

1. *Proper Motion.* 2. *Precession.*

The proper motions of the stars, and the mean effect of solar, lunar, and planetary precession, being each uniform during the small portions of time for which our tables are serviceable, may all be included in a

single expression, either in right ascension or declination. Their joint effect is generally known by the name of *variation,* and is given directly by observation. In fact, the proper motions of the stars are nothing else than the differences between their observed places, corrected for all the known periodical inequalities, and for the most probable value of their annual precession at the beginning and end of the year: and are therefore mere results of calculation, being found by subducting the *computed* precession from the *observed* variation. Calling then V and v the observed annual variations in right ascension and declination respectively, and t the time, in years and decimals of a year, from a given epoch (the commencement of the year) $+ V.\,t,$ and $+ v.\,t,$ will be the proportional variations for that time, and must be *added to* the mean place at the beginning of the year, or *subtracted from* the apparent place at the time of observation. We shall adhere to the former mode. The values of V and v in these tables are taken from the catalogue affixed to the *Nautical Almanac* for 1824.

3. *Aberration.*

The effects of aberration, depending on the motion of the spectator, are diurnal, annual, and systematic; the first has no effect in *declination* when a star is observed on the meridian, because then the motion of the observer is parallel to the tangent of a circle of declination, passing through the star. Its whole effect at that moment (and that is the only moment concerned here) is in right ascension. The diurnal velocity of the spectator being equal to the equatorial velocity × cosin of latitude; the aberration in Æ of an equatorial star will be

$$\frac{\text{Equatorial velocity}}{\text{Velocity of light}} \times \text{cos lat.}$$

and therefore that of a star, whose declination is δ, will be

$$\frac{\text{Equatorial velocity}}{\text{Velocity of light}} \times \text{cos lat.} \times \text{sec. decl. } (\delta).$$

The numerical co-efficient is no more than $0^s{,}0208$, when reduced to seconds of time.

This expression of the diurnal aberration in Æ therefore differs for every particular star, and for every different geographical situation (in latitude) of the observer. Being however always the same for the same star at the same

place, we need take no account of it here, as it must be regarded only in the light of a reduction from one observatory to another, to be applied as occasion may require in the form

$$a \times (\cos L - \cos L') \times \sec \delta,$$

L and L′ being the latitudes of the two observatories.

The systematic aberration (or that arising from the motion of the solar system in space) affects, in like manner, each star with a certain correction, which remains constant so long as the solar motion remains uniform. Mr. Pond has well remarked, too, that an *uniform acceleration* or *retardation* of the sun's velocity would produce an *uniform* increase or diminution in the constant of systematic aberration, and its effect would be to displace all the stars with an uniform motion. Its influence, therefore, would be confounded with their proper motions. We have, however, no certain ground to attribute any variable motion to the sun. Each star, then, is indeed seen by us out of its real place, but by a constant quantity, and one, of which we shall probably for ever remain ignorant.

The annual aberration alone remains. It may be regarded as consisting of two portions; one, which depends on the place of the sun's perigee, is invariable for each particular star in all places; and which, being of course necessarily included in the mean places as determined by observation, *ought* not to be again taken account of: the other, independent of this element, whose form, if we take account of only the first powers of very minute quantities, is, in Æ and Dec. as follows,

$$\left.\begin{aligned} Æ &= -\mu\{\sin \odot . \sin \alpha + \cos \odot . \cos \alpha . \cos \omega\} \times \sec . \delta \\ D &= -\mu\{\sin \odot . \cos \alpha . \sin \delta - \cos \odot (\sin \alpha . \cos \omega . \sin \delta - \sin \omega . \cos \delta)\} \end{aligned}\right\} (1)$$

μ representing the constant of aberration, α and δ the right ascension and declination of the star, ω the obliquity of the ecliptic, and $\odot$ the sun's true longitude.

The constant, μ, in the following tables is taken at 20″,5 of *space*, or 1ˢ,36667 of *time*. Bessel, in his reduction of Bradley's observations (*Fundamenta Astronomiæ*), has used the coefficient 20″,255; but there is every reason to believe this a little below the truth. He himself has called its exactness in question in the same valuable work; and by a collation of Bradley's observations (*Fundamenta*, pp. 112—123) has deduced the following results:

By 207 differences of Æ between Sirius and α Lyræ	$\mu = 20'',255 + 0'',6247 = 20'',8797$
By 200 ditto ditto between Procyon and α Aquilæ	$\mu = 20'',255 + 0'',0466 = 20'',3016$
By 117 observed right ascensions of Polaris	$\mu = 20'',255 + 0'',5001 = 20'',7551$
By 64 observed zenith distances of γ Draconis with the zenith sector	$\mu = 20'',255 + 0'',5423 = 20'',7973$

Each of these independent determinations indicates the necessity of an augmentation in the value of μ. If we take the mean, we find $\mu = 20'',68$.

The results of M. LINDENAU's labours correspond very nearly with the mean result here deduced. From a comparison of 810 observations of the right ascension of *Polaris* by BRADLEY, MASKELYNE, POND, and BESSEL, he obtained $\mu = 20'',6096$. (*Zeitschrift für Astronomie*, vol. i. p. 65.)

The observations of Dr. BRINKLEY confirm the necessity of an augmentation. By 166 observations of six circumpolar stars, he deduces (*Philosophical Transactions*, 1819) 20'',80 for the value of μ, exceeding even the result of BESSEL's computation; but recently, in a very elaborate paper printed in the *Transactions of the Royal Society* for 1821, he has reduced this value considerably, and assigned as his final result from no less than 2633 observations of a great multitude of stars, 0'',115 as the augmentation; or 20'',37 as the true value of μ.

This determination is doubtless to be preferred to the result of BRADLEY's observations, yet these cannot be entirely waved. A quantity, intermediate between them, will probably be nearer than either, and though 20'',5 is perhaps a very little too large, it may, in the present state of our knowledge, be assumed without apprehending an error of a tenth of a second either way.*

The expressions (1) are not however in the form best adapted to numerical computation, when many observations of the same star are to be reduced. To

* Since this was written, and the tables computed, the third volume of Mr. STRUVE's Dorpat observations has come to hand; in which, from a series of 693 observations of circumpolar stars, made with the utmost care, he assigns 20'',34929, or 20'',35, as the value of the constant of aberration. The admirable coincidence of this with Dr. BRINKLEY's result, is a strong proof, if any were wanted, of the accuracy of both observers. If we assign to all the determinations a weight proportional to the number of observations on which they rely, we shall have

bring them into such a form, a well-known trigonometrical transformation must be used. If we assume

$$\text{A}.\sin\theta + \text{B}.\cos\theta = \text{M}.\sin(\theta + \text{N}) \qquad (2)$$

we shall readily see that M and N must be such, that

$$\left.\begin{aligned} \tan \text{N} &= \frac{\text{B}}{\text{A}} \\ \text{M} &= \frac{\text{B}}{\sin \text{N}} = \frac{\text{A}}{\cos \text{N}} \end{aligned}\right\} \qquad (3)$$

To apply this to the expressions in (1), we have $\theta = \odot$, and (for the aberration in Æ)

$$\text{A} = -\mu.\sin\alpha.\sec\delta;$$
$$\text{B} = -\mu.\cos\omega.\cos\alpha.\sec\delta;$$

Hence we find

$$\left.\begin{aligned} \text{Aberration in Æ} &= \text{M}.\sin(\odot + \text{N}) \\ \tan \text{N} &= \cos\omega.\operatorname{cotan}\alpha \\ \text{M} &= -\mu.\frac{\sin\alpha}{\cos\delta.\cos\text{N}} \end{aligned}\right\} \qquad (4)$$

and similarly

$$\left.\begin{aligned} \text{Aberration in declination} &= m.\sin(\odot + n) \\ \tan n &= -\cos\omega.\tan\alpha + \sin\omega.\sec\alpha.\operatorname{cotan}\delta \\ m &= -\mu.\frac{\cos\alpha.\sin\delta}{\cos n} \end{aligned}\right\} \qquad (5)$$

ω representing 23° 27′ 40″, the mean obliquity of the ecliptic for 1830.

M. Bessel in the *Fundamenta Astronomiæ*, page 128, has investigated with great care the analytical values of the solar and lunar nutation; but the

---

Bradley,	20″,68	by 588 obs	=	12159″,84
Lindenau,	20,61	810	=	16694,10
Brinkley,	20,37	2633	=	53634,21
Struve,	20,35	693	=	14102,55
		4724		96590,70
			Mean =	20,447

Differing only 0″,053 from the value in the text.—For so minute a difference as this, it was not thought worth while to re-compute the table; especially as future determinations may be expected still to alter the value of μ by a few hundredths of a second.

reduction of his results to numbers pre-supposes an exact knowledge either of the moon's mass, or of the maximum effect of her attraction, in altering the obliquity of the ecliptic during one revolution of her node. The former may indeed be determined from other phænomena, but imperfectly: and it is undoubtedly preferable to have recourse to the other mode, and deduce the maximum of nutation from a series of observations of the places of proper stars. The observations of BRADLEY continued only through half a period of the moon's node not affording data sufficient, this point was left undetermined by M. BESSEL. It follows, however, from his investigations, that if we adopt 9″,6480 for the approximate major semiaxis of the ellipse of nutation, (or the maximum effect in question), as stated by Baron DE ZACH, the mass of the moon being $\frac{1}{69{\cdot}2376}$: then, if subsequent observations should fix the true value of the semi-axis at $(1 + i) \times 9'',6480$, the corresponding mass of the moon will be $\frac{1 + i}{69{\cdot}2376 - i \times 178{\cdot}2918}$. This relation between the two elements thus established, it remained only to go through the labour of determining the value of i. This has been performed by Mr. LINDENAU, who, from the above-mentioned 810 observations, embracing three complete periods of the moon's node, has fixed it at $-0{\cdot}069541$; so that we have 8″,977, and 6″·682, for the respective major and minor semi-axes of the ellipse, and $\frac{1}{87{\cdot}7}$ for the moon's mass.

The numerical results obtained by M. BESSEL from these data, with the analytical formulæ from which they are derived, will be found in the work above referred to. They have been revised with great care by Mr. BAILY, using the value of i above stated, and the mean obliquity of the ecliptic 23° 27′ 40″ for 1830, instead of the obliquity 23° 27′ 54″,8 for 1800, employed by M. BESSEL. In this revision some errors have been detected, and the numerical co-efficients of M. BESSEL are slightly altered, in consequence of this alteration of one of the data. The correct formulæ for nutation both in Æ and Decl. for 1830, will then stand as follows:

Nutation in Æ =

$$\left.\begin{array}{l} -\,(15'',39587 + 6'',68189 \sin\alpha.\tan\delta).\sin ☊ - 8'',97707 \cos\alpha.\tan\delta.\cos ☊ \\ +\,(0'',14763 + 0'',06408 \sin\alpha.\tan\delta).\sin 2☊ + 0'',08768 \cos\alpha.\tan\delta.\cos 2☊ \\ -\,(1'',22549 + 0'',53187 \sin\alpha.\tan\delta).\sin 2\odot - 0'',57992 \cos\alpha.\tan\delta.\cos 2\odot \\ -\,(0'',18464 + 0'',08013 \sin\alpha.\tan\delta).\sin 2☽ - 0'',08737 \cos\alpha.\tan\delta.\cos 2☽ \end{array}\right\} (6)$$

Nutation in Declination =

$$\left.\begin{array}{ll} -\ 6'',68189 \cos\alpha \times \sin \Omega & +\ 8'',97707 \sin\alpha \times \cos \Omega \\ +\ 0'',06408 \cos\alpha \times \sin 2\,\Omega & -\ 0'',08768 \sin\alpha \times \cos 2\,\Omega \\ -\ 0'',53187 \cos\alpha \times \sin 2\odot & +\ 0'',57992 \sin\alpha \times \cos 2\odot \\ -\ 0'',08013 \cos\alpha \times \sin 2\,☽ & +\ 0'',08737 \sin\alpha \times \cos 2\,☽ \end{array}\right\} \qquad (7)$$

Now it will appear, on executing the computations, that

$$\begin{array}{rcl} 0{\cdot}14763 & = 0{\cdot}009590 \times & 15{\cdot}39587 \\ 0{\cdot}06408 & = 0{\cdot}009590 \times & 6{\cdot}68189 \\ 0{\cdot}08768 & = 0{\cdot}009767 \times & 8{\cdot}97707 \\ & & \\ 1{\cdot}22549 & = 0{\cdot}079600 \times & 15{\cdot}39587 \\ 0{\cdot}53187 & = 0{\cdot}079600 \times & 6{\cdot}68189 \\ 0{\cdot}57992 & = 0{\cdot}064600 \times & 8{\cdot}97707 \\ & & \\ 0{\cdot}18464 & = 0{\cdot}011993 \times & 15{\cdot}39587 \\ 0{\cdot}08013 & = 0{\cdot}011993 \times & 6{\cdot}68189 \\ 0{\cdot}08737 & = 0{\cdot}009732 \times & 8{\cdot}97707 \end{array}$$

And moreover, that

$$\begin{array}{rcl} 0{\cdot}009767 & = 1{\cdot}018515 \times & 0{\cdot}009590 \\ 0{\cdot}064600 & = 0{\cdot}811555 \times & 0{\cdot}079600 \\ 0{\cdot}009732 & = 0{\cdot}811555 \times & 0{\cdot}011993 \end{array}$$

Assuming, therefore,

$$P = 15'',39587 + 6'',68189 \sin\alpha.\tan\delta; \qquad Q = 8'',97707 \cos\alpha.\tan\delta$$

$$p = 6'',68189 \cos\alpha; \qquad q = 8'',97707 \sin\alpha$$

the expressions above stated will take the following form:

$$\left.\begin{array}{l} \text{Nutation in Æ} = -\ (P.\sin\Omega + Q.\cos\Omega) \\ \qquad +\ 0{\cdot}009590\ (P.\sin 2\,\Omega + 1{\cdot}018515\,Q.\cos 2\,\Omega) \\ \qquad -\ 0{\cdot}079600\ (B.\sin 2\odot + 0{\cdot}811555\,Q.\cos 2\odot) \\ \qquad -\ 0{\cdot}011993\ (P.\sin 2\,☽ + 0{\cdot}811555\,Q.\cos 2\,☽) \end{array}\right\} \qquad (8)$$

$$\left.\begin{array}{l} \text{Nutation in Decl.} = -\,(p.\sin\Omega - q.\cos\Omega) \\ \quad + 0{\cdot}009590\,(p.\sin 2\Omega - 1{\cdot}018515\,q.\cos 2\Omega) \\ \quad - 0{\cdot}079600\,(p.\sin 2\odot - 0{\cdot}811555\,q.\cos 2\odot) \\ \quad - 0{\cdot}011993\,(p.\sin 2☽ - 0{\cdot}811555\,q.\cos 2☽) \end{array}\right\} \quad (9)$$

We have only now to reduce these expressions to the same form as was employed in the aberrations; viz. into terms of the form M. sin $(\theta + N)$.

Assuming, therefore, the indeterminate co-efficients M', N', M'', N'', M''', N''', M^{iv}, N^{iv}, and m', n', &c. so that they shall respectively coincide with

$M'.\sin(\Omega + N') + M''.\sin(2\odot + N'') + M'''.\sin(2☽ + N''') + M^{iv}.\sin(2\Omega + N^{iv})$;

and

$m'.\sin(\Omega + n') + m''.\sin(2\odot + n'') + m'''.\sin(2☽ + n'') + m^{iv}.\sin(2\Omega + n^{iv})$

in which the terms follow in the order of their magnitudes, we shall have (P and Q, p, and q, being determined as above)

$$\left.\begin{array}{l} \tan N' = \frac{Q}{P} \\ \tan N'' = 0{\cdot}811555\,\tan N' \\ \quad N''' = N'' \\ \tan N^{iv} = 1{\cdot}018515\,\tan N' \end{array}\right\} \quad (10)$$

$$\left.\begin{array}{l} M' = -\,\frac{Q}{\sin N'} = -\,\frac{8''{\cdot}97707\cos\alpha.\tan\delta}{\sin N'} \\ M'' = -\,\frac{0{\cdot}06460\,Q}{\sin N''} = -\,\frac{0''{\cdot}57992\cos\alpha.\tan\delta}{\sin N''} \\ M''' = -\,\frac{0{\cdot}00973\,Q}{\sin N'''} = -\,\frac{0''{\cdot}08737\cos\alpha.\tan\delta}{\sin N'''} = 0{\cdot}15066M'' \\ M^{iv} = +\,\frac{0{\cdot}00977\,Q}{\sin N^{iv}} = +\,\frac{0''{\cdot}08768\cos\alpha.\tan\delta}{\sin N^{iv}} \end{array}\right\} (11)$$

$$\left.\begin{array}{l} \tan n' = -\,\frac{q}{p} = -\,1{\cdot}34349\tan\alpha \\ \tan n'' = 0{\cdot}811555\tan n' = -\,1{\cdot}09030\tan\alpha \\ \quad n''' = n'' \\ \tan n^{iv} = 1{\cdot}018515\tan n' = -\,1{\cdot}36829\tan\alpha \end{array}\right\} \quad (12)$$

$$\left.\begin{aligned}
m' &= +\frac{q}{\sin n'} &&= +\frac{8''\cdot97707 \sin\alpha}{\sin n'} \\
m'' &= +\frac{0\cdot06460\, q}{\sin n''} &&= +\frac{0''\cdot57992 \sin\alpha}{\sin n''} \\
m''' &= +\frac{0\cdot00973\, q}{\sin n'''} &&= +\frac{0''\cdot08737 \sin\alpha}{\sin n'''} = 0\cdot15066\, m'' \\
m^{\text{iv}} &= -\frac{0\cdot00977\, q}{\sin n^{\text{iv}}} &&= -\frac{0''\cdot08768 \sin\alpha}{\sin n^{\text{iv}}}
\end{aligned}\right\} \quad (13)$$

These are all the terms of the first order, which can demonstrably, in the present state of the theory, affect the places of the stars, with the exception of some excessively minute terms, having $\odot + ☊$ for their argument. They are so minute, however, that even in the case of the pole star, their influence can never amount to $0'',06$ (of time) in Æ, and a quantity perfectly insensible in declination. These, of course, we reject as needless refinements.

But the nutation and aberration displace the stars in arcs of great circles, which, though exceedingly small, cannot be confounded with their differentials. Hence it may, in some cases of stars near the pole, become necessary to take account of the squares at least of these corrections. This has been done by M. Bessel, who finds that the following terms are to be added to the general formulæ for the corrections on this account.

In Right Ascension (in time)

$$+ \sec\delta^2\,(0'',00006082 \cos 2\alpha . \sin 2\odot - 0'',00006104 \sin 2\alpha . \cos 2\odot)$$
$$+ \tan\delta^2\,(0'',00000969 \cos 2\alpha . \sin 2☊ - 0'',00001012 \sin 2\alpha . \cos 2☊)$$
$$- 0'',00002233 \sin\alpha . \tan\delta . \sin 2☊ + 0'',00008312 \cos\alpha . \tan\delta . \cos 2☊$$

That is (since the co-efficients 0·00006082 and 0·00006104 differ exceedingly little from each other, and from their mean 0·00006093; and, in like manner, 0·00000969 and 0·00001012 differ by an exceedingly minute quantity from their mean 0·00000991)

$$+ 0\cdot00006093 \sec\delta^2\,(\cos 2\alpha . \sin 2\odot - \sin 2\alpha . \cos 2\odot)$$
$$+ 0\cdot00000991 \tan\delta^2\,(\cos 2\alpha . \sin 2☊ - \sin 2\alpha . \cos 2☊)$$
$$-(0\cdot00002333 \sin\alpha . \sin 2☊ - 0\cdot00008312 \cos\alpha . \cos 2☊) . \tan\delta$$

$$=0{\cdot}00006093\,(1+\tan\delta^2)\cdot\sin(2\odot-2\alpha)+0{\cdot}00000991\tan\delta^2.\sin(2\,☊-2\alpha)$$
$$-0{\cdot}00002233\sin\alpha.\sin 2\,☊.\tan\delta+0{\cdot}00008312\cos 2\alpha.\cos 2\,☊.\tan\delta.$$

Now this is necessarily less than what it would be were its terms all positive and at their maximum; that is, than

$$0{\cdot}00006093+0{\cdot}00010645\tan\delta+0{\cdot}00007084\tan\delta^2.$$

Let us suppose that we would carry our accuracy to the nearest thousandth of a second of time in Æ. If we put this expression $=0{\cdot}001$, we have a quadratic equation for determining the limit of δ, which resolved, will be found to give

$$\tan\delta=3{\cdot}712;\qquad \delta=74^\circ\,55'\,20''$$

The sum of all the terms is therefore, even in the most unfavourable cases, less than 0″,001 of time for any star whose polar distance is greater than 15° 4′ 40″. Among the forty-six principal stars which compose the Greenwich catalogue, there are only two which approach this limit; Polaris and β Ursæ Minoris: the latter just falls short of it, its polar distance being 15° 5′. For this then there is no occasion to encumber ourselves with such considerations. The pole star is the only one in which it can ever become worth while; the value of the above expression, even in its case, being less than $0^s,15$, a quantity not measurable in any single transit of that star, though it may become very sensible in the mean result of a long series of observations.

It may be demonstrated by reasonings nearly analogous, that the sum of the terms of the second order in declination, assigned by M. Bessel, can never, even in the most unfavourable cases, amount to a single hundredth of a second of space, unless for stars whose declination exceeds 86° 27′. These terms are perfectly insensible, even in the pole star itself, and it is therefore needless to set them down here.

Let us now assemble, in one formula, all the quantities above considered, and we shall have

$$\left.\begin{aligned}\text{Apparent Æ} = {} & \text{mean Æ at the beginning of the time } t\\ & + V.\,t\\ & + M.\sin(\odot+N)\\ & + M'.\sin(☊+N')\\ & + M''.\sin(2\odot+N'')\\ & + M'''.\sin(2\,☽+N''')\\ & + M^{iv}.\sin(2\,☊+N^{iv})\end{aligned}\right\}\qquad(14)$$

$$\left.\begin{aligned}\text{Apparent Decl.} = \text{Mean Decl. at the beginning of the time } t\\ + v.\,t\\ + m.\sin(\odot + n)\\ + m'.\sin(\Omega + n')\\ + m''.\sin(2\odot + n'')\\ + m'''.\sin(2\,☽ + n''')\\ + m^{iv}.\sin(2\,\Omega + n^{iv})\end{aligned}\right\}\quad(15)$$

The great importance of elements, so thoroughly ascertained as the places of the forty-six stars in the Greenwich catalogue, (as checks on all the observations which may be made in any observatory, or zero points in the heavens, quite independent of all causes of derangement, and all sources of deception,) cannot be too constantly borne in mind by every practical astronomer. Without constant and unremitting reference to these, his clocks, his circles, his levels, and plumb-lines, will all (unless of the very first class of excellence) be so many continual sources of annoyance and suspicion; while, on the other hand, if he will only consent to take for granted these invaluable data, which the labour of so many excellent observers, and the expenditure of so large a portion of the national wealth has been devoted to furnish; and waiving the fruitless (though I fear not uncommon) desire of rendering his observatory totally independent of all others, and ascertaining, with his own instruments, all the quantities he requires, content himself with referring all his observations to these standards, and these alone*; his work will flow easily under his hands, and he will find himself enabled to render his labours permanently useful to science, by devoting them to the accumulation of fresh results.

But in order to avail ourselves, as much as possible, of the advantages afforded by the Greenwich catalogue, it is essential to have them ready at command,—not merely embodied in formulæ, but developed into tables. It is for the purpose of facilitating the formation of such tables that the following are constructed. They are merely subsidiary, and afford all that is

* What is here said is not meant to apply to great establishments, which it is always desirable to keep independent of each other; or even (in all its force) to the observations of particular public bodies. But good instruments are now become so common, that there is hardly an amateur who might not chalk out a path of observation for himself, and pursue it with real advantage to science and honour to himself.

necessary for computing the corrections of the 46 stars, from the year 1820 to 1840 inclusive, or even a longer period, without the necessity of opening a table of logarithms, or any other book but an ephemeris. They are grounded on the foregoing formulæ, and computed from them. The values of M, N, *m, n,* &c. were computed by two independent computers, and the results being found to agree were adopted. The formation of the subsidiary tables from these elements is so simple, that one only (Mr. Richardson of the Royal Observatory) was employed in their construction.

Construction and Explanation of the Tables.

Table I. contains the variation in Æ and Decl.: or V. *t*, and *v. t*. The construction of this table is extremely simple. The annual variation of each star, divided by 365·25, gives the variation for a mean solar day. This multiplied by 5 gives the variation for five days, which added perpetually to the former gives the variation for 1, 6, 11, 16, &c. days to the end of the year. These numbers, with their proper signs, added to the mean Æ and Decl. for January $0^d\ 0^h\ 0^m$, will give the mean places for January 1, January 6, January 11, &c. at noon. The Æ is expressed in seconds of time throughout this and the following tables. The values of M, &c. above stated, are divided therefore by 15.

Table II. contains the aberration in Æ $=$ M. sin ($\odot + $ N). And

Table III. contains the lunar nutation in Æ $=$ M′ sin (☊ $+$ N′)

These two tables exhibit the values of the corrections, whose titles they bear for every 3° of their arguments. It will be observed that the arguments in these and the following tables are, not the longitudes ⊙ and ☊ of the sun, the moon's node, &c., but these longitudes *plus* the constant angles N, N′, &c. respectively. The other plan might have been adopted. We might have calculated them for every 3° of ⊙, ☊, &c. commencing from 0°; and this, at first sight, would appear more convenient for use, as we should then have no occasion to use the constants N, N′, &c. in the annual computations at all; but a little consideration will satisfy us that the construction actually followed possesses many advantages over the other. These are,

1st. Facility in the construction of the tables themselves, and of course less liability to error. In making the computations, the only logarithmic sines to be taken out are those of every third whole degree; and these once copied

out, the table of sines need not be again opened; whereas, had we used ☉ and ☊ themselves for the arguments, we should have to calculate the value of (☉ + N) &c. and look out for their logarithmic sines every time; thus increasing, on the whole, (as a little consideration will readily show,) the number of openings of the table of sines in the ratio of 368 to 1, a consideration of no small importance. But the actual saving of labour by the construction resorted to is twice as great, or in the proportion of 736 to 1. For,

2dly, This construction enables us to reduce the tables to one half of their magnitude. For the constants N, N′, &c. not being multiples of three degrees, the arcs in the first and third quadrants will not be exact supplements of any of those in the second and fourth. Thus, if $N = 1°$, the arcs in the first quadrant, if ☉ be taken for the argument, will be $(0° + 1°) = 1°$, $(3° + 1°) = 4°$ .... $(87° + 1°) = 88°$; while those in the second are 91°, 94°.... 178°; whose supplements fall between the arcs in the other series, and of course the table must be continued over 180° of the circumference in the place of 90.

To counterbalance these advantages, the only objection is, that, in the use of the tables when computed, the constants N, N′, &c. require to be added every time to the values of ☉, ☊, &c. as taken from an ephemeris; but this is so trifling an operation as scarcely to deserve notice; and as these values are rarely, if ever, exact multiples of 3°, no additional trouble in taking out proportional parts is thereby entailed. To facilitate this, the constant angles N, N′, &c. are entered at the foot of every column, and the proportional parts may be taken out almost by inspection by means of the column entitled Differences for 10′. If the argument with which the table is entered be found on the left side, or between 0 and 180°, the sign on the left side of the column must be taken; if between 180° and 360°, or, on the right, then the sign on the right hand must be used.

Tables IV. and V. contain the values of the solar nutation and lunar inequality: or M″. sin (2☉ + N″) and M‴. sin (2☾ + N‴) for the same values of the arguments (2☉ + N″) and (2☾ + N‴). For the convenience of the computer in the formation of annual tables, the values of N″, N‴ are placed opposite each star. If the argument be between 0 and 180°, the upper sign must be used; if between 180° and 360°, the lower. These tables are computed only for every 10° of the argument; the minuteness of the correction, which, at its maximum, rarely exceeds $\frac{1}{10}$ of a second for the solar nu-

tation, and (except for Polaris) never amounts to $\frac{1}{50}$ of a second in the lunar inequality, renders a closer subdivision quite superfluous.

The term $M^{iv}. \sin (2\Omega + N^{iv})$ scarcely ever amounts to $\frac{1}{50}$ of a second in Æ at its maximum; and instead of making a separate table for it, we may compute it by TABLE III. In fact, the last of our equations (10) shows that if we neglect very minute quantities, we have $\tan N^{iv} = \tan N'$, and $N^{iv} = N'$; moreover, the last of the equations (11) gives

$$M^{iv} = + \frac{0''\cdot 08737 \cos \alpha . \tan \delta}{\sin N^{iv}} = + \frac{0\cdot 08737}{8\cdot 97707} \cdot \frac{8''\cdot 97707 \cos \alpha . \tan \delta}{\sin N'} = -0\cdot 009733\ M'.$$

If then we take $M^{iv} = -0\cdot 01 \times M'$, the greatest error this can ever entail on us will never amount to $0\cdot 00037 \times M'$, which even in the case of the pole-star falls short of $\frac{1}{170}$th of a second, and for all the other stars is below $\frac{1}{2000}$. We have therefore only to remove the significant figures of TABLE III. two places further to the right to make it serve for the correction depending on 2Ω, the same value of N′ serving for both: observing, however, to alter the sign of the tabular value.

TABLES VI. and VII., entitled Aberration in declination, and Nutation in declination, are formed exactly in the same manner as TABLES II. and III., and require no further explanation.

TABLE VIII. gives the solar nutation in declination. On executing the computation of the values of m'' and n'' in the formula $m''. \sin (2\odot + n'')$ according to the expressions in (12), it will be found that m'' is very nearly the same for *all* the stars; its greatest value being $-0'',57987$ (for γ Draconis), and its least $-0'',53187$ (for α Andromedæ): the mean is $-0'',5559$, from which neither of the extremes differs more than $0'',024$, a quantity quite insensible. This peculiarity is taken advantage of to render one short table common to all the stars. A similar simplification takes place in the tables of the lunar inequality, the variation in the values of m''' being confined within still narrower limits: *viz.* $-0'',08015$, and $-0'',08736$, the mean being $-0'',0840$; and the greatest possible error which can arise from the use of this mean, as common to all the stars, being $0'',004$.

The values of n'' and n''' being equal, are to be taken from TABLE X.

The inequality in declination, depending on 2Ω, may be obtained from

the table of lunar nutation (Table VII.) in the same way as that in Æ from Table III., by merely entering the table with the argument ($2\,☊ + n'$), instead of ($☊ + n'$), removing the significant figure in the table two places to the right, and altering the sign of the tabular value.

The use of these tables is so simple as to need no explanation; the values of ☉, ☊, and ☾, only requiring to be taken from an ephemeris, for the time of the star's culmination, to give all we want.

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The day of the Year.

Argument.		1 γ Pegasi.		2 α Cassiop.		3 Polaris.		4 α Arietis.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0084	+0.0553	+0.0091	+0.0542	+0.0404	+0.0533	+0.0092	+0.0476
6	6	.0506	.3318	.0544	.3253	.2421	.3195	.0550	.2858
11	11	.0928	.6084	.0997	.5963	.4439	.5858	.1009	.5240
16	16	.1349	.8849	.1450	.8674	.6457	.8520	.1467	.7622
21	21	.1771	1.1614	.1903	1.1384	.8475	1.1183	.1926	1.0004
26	26	.2192	.4379	.2356	.4094	1.0493	.3845	.2385	.2386
31	31	.2614	.7144	.2809	.6805	.2510	.6508	.2843	.4768
Feb. 5	Feb. 5	.3036	.9910	.3262	.9515	.4528	.9170	.3302	.7150
10	10	.3437	2.2675	.3716	2.2226	.6546	2.1833	.3760	.9532
15	15	.3879	.5440	.4169	.4936	.8564	.4496	.4219	2.1914
20	20	.4301	.8205	.4622	.7647	2.0582	.7158	.4678	.4296
25	25	.4722	3.0971	.5075	3.0357	.2599	.9821	.5136	.6678
Mar. 2	Mar. 1	.5144	.3736	.5528	.3068	.4617	3.2483	.5595	.9060
7	6	.5566	.6501	.5981	.5778	.6635	.5146	.6053	3.1441
12	11	.5987	.9266	.6434	.8489	.8653	.7808	.6512	.3823
17	16	.6409	4.2032	.6887	4.1199	3.0670	4.0471	.6971	.6205
22	21	.6830	.4797	.7341	.3910	.2688	.3133	.7429	.8587
27	26	.7252	.7562	.7794	.6620	.4706	.5796	.7888	4.0969
April 1	31	.7674	5.0327	.8247	.9331	.6724	.8459	.8346	.3351
6	April 5	.8095	.3092	.8700	5.2041	.8742	5.1121	.8805	.5733
11	10	.8517	.5858	.9153	.4751	4.0759	.3784	.9264	.8115
16	15	.8939	.8623	.9606	.7462	.2777	.6446	.9722	5.0497
21	20	.9360	6.1388	1.0059	6.0172	.4795	.9109	1.0181	.2879
26	25	.9782	.4153	.0512	.2883	.6813	6.1771	.0639	.5261
May 1	30	1.0203	.6919	.0966	.5593	.8831	.4434	.1098	.7643
6	May 5	.0625	.9684	.1419	.8304	5.0848	.7097	.1556	6.0025
11	10	.1047	7.2449	.1872	7.1014	.2866	.9759	.2015	.2407
16	15	.1468	.5214	.2325	.3725	.4884	7.2422	.2474	.4789
21	20	.1890	.7979	.2778	.6435	.6902	.5084	.2932	.7170
26	25	.2312	8.0745	.3231	.9146	.8020	.7747	.3391	.9552
31	30	.2733	.3510	.3684	8.1856	6.0937	8.0409	.3849	7.1934
June 5	June 4	.3155	.6275	.4137	.4567	.2955	.3072	.4308	.4316
10	9	.3576	.9040	.4590	.7277	.4973	.5734	.4767	.6698
15	14	.3998	9.1806	.5044	.9988	.6991	.8397	.5225	.9080
20	19	+1.4420	+9.4571	+1.5497	+9.2698	+6.9009	+9.1060	+1.5684	+8.1462

Argument.		1 γ Pegasi.		2 α Cassiop.		3 Polaris.		4 α Arietis.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.4841	+9.7336	+1.5950	+9.5409	+7.1026	+9.3722	+1.6142	+8.3844
30	29	.5263	10.0101	.6403	.8119	.3044	.6385	.6601	.6226
July 5	July 4	.5685	.2867	.6856	10.0829	.5062	.9047	.7060	.8608
10	9	.6106	.5632	.7309	.3540	.7080	10.1710	.7518	9.0990
15	14	.6528	.8397	.7762	.6250	.9098	.4372	.7977	.3372
20	19	.6950	11.1162	.8215	.8961	8.1115	.7035	.8435	.5754
25	24	.7371	.3927	.8669	11.1671	.3133	.9697	.8894	.8136
30	29	.7793	.6693	.9122	.4382	.5151	11.2360	.9353	10.0517
Aug. 4	Aug. 3	.8214	.9458	.9575	.7092	.7169	.5023	.9811	.2899
9	8	.8636	12.2223	2.0028	.9803	.9187	.7685	2.0270	.5281
14	13	.9058	.4988	.0481	12.2513	9.1204	12.0348	.0728	.7663
19	18	.9479	.7754	.0934	.5224	.3222	.3010	.1187	11.0045
24	23	.9901	13.0519	.1387	.7934	.5240	.5673	.1645	.2427
29	28	2.0323	.3284	.1840	13.0645	.7258	.8335	.2104	.4809
Sept. 3	Sept. 2	.0744	.6049	.2294	.3355	.9276	13.0998	.2563	.7191
8	7	.1166	.8815	.2747	.6066	10.1293	.3661	.3021	.9573
13	12	.1587	14.1580	.3200	.8776	.3311	.6323	.3480	12.1955
18	17	.2009	.4345	.3653	14.1487	.5329	.8986	.3938	.4337
23	22	.2431	.7110	.4106	.4197	.7347	14.1648	.4397	.6719
28	27	.2852	.9875	.4559	.6907	.9365	.4311	.4856	.9101
Oct. 3	Oct. 2	.3274	15.2641	.5012	.9618	11.1382	.6973	.5314	13.1483
8	7	.3696	.5406	.5465	15.2328	.3400	.9636	.5773	.3864
13	12	.4117	.8171	.5918	.5039	.5418	15.2298	.6231	.6246
18	17	.4539	16.0936	.6372	.7749	.7436	.4961	.6690	.8628
23	22	.4961	.3702	.6825	16.0460	.9454	.7624	.7149	14.1010
28	27	.5382	.6467	.7278	.3170	12.1471	16.0286	.7607	.3392
Nov. 2	Nov. 1	.5804	.9232	.7731	.5881	.3489	.2949	.8066	.5774
7	6	.6225	17.1997	.8184	.8591	.5507	.5611	.8524	.8156
12	11	.6647	.4763	.8637	17.1302	.7525	.8274	.8983	15.0538
17	16	.7069	.7528	.9090	.4012	.9543	17.0936	.9441	.2920
22	21	.7490	18.0293	.9543	.6723	13.1560	.3599	.9900	.5302
27	26	.7912	.3058	.9997	.9433	.3578	.6261	3.0359	.7684
Dec. 2	Dec. 1	.8334	.5823	3.0450	18.2144	.5596	.8924	.0817	16.0066
7	6	.8755	.8589	.0903	.4854	.7614	18.1587	.1276	.2448
12	11	.9177	19.1354	.1356	.7565	.9631	.4249	.1734	.4830
17	16	.9598	.4119	.1809	19.0275	14.1649	.6912	.2193	.7211
22	21	3.0020	.6884	.2262	.2986	.3667	.9574	.2652	.9593
27	26	.0442	.9650	.2715	.5696	.5685	19.2237	.3110	17.1975
32	31	+3.0863	+20.2415	+3.3168	+19.8407	+14.7703	+19.4899	+3.3569	+17.4357

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The Day of the Year.

Argument.		5 α Ceti.		6 α Persei.		7 Aldebaran.		8 Capella.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0085	+0.0404	+0.0115	+0.0370	+0.0094	+0.0218	+0.0121	+0.0125
6	6	.0513	.2423	.0690	.2218	.0563	.1306	.0724	.0751
11	11	.0940	.4442	.1265	.4066	.1033	.2394	.1328	.1376
16	16	.1367	.6461	.1840	.5914	.1503	.3483	.1932	.2002
21	21	.1794	.8481	.2415	.7762	.1972	.4571	.2536	.2628
26	26	.2221	1.0500	.2990	.9610	.2442	.5659	.3139	.3253
31	31	.2648	.2519	.3565	1.1458	.2911	.6747	.3743	.3879
Feb. 5	Feb. 5	.3075	.4538	.4140	.3306	.3381	.7836	.4347	.4504
10	10	.3502	.6557	.4715	.5154	.3850	.8924	.4950	.5130
15	15	.3929	.8576	.5290	.7002	.4320	1.0012	.5554	.5756
20	20	.4357	2.0596	.5864	.8850	.4789	.1101	.6158	.6381
25	25	.4784	.2615	.6439	2.0698	.5259	.2189	.6761	.7007
Mar. 2	Mar. 1	.5211	.4634	.7014	.2546	.5728	.3277	.7365	.7632
7	6	.5638	.6653	.7589	.4394	.6198	.4366	.7969	.8258
12	11	.6065	.8672	.8164	.6242	.6667	.5454	.8573	.8884
17	16	.6492	3.0691	.8739	.8090	.7137	.6542	.9176	.9509
22	21	.6919	.2711	.9314	.9938	.7607	.7630	.9780	1.0135
27	26	.7346	.4730	.9889	3.1786	.8076	.8718	1.0384	.0760
April 1	31	.7773	.6749	1.0464	.3635	.8546	.9807	.0987	.1386
6	April 5	.8201	.8768	.1039	.5483	.9015	2.0895	.1591	.2012
11	10	.8628	4.0787	.1614	.7331	.9485	.1984	.2195	.2637
16	15	.9055	.2806	.2189	.9179	.9954	.3072	.2798	.3263
21	20	.9482	.4826	.2764	4.1027	1.0424	.4160	.3402	.3888
26	25	.9909	.6845	.3339	.2875	.0893	.5249	.4006	.4514
May 1	30	1.0336	.8864	.3914	.4723	.1363	.6337	.4610	.5140
6	May 5	.0763	5.0883	.4489	.6571	.1832	.7425	.5213	.5765
11	10	.1190	.2902	.5064	.8419	.2302	.8513	.5817	.6391
16	15	.1617	.4921	.5639	5.0267	.2771	.9602	.6421	.7016
21	20	.2045	.6941	.6214	.2115	.3241	3.0690	.7024	.7642
26	25	.2472	.8960	.6789	.3963	.3711	.1778	.7628	.8268
31	30	.2899	6.0979	.7363	.5811	.4180	.2867	.8232	.8893
June 5	June 4	.3326	.2998	.7938	.7659	.4650	.3955	.8835	.9519
10	9	.3753	.5017	.8513	.9507	.5119	.5043	.9439	2.0144
15	14	.4180	.7036	.9088	6.1355	.5589	.6132	2.0043	.0770
20	19	+1.4607	+6.9056	+1.9663	+6.3203	+1.6058	+3.7220	+2.0647	+2.1396

Argument.		5 α Ceti.		6 α Persei.		7 Aldebaran.		8 Capella.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.5034	+7.1075	+2.0238	+6.5051	+1.6528	+3.8308	+2.1250	+2.2021
30	29	.5461	.3094	.0813	.6899	.6997	.9396	.1854	.2647
July 5	July 4	.5888	.5113	.1388	.8747	.7467	4.0485	.2458	.3272
10	9	.6316	.7132	.1963	7.0596	.7936	.1573	.3061	.3898
15	14	.6743	.9151	.2538	.2444	.8406	.2661	.3665	.4524
20	19	.7170	8.1171	.3113	.4292	.8876	.3750	.4269	.5149
25	24	.7597	.3190	.3688	.6140	.9345	.4838	.4872	.5775
30	29	.8024	.5209	.4263	.7988	.9815	.5926	.5476	.6400
Aug. 4	Aug. 3	.8451	.7228	.4838	.9836	2.0284	.7015	.6080	.7026
9	8	.8878	.9247	.5413	8.1684	.0754	.8103	.6684	.7652
14	13	.9305	9.1266	.5988	.3532	.1223	.9191	.7287	.8277
19	18	.9732	.3286	.6563	.5380	.1693	5.0279	.7891	.8903
24	23	2.0160	.5305	.7138	.7228	.2162	.1368	.8495	.9528
29	28	.0587	.7324	.7713	.9076	.2632	.2456	.9098	3.0154
Sep. 3	Sep. 2	.1014	.9343	.8288	9.0924	.3101	.3544	.9702	.0780
8	7	.1441	10.1362	.8862	.2772	.3571	.4633	3.0306	.1405
13	12	.1868	.3382	.9437	.4620	.4040	.5721	.0909	.2031
18	17	.2295	.5401	3.0012	.6468	.4510	.6809	.1513	.2656
23	22	.2722	.7420	.0587	.8316	.4980	.7898	.2117	.3282
28	27	.3149	.9439	.1162	10.0164	.5449	.8986	.2721	.3908
Oct. 3	Oct. 2	.3576	11.1458	.1737	.2012	.5919	6.0074	.3324	.4533
8	7	.4004	.3477	.2312	.3860	.6388	.1162	.3928	.5159
13	12	.4431	.5497	.2887	.5708	.6858	.2251	.4532	.5784
18	17	.4858	.7516	.3462	.7557	.7327	.3339	.5135	.6410
23	22	.5285	.9535	.4037	.9405	.7797	.4427	.5739	.7036
28	27	.5712	12.1554	.4612	11.1253	.8266	.5516	.6343	.7661
Nov. 2	Nov. 1	.6139	.3573	.5187	.3101	.8736	.6604	.6946	.8287
7	6	.6566	.5592	.5762	.4949	.9205	.7692	.7550	.8912
12	11	.6993	.7612	.6337	.6797	.9675	.8781	.8154	.9538
17	16	.7420	.9631	.6912	.8645	3.0144	.9869	.8758	4.0164
22	21	.7848	13.1650	.7487	12.0493	.0614	7.0957	.9361	.0789
27	26	.8275	.3669	.8062	.2841	.1084	.2045	.9965	.1415
Dec. 2	Dec. 1	.8702	.5688	.8637	.4189	.1553	.3134	4.0569	.2040
7	6	.9129	.7707	.9212	.6037	.2023	.4222	.1172	.2666
12	11	.9556	.9727	.9787	.7885	.2492	.5310	.1776	.3292
17	16	.9983	14.1746	4.0361	.9733	.2962	.6399	.2380	.3917
22	21	3.0410	.3765	.0936	13.1581	.3431	.7487	.2983	.4543
27	26	.0837	.5784	.1511	.3429	.3901	.8575	.3587	.5168
32	31	+3.1264	+14.7803	+4.2086	+13.5277	+3.4370	+7.9664	+4.4191	+4.5794

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The Day of the Year.

Argument.		9 Rigel.		10 β Tauri.		11 α Orionis.		12 Sirius.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0079	−0.0135	+0.0103	+0.0105	+0.0089	+0.0038	+0.0071	+0.0119
6	6	.0473	.0808	.0621	.0629	.0534	.0225	.0429	.0716
11	11	.0867	.1482	.1138	.1153	.0979	.0413	.0786	.1313
16	16	.1262	.2155	.1656	.1678	.1424	.0600	.1143	.1910
21	21	.1656	.2829	.2173	.2202	.1869	.0788	.1501	.2507
26	26	.2050	.3502	.2691	.2726	.2313	.0975	.1858	.3104
31	31	.2444	.4176	.3208	.3251	.2758	.1163	.2215	.3700
Feb. 5	Feb. 5	.2839	.4849	.3726	.3775	.3203	.1350	.2572	.4297
10	10	.3233	.5523	.4243	.4299	.3648	.1538	.2930	.4894
15	15	.3627	.6196	.4761	.4824	.4093	.1725	.3287	.5491
20	20	.4021	.6870	.5278	.5348	.4538	.1913	.3644	.6088
25	25	.4416	.7543	.5795	.5872	.4983	.2101	.4002	.6685
Mar. 2	Mar. 1	.4810	.8217	.6313	.6396	.5428	.2288	.4359	.7282
7	6	.5204	.8890	.6830	.6921	.5873	.2476	.4716	.7878
12	11	.5598	.9564	.7348	.7445	.6318	.2663	.5074	8475
17	16	.5993	1.0237	.7865	.7969	.6762	.2851	.5431	.9072
22	21	.6387	.0911	.8383	.8494	.7207	.3038	.5788	.9669
27	26	.6781	.1584	.8900	.9018	.7652	.3226	.6145	1.0266
April 1	31	.7175	.2257	.9418	.9542	.8097	.3413	.6503	.0863
6	April 5	.7570	.2931	.9935	1.0067	.8542	.3601	.6860	.1460
11	10	.7964	.3605	1.0452	.0591	.8987	.3789	.7217	.2056
16	15	.8358	.4278	.0970	.1115	.9432	.3976	.7575	.2653
21	20	.8752	.4952	.1487	.1639	.9877	.4164	.7932	.3250
26	25	.9147	.5625	.2005	.2164	1.0322	.4351	.8289	.3847
May 1	30	.9541	.6299	.2522	.2688	.0767	.4539	.8646	.4444
6	May 5	.9935	.6972	.3040	.3212	.1211	.4726	.9004	.5041
11	10	1.0329	.7646	.3557	.3737	.1656	.4914	.9361	.5637
16	15	.0724	.8319	.4075	.4261	.2101	.5101	.9718	.6234
21	20	.1118	.8993	.4592	.4785	.2546	.5289	1.0076	.6831
26	25	.1512	.9666	.5110	.5310	.2991	.5476	.0433	.7428
31	30	.1906	2.0340	.5627	.5834	.3436	.5664	.0790	.8025
June 5	June 4	.2301	.1014	.6144	.6358	.3881	.5852	.1147	.8622
10	9	.2695	.1687	.6662	.6882	.4326	.6039	.1505	.9219
15	14	.3089	.2361	.7179	.7407	.4771	.6227	.1862	.9815
20	19	+1.3483	+2.3034	+1.7697	+1.7931	+1.5216	+0.6414	+1.2219	+2.0412

Argument.		9 Rigel.		10 β Tauri.		11 α Orionis.		12 Sirius.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.3878	−2.3708	+1.8214	+1.8455	+1.5660	+0.6602	+1.2577	+2.1009
30	29	.4272	.4381	.8732	.8980	.6105	.6789	.2934	.1606
July 5	July 4	.4666	.5055	.9249	.9504	.6550	.6977	.3291	.2203
10	9	.5060	.5728	.9767	2.0028	.6995	.7164	.3648	.2800
15	14	.5455	.6402	2.0284	.0553	.7440	.7352	.4006	.3397
20	19	.5849	.7075	.0801	.1077	.7885	.7540	.4363	.3993
25	24	.6243	.7749	.1319	.1601	.8330	.7727	.4720	.4590
30	29	.6637	.8422	.1836	.2125	.8775	.7915	.5078	.5187
Aug. 4	Aug. 3	.7032	.9096	.2354	.2650	.9220	.8102	.5435	.5784
9	8	.7426	.9769	.2871	.3174	.9665	.8290	.5792	.6381
14	13	.7820	3.0443	.3389	.3698	2.0109	.8477	.6150	.6978
19	18	.8214	.1116	.3906	.4223	.0554	.8665	.6507	.7574
24	23	.8609	.1790	.4424	.4747	.0999	.8852	.6864	.8171
29	28	.9003	.2463	.4941	.5271	.1444	.9040	.7221	.8768
Sept. 3	Sept. 2	.9397	.3137	.5459	.5796	.1889	.9227	.7579	.9365
8	7	.9791	.3810	.5976	.6320	.2334	.9415	.7936	.9962
13	12	2.0186	.4484	.6493	.6844	.2779	.9603	.8293	3.0559
18	17	.0580	.5157	.7011	.7368	.3224	.9790	.8651	.1156
23	22	.0974	.5831	.7528	.7893	.3669	.9978	.9008	.1752
28	27	.1368	.6504	.8046	.8417	.4114	1.0165	.9365	.2349
Oct. 3	Oct. 2	.1763	.7178	.8563	.8941	.4558	.0353	.9722	.2946
8	7	.2157	.7851	.9081	.9466	.5003	.0540	2.0080	.3543
13	12	.2551	.8525	.9598	.9990	.5448	.0728	.0437	.4140
18	17	.2945	.9198	3.0116	3.0514	.5893	.0915	.0794	.4737
23	22	.3340	.9872	.0633	.1039	.6338	.1103	.1152	.5334
28	27	.3734	4.0545	.1150	.1563	.6783	.1291	.1509	.5930
Nov. 2	Nov. 1	.4128	.1219	.1668	.2087	.7228	.1478	.1866	.6527
7	6	.4522	.1892	.2185	.2611	.7673	.1666	.2223	.7124
12	11	.4917	.2566	.2703	.3136	.8118	.1853	.2581	.7721
17	16	.5311	.3239	.3220	.3660	.8563	.2041	.2938	.8318
22	21	.5705	.3913	.3738	.4184	.9007	.2228	.3295	.8915
27	26	.6099	.4586	.4255	.4709	.9452	.2416	.3653	.9511
Dec. 2	Dec. 1	.6494	.5260	.4773	.5233	.9897	.2603	.4010	4.0108
7	6	.6888	.5933	.5290	.5757	3.0342	.2791	.4367	.0705
12	11	.7282	.6607	.5808	.6282	.0787	.2978	.4724	.1302
17	16	.7676	.7280	.6325	.6806	.1232	.3166	.5082	.1899
22	21	.8071	.7954	.6842	.7330	.1677	.3354	.5439	.2496
27	26	.8465	.8627	.7360	.7854	.2122	.3541	.5796	.3093
32	31	+2.8859	−4.9301	+3.7877	+3.8379	+3.2567	+1.3729	+2.6154	+4.3690

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The Day of the Year.

Argument.		13 Castor.		14 Procyon.		15 Pollux.		16 α Hydræ.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0105	−0.0193	+0.0086	−0.0234	+0.0101	−0.0219	+0.0081	+0.0416
6	6	.0632	.1160	.0517	.1403	.0606	.1314	.0485	.2495
11	11	.1159	.2126	.0949	.2572	.1111	.2409	.0888	.4575
16	16	.1687	.3093	.1380	.3741	.1616	.3504	.1292	.6654
21	21	.2214	.4059	.1811	.4910	.2122	.4600	.1696	.8733
26	26	.2741	.5026	.2242	.6079	.2627	.5695	.2100	1.0813
31	31	.3268	.5992	.2674	.7248	.3132	.6790	.2504	.2892
Feb. 5	Feb. 5	.3795	.6959	.3105	.8417	.3637	.7885	.2908	.4972
10	10	.4322	.7925	.3536	.9586	.4142	.8980	.3311	.7051
15	15	.4849	.8891	.3967	1.0755	.4647	1.0075	.3715	.9130
20	20	.5376	.9858	.4398	.1924	.5152	.1170	.4119	2.1210
25	25	.5903	1.0824	.4830	.3093	.5658	.2266	.4523	.3289
Mar. 2	Mar. 1	.6430	.1791	.5261	.4263	.6162	.3361	.4927	.5369
7	6	.6957	.2757	.5692	.5432	.6668	.4456	.5331	.7448
12	11	.7484	.3724	.6123	.6601	.7173	.5551	.5734	.9527
17	16	.8011	.4690	.6554	.7770	.7678	.6646	.6138	3.1607
22	21	.8538	.5657	.6986	.8939	.8183	.7741	.6542	.3686
27	26	.9065	.6623	.7417	2.0108	.8688	.8836	.6946	.5766
April 1	31	.9592	.7590	.7848	.1277	.9194	.9932	.7350	.7845
6	April 5	1.0119	.8556	.8279	.2446	.9699	2.1027	.7754	.9924
11	10	.0646	.9522	.8710	.3615	1.0204	.2122	.8157	4.2004
16	15	.1173	2.0489	.9142	.4784	.0709	.3217	.8561	.4083
21	20	.1700	.1455	.9573	.5953	.1214	.4312	.8965	.6163
26	25	.2227	.2422	1.0004	.7122	.1719	.5407	.9369	.8242
May 1	30	.2754	.3388	.0435	.8291	.2224	.6502	.9773	5.0321
6	May 5	.3281	.4355	.0866	.9460	.2730	.7598	1.0177	.2401
11	10	.3808	.5321	.1298	3.0629	.3235	.8693	.0580	.4480
16	15	.4335	.6288	.1729	.1798	.3740	.9788	.0984	.6560
21	20	.4863	.7254	.2160	.2967	.4245	3.0883	.1388	.8639
26	25	.5390	.8221	.2591	.4137	.4750	.1978	.1792	6.0718
31	30	.5917	.9187	.3023	.5306	.5255	.3073	.2196	.2798
June 5	June 4	.6444	3.0154	.3454	.6475	.5760	.4168	.2600	.4877
10	9	.6971	.1120	.3885	.7644	.6266	.5264	.3003	.6957
15	14	.7498	.2086	.4316	.8813	.6771	.6359	.3407	.9036
20	19	+1.8025	−3.3053	+1.4747	−3.9982	+1.7276	−3.7454	+1.3811	+7.1115

Argument.		13 Castor.		14 Procyon.		15 Pollux.		16 α Hydræ.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.8552	−3.4019	+1.5179	−4.1151	+1.7781	−3.8549	+1.4215	+7.3195
30	29	.9079	.4986	.5610	.2320	.8286	.9644	.4619	.5274
July 5	July 4	.9606	.5952	.6041	.3489	.8791	4.0739	.5022	.7354
10	9	2.0133	.6919	.6472	.4658	.9296	.1834	.5426	.9433
15	14	.0660	.7885	.6903	.5827	.9801	.2929	.5830	8.1512
20	19	.1187	.8852	.7335	.6996	2.0307	.4025	.6234	.3592
25	24	.1714	.9818	.7766	.8165	.0812	.5120	.6638	.5671
30	29	.2241	4.0785	.8197	.9334	.1317	.6215	.7042	.7751
Aug. 4	Aug. 3	.2768	.1751	.8628	5.0503	.1822	.7310	.7445	.9830
9	8	.3295	.2718	.9059	.1672	.2327	.8405	.7849	9.1909
14	13	.3822	.3684	.9491	.2842	.2832	.9500	.8253	.3989
19	18	.4349	.4650	.9922	.4011	.3337	5.0595	.8657	.6068
24	23	.4876	.5617	2.0353	.5180	.3843	.1691	.9061	.8148
29	28	.5403	.6583	.0784	.6349	.4348	.2786	.9465	10.0227
Sept. 3	Sept. 2	.5930	.7550	.1216	.7518	.4853	.3881	.9868	.2306
8	7	.6457	.8516	.1647	.8687	.5358	.4976	2.0272	.4386
13	12	.6984	.9483	.2078	.9856	.5863	.6071	.0676	.6465
18	17	.7511	5.0449	.2509	6.1025	.6368	.7166	.1080	.8545
23	22	.8039	.1416	.2940	.2194	.6873	.8261	.1484	11.0624
28	27	.8566	.2382	.3372	.3363	.7379	.9357	.1888	.2703
Oct. 3	Oct. 2	.9093	.3349	.3803	.4532	.7884	6.0452	.2291	.4783
8	7	.9620	.4315	.4234	.5701	.8389	.1547	.2695	.6862
13	12	3.0147	.5282	.4665	.6870	.8894	.2642	.3099	.8942
18	17	.0674	.6248	.5096	.8039	.9399	.3737	.3503	12.1021
23	22	.1201	.7214	.5528	.9208	.9904	.4832	.3907	.3100
28	27	.1728	.8181	.5959	7.0377	3.0409	.5927	.4311	.5180
Nov. 2	Nov. 1	.2255	.9147	.6390	.1546	.0915	.7023	.4714	.7259
7	6	.2782	6.0114	.6821	.2716	.1420	.8118	.5118	.9339
12	11	.3309	.1080	.7252	.3885	.1925	.9213	.5522	13.1418
17	16	.3836	.2047	.7684	.5054	.2430	7.0308	.5926	.3497
22	21	.4363	.3013	.8115	.6223	.2935	.1403	.6330	.5577
27	26	.4890	.3980	.8546	.7392	.3440	.2498	.6734	.7656
Dec. 2	Dec. 1	.5417	.4946	.8977	.8561	.3945	.3593	.7137	.9736
7	6	.5944	.5913	.9409	.9730	.4451	.4689	.7541	14.1815
12	11	.6471	.6880	.9840	8.0899	.4956	.5784	.7945	.3894
17	16	.6998	.7845	3.0271	.2068	.5461	.6879	.8349	.5974
22	21	.7525	.8812	.0702	.3237	.5966	.7974	.8753	.8053
27	26	.8052	.9778	.1133	.4406	.6471	.9069	.9157	15.0133
32	31	+3.8579	−7.0745	+3.1565	−8.5575	+3.6976	−8.0164	+2.9561	+15.2212

TABLE I. Annual Variation in Right Ascension and Declination.

Argument. The Day of the Year.

Argument.		17 Regulus.		18 α Ursæ Major.		19 β Leonis.		20 β Virginis.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0088	− 0.0474	+0.0105	−0.0528	+0.0084	−0.0549	+0.0085	−0.0548
6	6	.0527	.2847	.0629	.3170	.0504	.3292	.0513	.3285
11	11	.0967	.5219	.1153	.5812	.0925	.6035	.0940	.6023
16	16	.1406	.7592	.1677	.8455	.1345	.8779	.1367	.8761
21	21	.1846	.9964	.2201	1.1097	.1765	1.1522	.1794	1.1499
26	26	.2285	1.2336	.2725	.3739	.2185	.4265	.2221	.4237
31	31	.2724	.4709	.3249	.6381	.2606	.7009	.2648	.6975
Feb. 5	Feb. 5	.3164	.7081	.3773	.9023	.3026	.9752	.3075	.9713
10	10	.3603	.9453	.4297	2.1665	.3446	2.2495	.3502	2.2450
15	15	.4043	2.1826	.4821	.4307	.3866	.5239	.3929	.5188
20	20	.4482	.4198	.5345	.6949	.4287	.7982	.4356	.7926
25	25	.4922	.6570	.5869	.9591	.4707	3.0725	.4784	3.0664
Mar. 2	Mar. 1	.5361	.8943	.6393	3.2233	.5127	.3469	.5211	.3402
7	6	.5800	3.1315	.6917	.4875	.5547	.6212	.5638	.6140
12	11	.6240	.3687	.7441	.7517	.5968	.8955	.6065	.8877
17	16	.6679	.6060	.7965	4.0159	.6388	4.1699	.6492	4.1615
22	21	.7119	.8432	.8489	.2801	.6808	.4442	.6919	.4353
27	26	.7558	4.0804	.9013	.5443	.7228	.7185	.7346	.7091
April 1	31	.7998	.3177	.9537	.8085	.7649	.9929	.7773	.9829
6	April 5	.8437	.5549	1.0061	5.0727	.8069	5.2672	.8200	5.2567
11	10	.8876	.7921	.0585	.3369	.8489	.5415	.8627	.5305
16	15	.9316	5.0294	.1109	.6011	.8910	.8159	.9055	.8042
21	20	.9755	.2666	.1633	.8653	.9330	6.0902	.9482	6.0780
26	25	1.0195	.5039	.2157	6.1295	.9750	.3645	.9909	.3518
May 1	30	.0634	.7411	.2681	.3937	1.0170	.6389	1.0336	.6256
6	May 5	.1074	.9783	.3205	.6579	.0591	.9132	.0763	.8994
11	10	.1513	6.2156	.3729	.9221	.1011	7.1875	.1190	7.1732
16	15	.1952	.4528	.4253	7.1863	.1431	.4619	.1617	.4470
21	20	.2392	.6900	.4777	.4505	.1851	.7362	.2044	.7207
26	25	.2831	.9273	.5301	.7147	.2272	8.0105	.2471	.9945
31	30	.3271	7.1645	.5825	.9789	.2692	.2849	.2898	8.2683
June 5	June 4	.3710	.4017	.6349	8.2431	.3112	.5592	.3326	.5421
10	9	.4150	.6390	.6873	.5073	.3532	.8335	.3753	.8159
15	14	.4589	.8762	.7397	.7715	.3953	9.1079	.4180	9.0897
20	19	+1.5028	−8.1134	+1.7921	−9.0357	+1.4373	−9.3822	+1.4607	−9.3634

Argument.		17 Regulus.		18 α Ursæ Major.		19 β Leonis.		20 β Virginis.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.5468	−8.3507	+1.8445	−9.2999	+1.4793	−9.6565	+1.5034	−9.6372
30	29	.5907	.5879	.8969	.5641	.5213	.9309	.5461	.9110
July 5	July 4	.6347	.8251	.9493	.8284	.5634	10.2052	.5888	10.1848
10	9	.6786	9.0624	2.0017	10.0926	.6054	.4795	.6315	.4586
15	14	.7226	.2996	.0541	.3568	.6474	.7539	.6742	.7324
20	19	.7665	.5368	.1065	.6210	.6894	11.0282	.7169	11.0062
25	24	.8104	.7741	.1589	.8852	.7315	.3025	.7597	.2799
30	29	.8544	10.0113	.2113	11.1494	.7735	.5769	.8024	.5537
Aug. 4	Aug. 3	.8983	.2486	.2637	.4136	.8155	.8512	.8451	.8275
9	8	.9423	.4858	.3161	.6778	.8575	12.1255	.8878	12.1013
14	13	.9862	.7230	.3685	.9420	.8996	.3999	.9305	.3751
19	18	2.0302	.9603	.4209	12.2062	.9416	.6742	.9732	.6489
24	23	.0741	11.1975	.4733	.4704	.9836	.9485	2.0159	.9227
29	28	.1181	.4347	.5257	.7346	2.0257	13.2229	.0586	13.1964
Sept. 3	Sept. 2	.1620	.6720	.5781	.9988	.0677	.4972	.1013	.4702
8	7	.2059	.9092	.6305	13.2630	.1097	.7715	.1440	.7440
13	12	.2499	12.1464	.6829	.5272	.1517	14.0459	.1868	14.0178
18	17	.2938	.3837	.7353	.7914	.1938	.3202	.2295	.2916
23	22	.3378	.6209	.7877	14.0556	.2358	.5945	.2722	.5654
28	27	.3817	.8581	.8401	.3198	.2778	.8688	.3149	.8391
Oct. 3	Oct. 2	.4257	13.0954	.8925	.5840	.3198	15.1432	.3576	15.1129
8	7	.4696	.3326	.9449	.8482	.3619	.4175	.4003	.3867
13	12	.5135	.5698	.9973	15.1124	.4039	.6918	.4430	.6605
18	17	.5575	.8071	3.0497	.3766	.4459	.9662	.4857	.9343
23	22	.6014	14.0443	.1021	.6408	.4879	16.2405	.5284	16.2081
28	27	.6454	.2815	.1545	.9050	.5300	.5148	.5711	.4819
Nov. 2	Nov. 1	.6893	.5188	.2069	16.1692	.5720	.7892	.6139	.7556
7	6	.7333	.7560	.2593	.4334	.6140	17.0635	.6566	17.0294
12	11	.7772	.9933	.3117	.6976	.6560	.3378	.6993	.3032
17	16	.8211	15.2305	.3641	.9618	.6981	.6122	.7420	.5770
22	21	.8651	.4677	.4165	17.2260	.7401	.8865	.7847	.8508
27	26	.9090	.7050	.4689	.4902	.7821	18.1608	.8274	18.1246
Dec. 2	Dec. 1	.9530	.9422	.5213	.7544	.8241	.4352	.8701	.3984
7	6	.9969	16.1794	.5737	18.0186	.8662	.7095	.9128	.6721
12	11	3.0409	.4167	.6261	.2828	.9082	.9838	.9555	.9459
17	16	.0848	.6539	.6785	.5471	.9502	19.2582	.9982	19.2197
22	21	.1287	.8911	.7309	.8113	.9923	.5325	3.0410	.4935
27	26	.1727	17.1284	.7833	19.0755	3.0343	.8068	.0837	.7673
32	31	+3.2166	−17.3656	+3.8357	−19.3397	+3.0763	−20.0812	+3.1264	−20.0411

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The Day of the Year.

Argument.		21 γ Ursæ Major.		22 Spica Virginis.		23 η Ursæ Major.		24 Arcturus.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0088	−0.0548	+0.0086	+0.0519	+0.0065	−0.0498	+0.0075	−0.0520
6	6	.0526	.3285	.0516	.3113	.0391	.2990	.0448	.3120
11	11	.0964	.6023	.0946	.5707	.0717	.5481	.0822	.5719
16	16	.1402	.8761	.1375	.8301	.1043	.7973	.1196	.8319
21	21	.1840	1.1499	.1805	1.0895	.1368	1.0464	.1570	1.0918
26	26	.2278	.4237	.2235	.3489	.1694	.2956	.1943	.3518
31	31	.2716	.6975	.2665	.6083	.2020	.5447	.2317	.6117
Feb. 5	Feb. 5	.3154	.9713	.3095	.8678	.2346	.7938	.2691	.8717
10	10	.3592	2.2450	.3525	2.1272	.2672	2.0430	.3065	2.1317
15	15	.4030	.5188	.3955	.3866	.2997	.2921	.3438	.3916
20	20	.4468	.7926	.4384	.6460	.3323	.5413	.3812	.6516
25	25	.4906	3.0664	.4814	.9054	.3649	.7904	.4186	.9115
Mar. 2	Mar. 1	.5344	.3402	.5244	3.1648	.3975	3.0396	.4559	3.1715
7	6	.5782	.6140	.5674	.4242	.4301	.2887	.4933	.4315
12	11	.6220	.8877	.6104	.6836	.4626	.5379	.5307	.6914
17	16	.6659	4.1615	.6534	.9430	.4952	.7870	.5681	.9514
22	21	.7097	.4353	.6963	4.2025	.5278	4.0361	.6054	4.2113
27	26	.7535	.7091	.7393	.4619	.5604	.2853	.6428	.4713
April 1	31	.7973	.9829	.7823	.7213	.5930	.5344	.6802	.7313
6	April 5	.8411	5.2567	.8253	.9807	.6255	.7836	.7175	.9912
11	10	.8849	.5305	.8683	5.2401	.6581	5.0327	.7549	5.2512
16	15	.9287	.8042	.9113	.4995	.6907	.2819	.7923	.5111
21	20	.9725	6.0780	.9542	.7589	.7233	.5310	.8297	.7711
26	25	1.0163	.3518	.9972	6.0183	.7559	.7802	.8670	6.0310
May 1	30	.0601	.6256	1.0402	.2777	.7884	6.0293	.9044	.2910
6	May 5	.1039	.8994	.0832	.5372	.8210	.2785	.9418	.5510
11	10	.1477	7.1732	.1262	.7966	.8536	.5276	.9791	.8109
16	15	.1915	.4470	.1692	7.0560	.8862	.7767	1.0165	7.0709
21	20	.2353	.7207	.2121	.3154	.9188	7.0259	.0539	.3308
26	25	.2791	.9945	.2551	.5748	.9513	.2750	.0913	.5908
31	30	.3229	8.2683	.2981	.8342	.9839	.5242	.1286	.8508
June 5	June 4	.3667	.5421	.3411	8.0936	1.0165	.7733	.1660	8.1107
10	9	.4106	.8159	.3841	.3530	.0491	8.0225	.2034	.3707
15	14	.4544	9.0897	.4271	.6124	.0817	.2716	.2408	.6306
20	19	+1.4982	−9.3634	+1.4701	+8.8719	+1.1142	−8.5208	+1.2781	−8.8906

Argument.		21 γ Ursæ Major.		22 Spica Virginis.		23 η Ursæ Major.		24 Arcturus.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.5420	−9.6372	+1.5130	+9.1313	+1.1468	−8.7699	+1.3155	−9.1506
30	29	.5858	.9110	.5560	.3907	.1794	9.0190	.3529	.4105
July 5	July 4	.6296	10.1848	.5990	.6501	.2120	.2682	.3902	.6705
10	9	.6734	.4586	.6420	.9095	.2446	.5173	.4276	.9304
15	14	.7172	.7324	.6850	10.1689	.2771	.7665	.4650	10.1904
20	19	.7610	11.0062	.7280	.4283	.3097	10.0156	.5024	.4504
25	24	.8048	.2799	.7709	.6877	.3423	.2648	.5397	.7103
30	29	.8486	.5537	.8139	.9471	.3749	.5139	.5771	.9703
Aug. 4	Aug. 3	.8924	.8275	.8569	11.2066	.4076	.7631	.6145	11.2302
9	8	.9362	12.1013	.8999	.4660	.4400	11.0122	.6518	.4902
14	13	.9800	.3751	.9429	.7254	.4726	.2614	.6892	.7501
19	18	2.0238	.6489	.9859	.9848	.5052	.5105	.7266	12.0101
24	23	.0676	.9227	2.0288	12.2442	.5378	.7596	.7640	.2701
29	28	.1114	13.1964	.0718	.5036	.5704	12.0088	.8013	.5300
Sept. 3	Sept. 2	.1553	.4702	.1148	.7630	.6029	.2579	.8387	.7900
8	7	.1991	.7440	.1578	13.0224	.6355	.5071	.8761	13.0499
13	12	.2429	14.0178	.2008	.2818	.6681	.7562	.9134	.3099
18	17	.2867	.2916	.2438	.5413	.7002	13.0054	.9508	.5699
23	22	.3305	.5654	.2867	.8007	.7333	.2545	.9882	.8298
28	27	.3743	.8391	.3297	14.0601	.7658	.5037	2.0256	14.0898
Oct. 3	Oct. 2	.4181	15.1129	.3727	.3195	.7984	.7528	.0629	.3497
8	7	.4619	.3867	.4157	.5789	.8310	14.0019	.1003	.6097
13	12	.5057	.6605	.4587	.8383	.8636	.2511	.1377	.8697
18	17	.5495	.9343	.5017	15.0977	.8962	.5002	.1751	15.1296
23	22	.5933	16.2081	.5447	.3571	.9287	.7494	.2124	.3896
28	27	.6371	.4819	.5876	.6165	.9613	.9985	.2498	.6495
Nov. 2	Nov. 1	.6809	.7556	.6306	.8760	.9939	15.2477	.2872	.9095
7	6	.7247	17.0294	.6736	16.1354	2.0265	.4968	.3245	16.1695
12	11	.7685	.3032	.7166	.3948	.0591	.7460	.3619	.4294
17	16	.8125	.5770	.7596	.6542	.0916	.9951	.3993	.6894
22	21	.8562	.8508	.8026	.9136	.1242	16.2443	.4367	.9493
27	26	.9000	18.1246	.8455	17.1730	.1568	.4934	.4740	17.2093
Dec. 2	Dec. 1	.9438	.3984	.8885	.4324	.1894	.7425	.5114	.4692
7	6	.9876	.6721	.9315	.6918	.2220	.9917	.5488	.7292
12	11	3.0314	.9459	.9745	.9512	.2545	17.2408	.5861	.9892
17	16	.0752	19.2197	3.0175	18.2107	.2871	.4900	.6235	18.2491
22	21	.1190	.4935	.0605	.4701	.3197	.7391	.6609	.5091
27	26	.1628	.7673	.1034	.7295	.3523	.9883	.6983	.7690
32	31	+3.2066	−20.0411	+3.1464	+18.9889	+2.3849	−18.2374	+2.7356	−19.0290

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The Day of the Year.

Argument.		25, 26 α² Libræ.		27 β Ursæ Minor.		28 α Cor. Bor.		29 α Serpentis.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0090	+0.0416	−0.0009	−0.0402	+0.0069	−0.0342	+0.0080	−0.0321
6	6	.0540	.2497	.0053	.2415	.0416	.2052	.0483	.1927
11	11	.0991	.4578	.0096	.4427	.0762	.3762	.0885	.3533
16	16	.1441	.6658	.0140	.6439	.1108	.5471	.1288	.5138
21	21	.1892	.8739	.0184	.8452	.1455	.7181	.1690	.6744
26	26	.2342	1.0820	.0228	1.0464	.1801	.8891	.2093	.8350
31	31	.2792	.2901	.0272	.2476	.2147	1.0601	.2495	.9956
Feb. 5	Feb. 5	.3243	.4982	.0315	.4489	.2494	.2310	.2898	1.1561
10	10	.3693	.7062	.0359	.6501	.2840	.4020	.3300	.3167
15	15	.4144	.9143	.0403	.8513	.3186	.5730	.3703	.4773
20	20	.4594	2.1224	.0447	2.0526	.3533	.7440	.4105	.6379
25	25	.5044	.3305	.0491	.2538	.3879	.9150	.4508	.7984
Mar. 2	Mar. 1	.5495	.5385	.0534	.4550	.4225	2.0859	.4910	.9590
7	6	.5945	.7466	.0578	.6563	.4572	.2569	.5312	2.1196
12	11	.6395	.9547	.0622	.8575	.4918	.4279	.5715	.2802
17	16	.6846	3.1628	.0666	3.0587	.5264	.5989	.6117	.4407
22	21	.7296	.3708	.0710	.2600	.5611	.7699	.6520	.6013
27	26	.7747	.5789	.0754	.4612	.5957	.9408	.6922	.7619
April 1	31	.8197	.7870	.0797	.6624	.6303	3.1118	.7325	.9225
6	April 5	.8647	.9951	.0841	.8637	.6650	.2828	.7727	3.0830
11	10	.9098	4.2032	.0885	4.0649	.6996	.4538	.8130	.2436
16	15	.9548	.4112	.0929	.2661	.7342	.6248	.8532	.4042
21	20	.9998	.6193	.0973	.4674	.7689	.7957	.8935	.5648
26	25	1.0449	.8274	.1016	.6686	.8035	.9667	.9337	.7253
May 1	30	.0899	5.0355	.1060	.8698	.8381	4.1377	.9740	.8859
6	May 5	.1350	.2435	.1104	5.0710	.8728	.3087	1.0142	4.0465
11	10	.1800	.4516	.1148	.2723	.9074	.4797	.0544	.2071
16	15	.2250	.6597	.1192	.4735	.9420	.6506	.0947	.3676
21	20	.2701	.8678	.1235	.6747	.9767	.8216	.1349	.5282
26	25	.3151	6.0758	.1279	.8760	1.0113	.9926	.1752	.6888
31	30	.3601	.2839	.1323	6.0772	.0459	5.1636	.2154	.8494
June 5	June 4	.4052	.4920	.1367	.2784	.0806	.3345	.2557	5.0099
10	9	.4502	.7001	.1411	.4797	.1152	.5055	.2959	.1705
15	14	.4953	.9082	.1454	.6809	.1498	.6765	.3362	.3311
20	19	+1.5403	+7.1162	−0.1498	−6.8821	+1.1845	−5.8475	+1.3764	−5.4917

Argument.		25, 26 α² Libræ.		27 β Ursæ Minor.		28 α Cor. Bor.		29 α Serpentis.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.5853	+7.3243	−0.1542	−7.0834	+1.2191	−6.0185	+1.4167	−5.6522
30	29	.6304	.5324	.1586	.2846	.2538	.1894	.4569	.8128
July 5	July 4	.6754	.7405	.1630	.4858	.2884	.3604	.4972	.9734
10	9	.7205	.9485	.1674	.6871	.3230	.5314	.5374	6.1340
15	14	.7655	8.1566	.1717	.8883	.3577	.7024	.5776	.2945
20	19	.8105	.3647	.1761	8.0895	.3923	.8734	.6179	.4551
25	24	.8556	.5728	.1805	.2908	.4269	7.0443	.6581	.6157
30	29	.9006	.7808	.1849	.4920	.4616	.2153	.6984	.7763
Aug. 4	Aug. 3	.9456	.9889	.1893	.6932	.4962	.3863	.7386	.9368
9	8	.9907	9.1970	.1936	.8945	.5308	.5573	.7789	7.0974
14	13	2.0357	.4051	.1980	9.0957	.5655	.7283	.8191	.2580
19	18	.0808	.6132	.2024	.2969	.6001	.8992	.8594	.4186
24	23	.1258	.8212	.2068	.4982	.6347	8.0702	.8996	.5791
29	28	.1708	10.0293	.2112	.6994	.6694	.2412	.9399	.7397
Sept. 3	Sept. 2	.2159	.2374	.2155	.9006	.7040	.4122	.9801	.9003
8	7	.2609	.4455	.2199	10.1018	.7386	.5831	2.0203	8.0609
13	12	.3059	.6535	.2243	.3031	.7733	.7541	.0606	.2214
18	17	.3510	.8616	.2287	.5043	.8079	.9251	.1008	.3820
23	22	.3960	11.0697	.2331	.7055	.8425	9.0961	.1411	.5426
28	27	.4411	.2778	.2375	.9068	.8772	.2671	.1813	.7032
Oct. 3	Oct. 2	.4861	.4859	.2418	11.1080	.9118	.4380	.2216	.8637
8	7	.5311	.6939	.2462	.3092	.9464	.6090	.2618	9.0243
13	12	.5762	.9020	.2506	.5105	.9811	.7800	.3021	.1849
18	17	.6212	12.1101	.2550	.7117	2.0157	.9510	.3423	.3455
23	22	.6663	.3182	.2594	.9129	.0503	10.1220	.3826	.5060
28	27	.7113	.5262	.2637	12.1142	.0850	.2929	.4228	.6666
Nov. 2	Nov. 1	.7563	.7343	.2681	.3154	.1196	.4639	.4631	.8272
7	6	.8014	.9424	.2725	.5166	.1542	.6349	.5033	.9878
12	11	.8464	13.1505	.2769	.7179	.1889	.8059	.5435	10.1483
17	16	.8914	.3585	.2813	.9191	.2235	.9769	.5838	.3089
22	21	.9365	.5666	.2856	13.1203	.2581	11.1478	.6240	.4695
27	26	.9815	.7747	.2900	.3216	.2928	.3188	.6643	.6301
Dec. 2	Dec. 1	3.0266	.9828	.2944	.5228	.3274	.4898	.7045	.7906
7	6	.0716	14.1909	.2988	.7240	.3620	.6608	.7448	.9512
12	11	.1166	.3989	.3032	.9253	.3967	.8317	.7850	11.1118
17	16	.1617	.6070	.3075	14.1265	.4313	12.0027	.8253	.2724
22	21	.2067	.8151	.3119	.3277	.4659	.1737	.8655	.4329
27	26	.2517	15.0232	.3163	.5290	.5006	.3447	.9058	.5935
32	31	+3.2968	+15.2312	−0.3207	−14.7302	+2.5352	−12.5157	+2.9460	−11.7541

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The Day of the Year.

Argument.		30 Antares.		31 α Herculis.		32 α Ophiuchi.		33 γ Draconis.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0100	+0.0236	+0.0075	−0.0123	+0.0076	−0.0085	+0.0038	−0.0019
6	6	.0601	.1416	.0448	.0736	.0455	.0509	.0227	.0115
11	11	.1102	.2596	.0822	.1349	.0834	.0934	.0416	.0211
16	16	.1603	.3776	.1196	.1963	.1213	.1358	.0605	.0307
21	21	.2104	.4956	.1570	.2576	.1593	.1782	.0793	.0402
26	26	.2605	.6136	.1943	.3189	.1972	.2207	.0982	.0498
31	31	.3106	.7316	.2317	.3802	.2351	.2631	.1171	.0594
Feb. 5	Feb. 5	.3607	.8496	.2691	.4416	.2730	.3055	.1360	.0690
10	10	.4108	.9676	.3065	.5029	.3109	.3480	.1549	.0786
15	15	.4609	1.0856	.3438	.5642	.3489	.3904	.1738	.0882
20	20	.5111	.2036	.3812	.6255	.3868	.4329	.1927	.0977
25	25	.5612	.3216	.4186	.6869	.4247	.4753	.2116	.1073
Mar. 2	Mar. 1	.6113	.4396	.4559	.7482	.4626	.5177	.2305	.1169
7	6	.6614	.5576	.4933	.8095	.5005	.5602	.2494	.1265
12	11	.7115	.6756	.5307	.8709	.5385	.6026	.2683	.1361
17	16	.7616	.7936	.5681	.9322	.5764	.6450	.2871	.1456
22	21	.8117	.9116	.6054	.9935	.6143	.6875	.3060	.1552
27	26	.8618	2.0296	.6428	1.0548	.6522	.7299	.3249	.1648
April 1	31	.9119	.1476	.6802	.1162	.6901	.7724	.3438	.1744
6	April 5	.9620	.2656	.7175	.1775	.7280	.8148	.3627	.1840
11	10	1.0121	.3836	.7549	.2388	.7660	.8572	.3816	.1936
16	15	.0622	.5016	.7923	.3002	.8039	.8997	.4005	.2031
21	20	.1123	.6196	.8297	.3615	.8418	.9421	.4194	.2127
26	25	.1624	.7376	.8670	.4228	.8797	.9845	.4383	.2223
May 1	30	.2125	.8556	.9044	.4841	.9176	1.0270	.4572	.2319
6	May 5	.2626	.9736	.9418	.5455	.9556	.0694	.4761	.2415
11	10	.3127	3.0916	.9791	.6068	.9935	.1118	.4949	.2510
16	15	.3628	.2096	1.0165	.6681	1.0314	.1543	.5138	.2606
21	20	.4129	.3276	.0539	.7295	.0693	.1967	.5327	.2702
26	25	.4630	.4456	.0913	.7908	.1072	.2392	.5516	.2798
31	30	.5131	.5636	.1286	.8521	.1452	.2816	.5705	.2894
June 5	June 4	.5632	.6816	.1660	.9134	.1831	.3240	.5894	.2990
10	9	.6133	.7996	.2034	.9748	.2210	.3665	.6083	.3085
15	14	.6634	.9176	.2408	2.0361	.2589	.4089	.6272	.3181
20	19	+1.7135	+4.0356	+1.2781	−2.0974	+1.2968	−1.4513	+0.6461	−0.3277

Argument.		30 Antares.		31 α Herculis.		32 α Ophiuchi.		33 γ Draconis.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.7636	+4.1536	+1.3155	−2.1587	+1.3347	−1.4938	+0.6650	−0.3373
30	29	.8137	.2716	.3529	.2201	.3727	.5362	.6839	.3469
July 5	July 4	.8638	.3896	.3902	.2814	.4106	.5787	.7027	.3565
10	9	.9139	.5076	.4276	.3427	.4485	.6211	.7216	.3660
15	14	.9640	.6256	.4650	.4041	.4864	.6635	.7405	.3756
20	19	2.0141	.7436	.5024	.4654	.5243	.7060	.7594	.3852
25	24	.0642	.8616	.5397	.5267	.5623	.7484	.7783	.3948
30	29	.1143	.9796	.5771	.5880	.6002	.7908	.7972	.4044
Aug. 4	Aug. 3	.1645	5.0976	.6145	.6494	.6381	.8333	.8161	.4139
9	8	.2146	.2156	.6518	.7107	.6760	.8757	.8350	.4235
14	13	.2647	.3336	.6892	.7720	.7139	.9182	.8539	.4331
19	18	.3148	.4516	.7266	.8334	.7519	.9606	.8728	.4427
24	23	.3649	.5696	.7640	.8947	.7898	2.0030	.8917	.4523
29	28	.4150	.6876	.8013	.9560	.8277	.0455	.9105	.4619
Sept. 3	Sept. 2	.4651	.8056	8387	3.0173	.8656	.0879	.9294	.4714
8	7	.5152	.9237	.8761	.0787	.9035	.1303	.9483	.4810
13	12	.5653	6.0417	.9134	.1400	.9415	.1728	.9672	.4906
18	17	.6154	.1597	.9508	.2013	.9794	.2152	.9861	.5002
23	22	.6655	.2777	.9882	.2627	2.0173	.2576	1.0050	.5098
28	27	.7156	.3957	2.0256	.3240	.0552	.3001	.0239	.5193
Oct. 3	Oct. 2	.7657	.5137	.0629	.3853	.0931	.3425	.0428	.5289
8	7	.8158	.6317	.1003	.4466	.1310	.3850	.0617	.5385
13	12	.8659	.7497	.1377	.5080	.1690	.4274	.0806	.5481
18	17	.9160	.8677	.1751	.5693	.2069	.4698	.0995	.5577
23	22	.9661	.9857	.2124	.6306	.2448	.5123	.1183	.5673
28	27	3.0162	7.1037	.2498	.6919	.2827	.5547	.1372	.5768
Nov. 2	Nov. 1	.0663	.2217	.2872	.7533	.3206	.5971	.1561	.5864
7	6	.1164	.3397	.3245	.8146	.3586	.6396	.1750	.5960
12	11	.1665	.4577	.3619	.8759	.3965	.6820	.1939	.6056
17	16	.2166	.5757	.3993	.9373	.4344	.7245	.2128	.6152
22	21	.2667	.6937	.4367	.9986	.4723	.7669	.2317	.6247
27	26	.3168	.8117	.4740	4.0599	.5102	.8093	.2506	.6343
Dec. 2	Dec. 1	.3669	.9297	.5114	.1212	.5482	.8518	.2695	.6439
7	6	.4170	8.0477	.5488	.1826	.5861	.8942	.2884	.6535
12	11	.4671	.1657	.5861	.2439	.6240	.9366	.3073	.6631
17	16	.5172	.2837	.6235	.3052	.6619	.9791	.3261	.6727
22	21	.5673	.4017	.6609	.3666	.6998	3.0215	.3450	.6822
27	26	.6174	.5197	.6983	.4279	.7378	.0640	.3639	.6918
32	31	+3.6675	+8.6377	+2.7356	−4.4892	+2.7757	−3.1064	+1.3828	−0.7014

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The Day of the Year.

Argument.		34 α Lyræ.		35 γ Aquilæ.		36 α Aquilæ.		37 β Aquilæ.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0056	+0.0082	+0.0078	+0.0229	+0.0080	+0.0248	+0.0081	+0.0235
6	6	.0333	.0493	.0468	.1377	.0481	.1488	.0485	.1408
11	11	.0611	.0904	.0858	.2524	.0882	.2729	.0888	.2581
16	16	.0889	.1314	.1248	.3671	.1284	.3969	.1292	.3754
21	21	.1167	.1725	.1639	.4818	.1685	.5209	.1696	.4927
26	26	.1445	.2136	.2029	.5965	.2086	.6449	.2100	.6100
31	31	.1723	.2546	.2419	.7112	.2487	.7690	.2504	.7274
Feb. 5	Feb. 5	.2001	.2957	.2809	.8260	.2888	.8930	.2908	.8447
10	10	.2279	.3368	.3199	.9407	.3289	1.0170	.3311	.9620
15	15	.2557	.3778	.3589	1.0554	.3690	.1410	.3715	1.0793
20	20	.2834	.4189	.3980	.1701	.4091	.2651	.4119	.1966
25	25	.3112	.4600	.4370	.2848	.4492	.3891	.4523	.3140
Mar. 2	Mar. 1	.3390	.5010	.4760	.3995	.4893	.5131	.4927	.4313
7	6	.3668	.5421	.5150	.5143	.5295	.6371	.5331	.5486
12	11	.3946	.5832	.5540	.6290	.5696	.7612	.5734	.6659
17	16	.4224	.6242	.5930	.7437	.6097	.8852	.6138	.7832
22	21	.4502	.6653	.6320	.8584	.6498	2.0092	.6542	.9005
27	26	.4780	.7064	.6711	.9731	.6899	.1332	.6946	2.0179
April 1	31	.5058	.7474	.7101	2.0878	.7300	.2573	.7350	.1352
6	April 5	.5335	.7885	.7491	.2025	.7701	.3813	.7754	.2525
11	10	.5613	.8296	.7881	.3173	.8102	.5053	.8157	.3698
16	15	.5891	.8706	.8271	.4320	.8503	.6293	.8561	.4871
21	20	.6169	.9117	.8661	.5467	.8904	.7534	.8965	.6044
26	25	.6447	.9528	.9051	.6614	.9306	.8774	.9369	.7218
May 1	30	.6725	.9938	.9442	.7761	.9707	3.0014	.9773	.8391
6	May 5	.7003	1.0349	.9832	.8908	1.0108	.1254	1.0177	.9564
11	10	.7281	.0760	1.0222	3.0056	.0509	.2495	.0580	3.0737
16	15	.7559	.1171	.0612	.1203	.0910	.3735	.0984	.1910
21	20	.7837	.1581	.1002	.2350	.1311	.4975	.1388	.3083
26	25	.8114	.1992	.1392	.3497	.1712	.6215	.1792	.4257
31	30	.8392	.2403	.1783	.4644	.2113	.7456	.2196	.5430
June 5	June 4	.8670	.2813	.2173	.5791	.2514	.8696	.2600	.6603
10	9	.8948	.3224	.2563	.6939	.2915	.9936	.3003	.7776
15	14	.9226	.3635	.2953	.8086	.3317	4.1176	.3407	.8949
20	19	+0.9504	+1.4045	+1.3343	+3.9233	+1.3718	+4.2417	+1.3811	+4.0122

Argument.		34 α Lyræ.		35 Aquilæ.		36 α Aquilæ.		37 β Aquilæ.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+0.9782	+1.4456	+1.3733	+4.0380	+1.4119	+4.3657	+1.4215	+4.1296
30	29	1.0060	.4867	.4123	.1527	.4520	.4897	.4619	.2469
July 5	July 4	.0338	.5277	.4514	.2674	.4921	.6137	.5022	.3642
10	9	.0615	.5688	.4904	.3822	.5322	.7378	.5426	.4815
15	14	.0893	.6099	.5294	.4969	.5723	.8618	.5830	.5988
20	19	.1171	.6509	.5684	.6116	.6124	.9858	.6234	.7161
25	24	.1449	.6920	.6074	.7263	.6525	5.1098	.6638	.8335
30	29	.1727	.7331	.6464	.8410	.6926	.2339	.7042	.9508
Aug. 4	Aug. 3	.2005	.7741	.6854	.9557	.7328	.3579	.7445	5.0681
9	8	.2283	.8152	.7245	5.0704	.7729	.4819	.7849	.1854
14	13	.2561	.8563	.7635	.1852	.8130	.6059	.8253	.3027
19	18	.2838	.8973	.8025	.2999	.8531	.7300	.8657	.4200
24	23	.3116	.9384	.8415	.4146	.8932	.8540	.9061	.5374
29	28	.3394	.9795	.8805	.5293	.9333	.9780	.9465	.6547
Sept. 3	Sept. 2	.3672	2.0205	.9195	.6440	.9734	6.1020	.9868	.7720
8	7	.3950	.0616	.9586	.7587	2.0135	.2261	2.0272	.8893
13	12	.4228	.1027	.9976	.8735	.0536	.3501	.0676	6.0066
18	17	.4506	.1438	2.0366	.9882	.0937	.4741	.1080	.1239
23	22	.4784	.1848	.0756	6.1029	.1339	.5981	.1484	.2413
28	27	.5062	.2259	.1146	.2176	.1740	.7222	.1888	.3586
Oct. 3	Oct. 2	.5340	.2670	.1536	.3323	.2141	.8462	.2291	.4759
8	7	.5617	.3080	.1926	.4470	.2542	.9702	.2695	.5932
13	12	.5895	.3491	.2317	.5618	.2943	7.0942	.3099	.7105
18	17	.6173	.3902	.2707	.6765	.3344	.2183	.3503	.8278
23	22	.6451	.4312	.3097	.7912	.3745	.3423	.3907	.9452
28	27	.6729	.4723	.3487	.9059	.4146	.4663	.4311	7.0625
Nov. 2	Nov. 1	.7007	.5134	.3877	7.0206	.4547	.5903	.4714	.1798
7	6	.7285	.5544	.4267	.1353	.4948	.7144	.5118	.2971
12	11	.7563	.5955	.4657	.2501	.5350	.8384	.5522	.4144
17	16	.7841	.6366	.5048	.3648	.5751	.9624	.5926	.5318
22	21	.8118	.6776	.5438	.4795	.6152	8.0864	.6330	.6491
27	26	.8396	.7187	.5828	.5942	.6553	.2105	.6734	.7664
Dec. 2	Dec. 1	.8674	.7598	.6218	.7089	.6954	.3345	.7137	.8837
7	6	.8952	.8008	.6608	.8236	.7355	.4585	.7541	8.0010
12	11	.9230	.8419	.6998	.9383	.7756	.5825	.7945	.1183
17	16	.9508	.8830	.7389	8.0531	.8157	.7066	.8349	.2357
22	21	.9786	.9240	.7779	.1678	.8558	.8306	.8753	.3530
27	26	2.0064	.9651	.8169	.2825	.8959	.9546	.9157	.4703
32	31	+2.0342	+3.0062	+2.8559	+8.3972	+2.9361	+9.0786	+2.9560	+8.5876

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The Day of the Year.

Argument.		38, 39 α^2 Capricorni.		40 α Cygni.		41 α Cephei.		42 β Cephei.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0091	−0.0296	+0.0056	+0.0346	+0.0039	+0.0408	+0.0022	+0.0430
6	6	.0547	.1774	.0335	.2075	.0233	.2448	.0133	.2579
11	11	.1003	.3253	.0614	.3804	.0428	.4487	.0244	.4728
16	16	.1459	.4731	.0894	.5533	.0622	.6527	.0355	.6877
21	21	.1915	.6209	.1173	.7262	.0816	.8567	.0466	.9027
26	26	.2370	.7688	.1452	.8991	.1011	1.0606	.0577	1.1176
31	31	.2826	.9166	.1731	1.0719	.1205	.2646	.0687	.3325
Feb. 5	Feb. 5	.3282	1.0645	.2011	.2448	.1400	.4686	.0798	.5474
10	10	.3738	.2123	.2290	.4177	.1594	.6726	.0909	.7624
15	15	.4194	.3602	.2569	.5906	.1788	.8765	.1020	.9773
20	20	.4650	.5080	.2848	.7635	.1983	2.0805	.1131	2.1922
25	25	.5106	.6559	.3128	.9364	.2177	.2845	.1242	.4071
Mar. 2	Mar. 1	.5561	.8037	.3407	2.1093	.2372	.4884	.1353	.6220
7	6	.6017	.9515	.3686	.2822	.2566	.6924	.1464	.8370
12	11	.6473	2.0994	.3965	.4551	.2760	.8964	.1575	3.0519
17	16	.6929	.2472	.4245	.6280	.2955	3.1003	.1685	.2668
22	21	.7385	.3951	.4524	.8009	.3149	.3043	.1796	.4817
27	26	.7841	.5429	.4803	.9738	.3344	.5083	.1907	.6966
April 1	31	.8296	.6908	.5083	3.1467	.3538	.7123	.2018	.9116
6	April 5	.8752	.8386	.5362	.3196	.3732	.9162	.2129	4.1265
11	10	.9208	.9864	.5641	.4925	.3927	4.1202	.2240	.3414
16	15	.9664	3.1343	.5920	.6654	.4121	.3242	.2351	.5563
21	20	1.0120	.2821	.6200	.8383	.4315	.5282	.2462	.7712
26	25	.0576	.4300	.6479	4.0112	.4510	.7321	.2572	.9862
May 1	30	.1032	.5778	.6758	.1841	.4704	.9361	.2683	5.2011
6	May 5	.1487	.7257	.7037	.3570	.4899	5.1400	.2794	.4160
11	10	.1943	.8735	.7317	.5298	.5093	.3440	.2905	.6309
16	15	.2399	4.0213	.7596	.7027	.5287	.5480	.3016	.8459
21	20	.2855	.1692	.7875	.8756	.5482	.7520	.3127	6.0608
26	25	.3311	.3170	.8154	5.0485	.5676	.9559	.3238	.2757
31	30	.3767	.4649	.8434	.2214	.5871	6.1599	.3349	.4906
June 5	June 4	.4223	.6127	.8713	.3943	.6065	.3639	.3459	.7055
10	9	.4678	.7606	.8992	.5672	.6259	.5678	.3570	.9205
15	14	.5134	.9084	.9271	.7401	.6454	.7718	.3681	7.1354
20	19	+1.5590	−5.0563	+0.9551	+5.9130	+0.6648	+6.9758	+0.3792	+7.3503

Argument.		38} 39} α^2 Capricorni.		40 α Cygni.		41 α Cephei.		42 β Cephei.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.6046	−5.2041	+0.9830	+6.0859	+0.6843	+7.1797	+0.3903	+7.5652
30	29	.6502	.3520	1.0109	.2588	.7037	.3837	.4014	.7801
July 5	July 4	.6958	.4998	.0388	.4317	.7231	.5877	.4125	.9951
10	9	.7413	.6476	.0668	.6046	.7426	.7917	.4236	8.2100
15	14	.7869	.7955	.0947	.7775	.7620	.9956	.4347	.4249
20	19	.8325	.9433	.1226	.9504	.7814	8.1996	.4457	.6398
25	24	.8781	6.0912	.1506	7.1233	.8009	.4036	.4568	.8547
30	29	.9237	.2390	.1785	.2962	.8203	.6075	.4679	9.0697
Aug. 4	Aug. 3	.9693	.3869	.2064	.4691	.8398	.8115	.4790	.2846
9	8	2.0149	.5347	.2343	.6420	.8592	9.0155	.4901	.4995
14	13	.0604	.6825	.2623	.8149	.8786	.2194	.5012	.7144
19	18	.1060	.8304	.2902	.9877	.8981	.4234	.5123	.9294
24	23	.1516	.9782	.3181	8.1606	.9175	.6274	.5234	10.1443
29	28	.1972	7.1261	.3460	.3335	.9370	.8314	.5344	.3592
Sept. 3	Sept. 2	.2428	.2739	.3740	.5064	.9564	10.0353	.5455	.5741
8	7	.2884	.4218	.4019	.6793	.9758	.2393	.5566	.7890
13	12	.3340	.5696	.4298	.8522	.9953	.4433	.5677	11.0040
18	17	.3795	.7175	.4577	9.0251	1.0147	.6472	.5788	.2189
23	22	.4251	.8653	.4857	.1980	.0342	.8512	.5899	.4338
28	27	.4707	8.0131	.5136	.3709	.0536	11.0552	.6010	.6487
Oct. 3	Oct. 2	.5163	.1610	.5415	.5438	.0730	.2591	.6121	.8636
8	7	.5619	.3088	.5694	.7167	.0925	.4631	.6231	12.0786
13	12	.6075	.4567	.5974	.8896	.1119	.6671	.6342	.2935
18	17	.6530	.6045	.6253	10.0625	.1314	.8711	.6453	.5084
23	22	.6986	.7524	.6532	.2354	.1508	12.0750	.6564	.7233
28	27	.7442	.9002	.6811	.4083	.1702	.2790	.6675	.9382
Nov. 2	Nov. 1	.7898	9.0481	.7091	.5812	.1897	.4830	.6786	13.1532
7	6	.8354	.1959	.7370	.7541	.2091	.6869	.6897	.3681
12	11	.8810	.3437	.7649	.9270	.2285	.8909	.7008	.5830
17	16	.9266	.4916	.7928	11.0999	.2480	13.0949	.7119	.7979
22	21	.9721	.6394	.8208	.2728	.2674	.2988	.7229	14.0128
27	26	3.0177	.7873	.8487	.4456	.2869	.5028	.7340	.2278
Dec. 2	Dec. 1	.0633	.9351	.8766	.6185	.3063	.7068	.7451	.4427
7	6	.1089	10.0830	.9046	.7914	.3257	.9108	.7562	.6576
12	11	.1545	.2308	.9325	.9643	.3452	14.1147	.7673	.8725
17	16	.2001	.3786	.9604	12.1372	.3646	.3187	.7784	15.0875
22	21	.2457	.5265	.9883	.3101	.3841	.5227	.7895	.3024
27	26	.2912	.6743	2.0163	.4830	.4035	.7266	.8006	.5173
32	31	+3.3368	−10.8222	+2.0442	+12.6559	+1.4229	+14.9306	+0.8116	+15.7322

TABLE I. Annual Variation in Right Ascension and Declination.

ARGUMENT. The Day of the Year.

Argument.		43 α Aquarii.		44 Fomalhaut.		45 α Pegasi.		46 α Andromedæ.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
Jan. 1	Jan. 1	+0.0085	− 0.0476	+0.0091	−0.0523	+0.0082	+0.0532	+0.0084	+0.0547
6	6	.0508	.2853	.0549	.3138	.0490	.3192	.0506	.3284
11	11	.0931	.5231	.1006	.5752	.0897	.5852	.0928	.6020
16	16	.1354	.7609	.1463	.8367	.1305	.8511	.1349	.8757
21	21	.1777	.9987	.1920	1.0982	.1713	1.1171	.1771	1.1493
26	26	.2200	1.2365	.2378	.3596	.2121	.3831	.2192	.4230
31	31	.2623	.4742	.2835	.6211	.2529	.6491	.2614	.6966
Feb. 5	Feb. 5	.3046	.7120	.3292	.8825	.2937	.9151	.3036	.9703
10	10	.3469	.9498	.3749	2.1440	.3345	2.1811	.3457	2.2439
15	15	.3892	2.1876	.4206	.4055	.3753	.4470	.3879	.5176
20	20	.4315	.4254	.4664	.6669	.4161	.7130	.4301	.7912
25	25	.4738	.6632	.5121	.9284	.4569	.9790	.4722	3.0649
Mar. 2	Mar. 1	.5161	.9009	.5578	3.1899	.4977	3.2450	.5144	.3385
7	6	.5584	3.1387	.6035	.4513	.5385	.5110	.5566	.6122
12	11	.6007	.3765	.6493	.7128	.5793	.7769	.5987	.8858
17	16	.6430	.6143	.6950	.9743	.6201	4.0429	.6409	4.1595
22	21	.6853	.8521	.7407	4.2357	.6609	.3089	.6830	.4331
27	26	.7276	4.0899	.7864	.4972	.7017	.5749	.7252	.7067
April 1	31	.7699	.3276	.8321	.7587	.7425	.8409	.7674	.9804
6	April 5	.8122	.5654	.8779	5.0201	.7832	5.1069	.8095	5.2540
11	10	.8545	.8032	.9236	.2816	.8240	.3728	.8517	.5277
16	15	.8968	5.0410	.9693	.5431	.8648	.6388	.8939	.8013
21	20	.9391	.2788	1.0150	.8045	.9056	.9048	.9360	6.0750
26	25	.9814	.5165	.0608	6.0660	.9464	6.1708	.9782	.3486
May 1	30	1.0237	.7543	.1065	.3275	.9872	.4368	1.0203	.6223
6	May 5	.0660	.9921	.1522	.5889	1.0280	.7027	.0625	.8959
11	10	.1083	6.2299	.1979	.8504	.0688	.9687	.1047	7.1696
16	15	.1506	.4677	.2436	7.1118	.1096	7.2347	.1468	.4432
21	20	.1929	.7055	.2894	.3733	.1504	.5007	.1890	.7169
26	25	.2352	.9432	.3351	.6348	.1912	.7667	.2312	.9905
31	30	.2775	7.1810	.3808	.8962	.2320	8.0327	.2733	8.2642
June 5	June 4	.3198	.4188	.4265	8.1577	.2728	.2986	.3155	.5378
10	9	.3621	.6566	.4722	.4192	.3136	.5646	.3576	.8115
15	14	.4044	.8944	.5180	.6806	.3544	.8306	.3998	[illegible]
20	19	+1.4467	−8.1321	+1.5637	−8.9421	+1.3952	+9.0966	+1.4420	[illegible]

Argument.		43 α Aquarii.		44 Fomalhaut.		45 α Pegasi.		46 α Andromedæ.	
Common Years.	Bissextiles.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.	Æ in time.	Dec. in space.
June 25	June 24	+1.4890	−8.3699	+1.6094	−9.2036	+1.4359	+9.3626	+1.4841	+9.6324
30	29	.5313	.6077	.6551	.4650	.4767	.6285	.5263	.9061
July 5	July 4	.5736	.8455	.7009	.7265	.5175	.8945	.5685	10.1797
10	9	.6159	9.0833	.7466	.9880	.5583	10.1605	.6106	.4534
15	14	.6582	.3211	.7923	10.2494	.5991	.4265	.6528	.7270
20	19	.7005	.5588	.8380	.5109	.6399	.6925	.6950	11.0007
25	24	.7428	.7966	.8837	.7724	.6807	.9585	.7371	.2743
30	29	.7851	10.0344	.9295	11.0338	.7215	11.2244	.7793	.5479
Aug. 4	Aug. 3	.8274	.2722	.9752	.2953	.7623	.4904	.8214	.8216
9	8	.8697	.5100	2.0209	.5568	.8031	.7564	.8636	12.0952
14	13	.9120	.7477	.0666	.8182	.8439	12.0224	.9058	.3689
19	18	.9543	.9855	.1124	12.0797	.8847	.2884	.9479	.6425
24	23	.9966	11.2233	.1581	.3411	.9255	.5544	.9901	.9162
29	28	2.0389	.4611	.2038	.6026	.9663	.8203	2.0323	13.1898
Sept. 3	Sept. 2	.0812	.6989	.2495	.8641	2.0071	13.0863	.0744	.4635
8	7	.1235	.9367	.2952	13.1255	.0479	.3523	.1166	.7371
13	12	.1658	12.1744	.3410	.3870	.0887	.6183	.1587	14.0108
18	17	.2081	.4122	.3867	.6485	.1294	.8843	.2009	.2844
23	22	.2504	.6500	.4324	.9099	.1702	14.1502	.2431	.5581
28	27	.2927	.8878	.4781	14.1714	.2110	.4162	.2852	.8317
Oct. 3	Oct. 2	.3350	13.1256	.5239	.4329	.2518	.4822	.3274	15.1054
8	7	.3773	.3633	.5696	.6943	.2926	.9482	.3696	.3790
13	12	.4196	.6011	.6153	.9558	.3334	15.2142	.4117	.6527
18	17	.4619	.8389	.6610	15.2173	.3742	.4802	.4539	.9263
23	22	.5042	14.0767	.7067	.4787	.4150	.7461	.4961	16.2000
28	27	.5465	.3145	.7525	.7402	.4558	16.0121	.5382	.4736
Nov. 2	Nov. 1	.5888	.5523	.7982	16.0017	.4966	.2781	.5804	.7473
7	6	.6311	.7900	.8439	.2631	.5374	.5441	.6225	17.0209
12	11	.6734	15.0278	.8896	.5246	.5782	.8101	.6647	.2946
17	16	.7157	.2656	.9354	.7861	.6190	17.0760	.7069	.5682
22	21	.7580	.5034	.9811	17.0475	.6598	.3420	.7490	.8419
27	26	.8003	.7412	3.0268	.3090	.7006	.6080	.7912	18.1155
Dec. 2	Dec. 1	.8426	.9790	.0725	.5704	.7414	.8740	.8334	.3891
7	6	.8849	16.2167	.1182	.8319	.7822	18.1400	.8755	.6628
12	11	.9272	.4545	.1640	18.0934	.8229	.4060	.9177	.9364
17	16	.9695	.6923	.2097	.3548	.8637	.6719	.9598	19.2101
22	21	3.0118	.9301	.2554	.6163	.9045	.9379	3.0020	.4837
27	26	.0541	17.1679	.3011	.8778	.9453	19.2039	.0442	.7574
32	31	+3.0964	−17.4056	+3.3469	−19.1392	+2.9861	+19.4689	+3.0863	+20.0310

TABLE II. Aberration in Right Ascension, *in time.*

M. sin (⊙ + N)

Argument (⊙ + N)	1 γ Pegasi M = −1.2935 N = 88° 47′	Diff. for 10′	2 α Cassiop. M = −2.2230 N = 81° 35′	Diff. for 10′	3 Polaris M = −45.2486 N = 73° 51′	Diff. for 10′	4 α Arietis M = −1.3890 N = 58° 26′	Diff. for 10′	5 α Ceti M = −1.3105 N = 44° 11′	Diff. for 10′	Argument (⊙ + N)
0° 180°	−0.0000″+	37,6	−0.0000″+	64,6	−0.0000″+	1315,6	−0.0000″+	40,4	−0.0000″+	38,1	180° 360°
3 177	0.0677	37,5	0.1163	64,5	2.3681	1312,0	0.0727	40,3	0.0686	38,0	183 357
6 174	0.1352	37,3	0.2324	64,1	4.7297	1304,8	0.1452	40,1	0.1370	37,8	186 354
9 171	0.2023	37,0	0.3478	63,6	7.0784	1294,1	0.2173	39,7	0.2050	37,5	189 351
12 168	0.2689	36,6	0.4622	62,9	9.4077	1279,7	0.2888	39,3	0.2725	37,1	192 348
15 165	0.3348	36,1	0.5754	61,9	11.7112	1261,8	0.3595	38,7	0.3392	36,6	195 345
18 162	0.3997	35,4	0.6869	61,0	13.9825	1240,7	0.4292	38,1	0.4050	35,9	198 342
21 159	0.4635	34,8	0.7967	59,7	16.2157	1215,8	0.4978	37,3	0.4696	35,2	201 339
24 156	0.5261	33,9	0.9042	58,4	18.4041	1188,1	0.5650	36,4	0.5330	34,4	204 336
27 153	0.5872	33,1	1.0093	56,8	20.5426	1156,5	0.6306	35,5	0.5949	33,5	207 333
30 150	0.6467	32,1	1.1115	55,1	22.6243	1122,2	0.6945	34,4	0.6552	32,5	210 330
33 147	0.7045	31,0	1.2107	53,3	24.6442	1084,6	0.7565	33,3	0.7137	31,4	213 327
36 144	0.7603	29,8	1.3067	51,3	26.5965	1043,0	0.8164	32,1	0.7703	30,2	216 324
39 141	0.8140	28,6	1.3990	49,2	28.4739	1001,8	0.8741	30,7	0.8247	29,0	219 321
42 138	0.8655	27,3	1.4875	46,9	30.2772	954,4	0.9294	29,3	0.8769	27,6	222 318
45 135	0.9146	25,9	1.5719	44,5	31.9952	906,0	0.9822	27,8	0.9266	26,3	225 315
48 132	0.9612	24,4	1.6520	42,0	33.6260	854,7	1.0322	26,2	0.9739	24,7	228 312
51 129	1.0052	22,9	1.7276	39,4	35.1645	801,4	1.0794	24,6	1.0184	23,2	231 309
54 126	1.0464	21,3	1.7985	36,6	36.6071	745,2	1.1237	22,9	1.0602	21,6	234 306
57 123	1.0848	19,7	1.8644	33,8	37.9485	687,7	1.1649	21,1	1.0991	19,9	237 303
60 120	1.1202	17,9	1.9252	30,8	39.1864	627,9	1.2029	19,3	1.1349	18,2	240 300
63 117	1.1525	16,2	1.9807	27,8	40.3167	566,6	1.2376	17,4	1.1677	16,4	243 297
66 114	1.1816	14,4	2.0308	24,7	41.3366	503,6	1.2689	15,4	1.1972	14,6	246 294
69 111	1.2075	12,6	2.0753	21,1	42.2430	439,6	1.2967	13,5	1.2234	12,8	249 291
72 108	1.2302	10,7	2.1142	18,3	43.0343	373,4	1.3210	11,5	1.2464	10,8	252 288
75 105	1.2494	8,8	2.1472	15,1	43.7064	307,2	1.3417	9,4	1.2658	8,9	255 285
78 102	1.2652	6,8	2.1744	11,8	44.2594	240,1	1.3586	7,4	1.2818	6,9	258 282
81 99	1.2775	4,9	2.1956	8,4	44.6915	171,6	1.3719	5,3	1.2943	5,0	261 279
84 96	1.2864	2,9	2.2108	5,1	45.0003	103,2	1.3814	3,2	1.3033	3,0	264 276
87 93	1.2917	1,0	2.2199	1,7	45.1861	34,7	1.3871	1,1	1.3087	1,0	267 273
90 90	−1.2935 +		−2.2230 +		−45.2486 +		−1.3890 +		−1.3105 +		270 270
(⊙ + N) Argument	γ Pegasi		α Cassiop.		Polaris		α Arietis		α Ceti		(⊙ + N) Argument

TABLE II. Aberration in Right Ascension, *in time.*

M. sin (⊙ + N)

Argument (⊙+N)	6 α Persei M=−2.0181 N=39° 30′	Diff. for 10′	7 Aldebaran M=−1.4049 N=21° 42′	Diff. for 10′	8 Capella M=−1.9518 N=12° 51′	Diff. for 10′	9 Rigel M=−1.3756 N=12° 20′	Diff. for 10′	10 β Tauri M=−1.5499 N=10° 13′	Diff. for 10′	Argument (⊙+N)
0° 180°	0″.0000+	58,7	−0″.0000+	40,8	−0″.0000+	56,7	−0″.0000+	40,0	−0″.0000+	45,1	180° 360°
3 177	0.1056	58,6	0.0735	40,8	0.1021	56,6	0.0720	39,9	0.0811	44,9	183 357
6 174	0.2110	58,2	0.1469	40,5	0.2040	56,3	0.1438	39,7	0.1620	44,7	186 354
9 171	0.3157	57,7	0.2198	40,2	0.3053	55,8	0.2152	39,3	0.2425	44,3	189 351
12 168	0.4196	57,1	0.2921	39,7	0.4058	55,2	0.2860	38,9	0.3222	43,8	192 348
15 165	0.5223	56,3	0.3636	39,2	0.5052	54,4	0.3560	38,4	0.4011	43,2	195 345
18 162	0.6236	55,3	0.4341	38,6	0.6031	53,6	0.4251	37,7	0.4789	42,5	198 342
21 159	0.7232	54,2	0.5035	37,7	0.6995	52,4	0.4930	36,9	0.5554	41,7	201 339
24 156	0.8208	53,0	0.5714	36,9	0.7939	51,2	0.5595	36,1	0.6304	40,7	204 336
27 153	0.9162	51,6	0.6378	35,9	0.8861	49,9	0.6245	35,2	0.7036	39,6	207 333
30 150	1.0091	50,1	0.7025	34,9	0.9759	48,4	0.6878	34,1	0.7749	38,4	210 330
33 147	1.0992	48,3	0.7652	33,7	1.0630	46,8	0.7492	33,0	0.8441	37,2	213 327
36 144	1.1862	46,6	0.8258	32,4	1.1472	45,1	0.8086	31,7	0.9110	35,8	216 324
39 141	1.2701	44,6	0.8841	31,1	1.2283	43,2	0.8657	30,4	0.9754	34,3	219 321
42 138	1.3504	42,6	0.9401	29,6	1.3060	41,2	0.9204	29,1	1.0371	32,7	222 318
45 135	1.4270	40,4	0.9934	28,1	1.3801	39,1	0.9727	27,6	1.0959	31,1	225 315
48 132	1.4998	38,1	1.0440	26,6	1.4504	36,9	1.0223	25,9	1.1518	29,3	228 312
51 129	1.5684	35,7	1.0918	24,9	1.5168	34,6	1.0690	24,4	1.2045	27,4	231 309
54 126	1.6327	33,2	1.1366	23,2	1.5790	32,2	1.1129	22,7	1.2539	25,5	234 306
57 123	1.6925	30,7	1.1783	21,3	1.6369	29,7	1.1537	20,9	1.2998	23,2	237 303
60 120	1.7478	28,0	1.2167	19,5	1.6903	27,1	1.1913	19,1	1.3422	21,6	240 300
63 117	1.7982	25,3	1.2518	17,6	1.7390	24,4	1.2257	17,2	1.3810	19,4	243 297
66 114	1.8437	22,4	1.2835	15,6	1.7830	21,7	1.2567	15,3	1.4159	17,2	246 294
69 111	1.8841	19,6	1.3116	13,7	1.8221	19,0	1.2842	13,4	1.4469	15,1	249 291
72 108	1.9194	16,7	1.3362	11,6	1.8563	16,1	1.3083	11,3	1.4740	12,8	252 288
75 105	1.9494	13,7	1.3570	9,6	1.8853	13,2	1.3287	9,3	1.4971	10,5	255 285
78 102	1.9740	10,7	1.3742	7,4	1.9091	10,3	1.3455	7,3	1.5160	8,2	258 282
81 99	1.9933	7,7	1.3876	5,3	1.9277	7,4	1.3587	5,2	1.5308	5,9	261 279
84 96	2.0071	4,6	1.3972	3,2	1.9411	4,4	1.3680	3,2	1.5414	3,5	264 276
87 93	2.0153	1,6	1.4030	1,1	1.9491	1,5	1.3737	1,1	1.5477	1,2	267 273
90 90	−2.0181+		−1.4049+		−1.9518+		−1.3756+		−1.5499+		270 270
(⊙ + N) Argument	α Persei		Aldebaran		Capella		Rigel		β Tauri		(⊙ + N) Argument

TABLE II. Aberration in Right Ascension, *in time.*

M. sin (⊙ + N)

Argument (⊙ + N)	11 α Orionis M = −1.3776 N = 3° 13′	Diff. for 10′	12 Sirius M = −1.4222 N = 351° 21′	Diff. for 10′	13 Castor M = −1.5996 N = 340° 40′	Diff. for 10′	14 Procyon M = −1.3572 N = 339° 6′	Diff. for 10′	15 Pollux M = −1.5340 N = 338° 2′	Diff. for 10′	Argument (⊙ + N)
0° 180°	−0″.0000+	40,1	−0″.0000+	41,3	−0″.0000+	46,5	−0″.0000+	39,4	−0.0000+	44,6	180° 360°
3 177	0.0721	39,9	0.0744	41,3	0.0837	46,4	0.0710	39,4	0.0803	44,4	183 357
6 174	0.1440	39,7	0.1487	41,0	0.1672	46,1	0.1419	39,1	0.1603	44,3	186 354
9 171	0.2155	39,4	0.2225	40,7	0.2502	45,8	0.2123	38,8	0.2400	43,8	189 351
12 168	0.2864	39,0	0.2957	40,2	0.3326	45,2	0.2822	38,4	0.3189	43,4	192 348
15 165	0.3566	38,4	0.3681	39,7	0.4140	44,6	0.3513	37,8	0.3970	42,8	195 345
18 162	0.4257	37,8	0.4395	39,0	0.4943	43,8	0.4194	37,2	0.4740	42,1	198 342
21 159	0.4937	37,0	0.5097	38,2	0.5732	43,0	0.4864	36,4	0.5497	41,2	201 339
24 156	0.5603	36,2	0.5785	37,3	0.6506	42,0	0.5520	36,2	0.6239	40,3	204 336
27 153	0.6254	35,2	0.6457	36,3	0.7262	40,9	0.6162	34,7	0.6964	39,2	207 333
30 150	0.6888	34,2	0.7111	35,3	0.7998	39,7	0.6786	33,7	0.7670	38,1	210 330
33 147	0.7503	33,0	0.7746	34,1	0.8712	38,3	0.7392	32,5	0.8355	36,8	213 327
36 144	0.8097	31,8	0.8360	32,8	0.9402	36,9	0.7977	31,3	0.9017	35,4	216 324
39 141	0.8670	30,4	0.8950	31,5	1.0066	35,4	0.8541	30,0	0.9654	33,9	219 321
42 138	0.9218	29,1	0.9517	30,0	1.0703	33,7	0.9081	28,7	1.0265	32,3	222 318
45 135	0.9741	27,6	1.0057	28,4	1.1310	32,1	0.9597	27,2	1.0847	30,7	225 315
48 132	1.0238	26,0	1.0569	26,9	1.1887	30,2	1.0086	25,6	1.1400	28,9	228 312
51 129	1.0706	24,4	1.1053	25,2	1.2431	28,3	1.0547	24,1	1.1921	27,2	231 309
54 126	1.1145	22,7	1.1506	23,4	1.2941	26,3	1.0980	22,3	1.2411	25,2	234 306
57 123	1.1554	20,9	1.1928	21,6	1.3415	24,3	1.1382	20,7	1.2865	23,3	237 303
60 120	1.1931	19,1	1.2317	19,7	1.3853	22,2	1.1754	18,8	1.3285	21,3	240 300
63 117	1.2275	17,2	1.2672	17,8	1.4252	20,1	1.2093	17,0	1.3668	19,2	243 297
66 114	1.2585	15,3	1.2993	15,8	1.4613	17,8	1.2399	15,1	1.4014	17,1	246 294
69 111	1.2861	13,4	1.3278	13,8	1.4933	15,6	1.2670	13,2	1.4321	14,9	249 291
72 108	1.3102	11,4	1.3526	11,8	1.5213	13,2	1.2908	11,2	1.4590	12,6	252 288
75 105	1.3307	9,3	1.3738	9,6	1.5450	10,9	1.3109	9,2	1.4817	10,4	255 285
78 102	1.3475	7,3	1.3911	7,6	1.5646	8,5	1.3275	7,2	1.5005	8,1	258 282
81 99	1.3607	5,2	1.4047	5,4	1.5799	6,1	1.3405	5,1	1.5151	5,8	261 279
84 96	1.3701	3,1	1.4144	3,3	1.5908	3,6	1.3497	3,1	1.5256	3,5	264 276
87 93	1.3757	1,1	1.4203	1,1	1.5973	1,3	1.3553	1,1	1.5319	1,2	267 273
90 90	−1.3776+		−1.4222+		−1.5996+		−1.3572+		−1.5340+		270 270
(⊙ + N) Argument	α Orionis		Sirius		Castor		Procyon		Pollux		(⊙ + N) Argument

TABLE II. Aberration in Right Ascension, *in time.*

M. sin (⊙ + N)

Argument	16 α Hydræ		17 Regulus		18 α Ursæ Major.		19 β Leonis		20 β Virginis		Argument
(⊙ + N)	M = −1.3145 N = 312° 39′	Diff. for 10′	M = −1.3158. N = 302° 22′	Diff. for 10′	M = −2.7516 N = 288° 7′	Diff. for 10′	M = −1.3020 N = 275° 21′	Diff. for 10′	M = −1.2559 N = 274 57′	Diff. for 10′	(⊙ + N)
0 180°	−0.″0000+	38,2	−0.″0000+	38,3	−0.″0000+	80,0	−0.″0000+	37,8	−0.″0000+	36,5	180° 360°
3 177	0.0688	38,1	0.0689	38,1	0.1440	79,8	0.0681	37,8	0.0657	36,4	183 357
6 174	0.1374	37,9	0.1375	37,9	0.2876	79,3	0.1361	37,6	0.1313	36,2	186 354
9 171	0.2056	37,6	0.2058	37,7	0.4304	78,7	0.2037	37,2	0.1965	35,9	189 351
12 168	0.2733	37,2	0.2736	37,2	0.5721	77,8	0.2707	36,8	0.2611	35,5	192 348
15 165	0.3402	36,7	0.3406	36,7	0.7122	76,7	0.3370	36,3	0.3250	35,1	195 345
18 162	0.4062	36,1	0.4066	36,1	0.8503	75,4	0.4023	35,7	0.3881	34,4	198 342
21 159	0.4711	35,3	0.4716	35,3	0.9861	73,9	0.4666	35,0	0.4501	33,7	201 339
24 156	0.5346	34,6	0.5352	34,6	1.1192	72,2	0.5296	34,2	0.5108	33,0	204 336
27 153	0.5968	33,6	0.5974	33,6	1.2492	70,3	0.5911	33,3	0.5702	32,1	207 333
30 150	0.6572	32,6	0.6579	32,7	1.3758	68,2	0.6510	32,3	0.6279	31,2	210 330
33 147	0.7159	31,5	0.7167	31,5	1.4986	65,9	0.7091	31,2	0.6840	30,1	213 327
36 144	0.7726	30,3	0.7734	30,4	1.6173	63,5	0.7653	30,1	0.7382	28,9	216 324
39 141	0.8272	29,1	0.8281	29,1	1.7316	60,9	0.8194	28,8	0.7903	27,8	219 321
42 138	0.8795	27,8	0.8805	27,7	1.8412	58,0	0.8712	27,5	0.8403	26,5	222 318
45 135	0.9295	26,3	0.9304	26,3	1.9456	55,1	0.9207	26,1	0.8880	25,2	225 315
48 132	0.9768	24,8	0.9778	24,9	2.0448	52,0	0.9676	24,6	0.9333	23,7	228 312
51 129	1.0215	23,3	1.0226	23,3	2.1384	48,7	1.0119	23,1	0.9760	22,2	231 309
54 126	1.0635	21,6	1.0645	21,7	2.2261	45,3	1.0534	21,4	1.0160	20,7	234 306
57 123	1.1024	20,0	1.1035	20,0	2.3077	41,8	1.0920	19,8	1.0532	19,1	237 303
60 120	1.1384	18,2	1.1395	18,3	2.3829	38,2	1.1276	18,1	1.0876	17,4	240 300
63 117	1.1712	16,4	1.1724	16,5	2.4517	34,4	1.1601	16,3	1.1190	15,7	243 297
66 114	1.2008	14,7	1.2021	14,6	2.5137	30,6	1.1895	14,4	1.1473	13,9	246 294
69 111	1.2272	12,7	1.2284	12,8	2.5688	26,7	1.2155	12,7	1.1724	12,2	249 291
72 108	1.2501	10,9	1.2514	10,9	2.6169	22,7	1.2383	10,8	1.1944	10,4	252 288
75 105	1.2697	8,9	1.2710	8,9	2.6578	18,7	1.2577	8,8	1.2131	8,5	255 285
78 102	1.2857	7,0	1.2871	6,9	2.6914	14,6	1.2736	6,9	1.2284	6,7	258 282
81 99	1.2983	5,0	1.2996	5,0	2.7177	10,4	1.2860	4,9	1.2404	4,8	261 279
84 96	1.3073	3,0	1.3086	3,0	2.7365	6,3	1.2949	2,9	1.2490	2,8	264 276
87 93	1.3127	1,0	1.3140	1,0	2.7478	2,1	1.3002	1,0	1.2541	1,0	267 273
90 90	−1.3145+		−1.3158+		−2.7516+		−1.3020+		−1.2559+		270 270
(⊙ + N) Argument	α Hydræ		Regulus		α Ursæ Major.		β Leonis		β Virginis		(⊙ + N) Argument

TABLE II. Aberration in Right Ascension, *in time.*

M. sin (⊙ + N)

Argument (⊙ + N)	21 γ Ursæ Major. M = −2.1671 N = 274° 8′	Diff. for 10′	22 Spica Virginis M = +1.2868 N = 69° 22′	Diff. for 10′	23 η Ursæ Major. M = +1.9903 N = 62° 50′	Diff. for 10′	24 Arcturus M = +1.3695 N = 55° 46′	Diff. for 10′	25, 26 α² Libræ M = +1.3503 N = 41° 10′	Diff. for 10′	Argument (⊙ + N)
0° 180°	−0″.0000+	63,0	+0″.0000−	37,4	+0″.0000−	57,9	+0″.0000−	39,8	+0″.0000−	39,3	180° 360°
3 177	0.1134	62,8	0.0673	37,3	0.1042	57,7	0.0717	39,7	0.0707	39,1	183 357
6 174	0.2265	62,5	0.1345	37,1	0.2080	57,4	0.1432	39,4	0.1411	38,9	186 354
9 171	0.3390	62,0	0.2013	36,8	0.3113	56,9	0.2142	39,2	0.2112	38,7	189 351
12 168	0.4506	61,3	0.2675	36,4	0.4138	56,3	0.2847	38,8	0.2808	38,2	192 348
15 165	0.5609	60,4	0.3331	35,8	0.5151	55,5	0.3545	38,2	0.3495	37,7	195 345
18 162	0.6697	59,4	0.3976	35,3	0.6150	54,6	0.4232	37,6	0.4173	37,0	198 342
21 159	0.7766	58,3	0.4612	34,6	0.7132	53,5	0.4908	36,8	0.4839	36,3	201 339
24 156	0.8815	56,9	0.5234	33,8	0.8095	52,3	0.5570	36,0	0.5492	35,4	204 336
27 153	0.9839	55,4	0.5842	32,9	0.9036	50,8	0.6218	35,0	0.6130	34,6	207 333
30 150	1.0836	53,7	0.6434	31,9	0.9951	49,4	0.6848	33,9	0.6752	33,4	210 330
33 147	1.1803	51,9	0.7009	30,8	1.0840	47,7	0.7459	32,9	0.7354	32,4	213 327
36 144	1.2738	50,0	0.7564	29,7	1.1699	45,9	0.8050	31,6	0.7937	31,2	216 324
39 141	1.3638	48,9	0.8098	28,5	1.2525	44,0	0.8619	30,3	0.8498	29,8	219 321
42 138	1.4501	45,7	0.8611	27,1	1.3317	42,0	0.9164	28,9	0.9035	28,5	222 318
45 135	1.5324	43,4	0.9099	25,8	1.4073	39,8	0.9684	27,4	0.9548	27,1	225 315
48 132	1.6105	40,9	0.9563	24,3	1.4790	37,6	1.0178	25,8	1.0035	25,5	228 312
51 129	1.6842	38,4	1.0000	22,8	1.5467	35,3	1.0643	24,3	1.0494	24,4	231 309
54 126	1.7533	35,7	1.0411	21,2	1.6102	32,8	1.1080	22,6	1.0924	22,3	234 306
57 123	1.8175	32,9	1.0792	19,6	1.6692	30,2	1.1486	20,8	1.1325	20,5	237 303
60 120	1.8768	30,1	1.1144	17,9	1.7236	27,6	1.1861	19,0	1.1694	18,8	240 300
63 117	1.9309	27,2	1.1466	16,1	1.7733	24,9	1.2203	17,2	1.2034	16,9	243 297
66 114	1.9798	24,1	1.1756	14,3	1.8182	22,2	1.2513	15,2	1.2336	15,0	246 294
69 111	2.0232	21,1	1.2013	12,6	1.8581	19,3	1.2786	13,3	1.2606	13,3	249 291
72 108	2.0611	17,9	1.2239	10,6	1.8929	16,4	1.3026	11,3	1.2845	11,0	252 288
75 105	2.0933	14,7	1.2430	8,7	1.9224	13,5	1.3229	9,3	1.3043	9,2	255 285
78 102	2.1198	11,5	1.2587	6,8	1.9467	10,6	1.3396	7,3	1.3208	7,2	258 282
81 99	2.1405	8,2	1.2710	4,9	1.9658	7,5	1.3527	5,2	1.3337	5,1	261 279
84 96	2.1553	4,9	1.2798	2,9	1.9793	4,6	1.3620	3,1	1.3429	3,1	264 276
87 93	2.1642	1,6	1.2851	0,9	1.9875	1,6	1.3676	1,1	1.3485	1,0	267 273
90 90	−2.1671+		+1.2868−		+1.9903−		+1.3695−		+1.3503 −		270 270
(⊙ + N) Argument	γ Ursæ Major.		Spica Virginis		η Ursæ Major.		Arcturus		α² Libræ		(⊙ + N) Argument

TABLE II. Aberration in Right Ascension, *in time.*

M. sin (⊙ + N)

Argument (⊙ + N)	27 β Ursæ Minor. M = +5.0016 N = 44° 43′	Diff. for 10′	28 α Cor. Bor. M = +1.4907 N = 35° 45′	Diff. for 10′	29 Serpentis M + = 1.3385 N = 33° 42′	Diff. for 10′	30 Antares M = +1.4990 N = 23° 24′	Diff. for 10′	31 α Herculis M = +1.4063 N = 12° 13′	Diff. for 10′	Argument (⊙ + N)
0° 180°	+0″.0000 –	145,4	+0″.0000 –	43,3	+0″.0000 –	38,9	+0″.0000 –	43,6	+0″.0000 –	40,9	180° 360°
3 177	0.2618	145,0	0.0780	43,2	0.0701	38,8	0.0785	43,4	0.0736	40,8	183 357
6 174	0.5228	144,2	0.1558	43,0	0.1399	38,6	0.1567	43,2	0.1470	40,6	186 354
9 171	0.7824	143,1	0.2332	42,6	0.2094	38,3	0.2345	42,9	0.2200	40,2	189 351
12 168	1.0399	141,4	0.3099	42,2	0.2783	37,8	0.3117	42,4	0.2924	39,8	192 348
15 165	1.2945	139,5	0.3858	41,6	0.3464	37,3	0.3880	41,8	0.3640	39,2	195 345
18 162	1.5456	137,1	0.4606	40,9	0.4136	36,7	0.4632	41,1	0.4346	38,6	198 342
21 159	1.7924	134,4	0.5342	40,1	0.4797	35,9	0.5372	40,3	0.5040	37,8	201 339
24 156	2.0343	131,3	0.6063	39,2	0.5444	35,2	0.6097	39,3	0.5720	36,9	204 336
27 153	2.2707	127,8	0.6768	38,1	0.6077	34,2	0.6805	38,3	0.6385	35,9	207 333
30 150	2.5008	124,1	0.7453	37,0	0.6693	33,2	0.7495	37,2	0.7032	34,8	210 330
33 147	2.7241	119,9	0.8119	35,7	0.7290	32,1	0.8164	35,9	0.7659	33,7	213 327
36 144	2.9399	115,4	0.8762	34,4	0.7868	30,9	0.8811	34,6	0.8266	32,4	216 324
39 141	3.1476	110,6	0.9381	33,0	0.8424	29,6	0.9434	33,1	0.8850	31,1	219 321
42 138	3.3467	105,5	0.9975	31,4	0.8957	28,2	1.0030	31,7	0.9410	29,7	222 318
45 135	3.5366	100,2	1.0541	29,8	0.9465	26,8	1.0600	30,0	0.9944	28,2	225 315
48 132	3.7169	94,5	1.1078	28,2	0.9947	25,3	1.1140	28,3	1.0451	26,6	228 312
51 129	3.8870	88,6	1.1585	26,4	1.0403	23,7	1.1650	26,5	1.0929	24,9	231 309
54 126	4.0464	82,4	1.2060	24,5	1.0829	22,1	1.2127	24,7	1.1377	23,2	234 306
57 123	4.1947	76,0	1.2501	22,7	1.1226	20,3	1.2572	22,8	1.1794	21,4	237 303
60 120	4.3315	69,4	1.2910	20,7	1.1592	18,6	1.2982	20,8	1.2179	19,5	240 300
63 117	4.4565	62,6	1.3282	18,7	1.1927	16,7	1.3356	18,8	1.2530	17,6	243 297
66 114	4.5692	55,7	1.3618	16,6	1.2228	14,9	1.3694	16,7	1.2847	15,7	246 294
69 111	4.6694	48,6	1.3917	14,5	1.2497	13,0	1.3995	14,6	1.3129	13,7	249 291
72 108	4.7569	41,2	1.4178	12,3	1.2731	11,0	1.4257	12,3	1.3375	11,6	252 288
75 105	4.8311	34,0	1.4399	10,1	1.2929	9,1	1.4479	10,2	1.3584	9,6	255 285
78 102	4.8923	26,5	1.4581	7,9	1.3093	7,1	1.4663	7,9	1.3756	7,4	258 282
81 99	4.9400	19,0	1.4723	5,7	1.3221	5,1	1.4806	5,7	1.3890	5,3	261 279
84 96	4.9742	11,4	1.4825	3,4	1.3312	3,1	1.4908	3,4	1.3986	3,2	264 276
87 93	4.9947	3,8	1.4886	1,2	1.3367	1,0	1.4970	1,1	1.4044	1,1	267 273
90 90	+5.0016 –		+1.4907 –		+1.3385 –		+1.4990 –		+1.4063 –		270 270
(⊙ + N) Argument	β Ursæ Minor.		α Cor. Bor.		α Serpentis		Antares		α Herculis		(⊙ + N) Argument

TABLE II. Aberration in Right Ascension, *in time.*

M. sin (⊙ + N)

Argument (⊙ + N)	32 α Ophiuchi M = + 1.3986 N = 7° 34′	Diff. for 10′	33 γ Draconis M = + 2.1958 N = 1° 41′	Diff. for 10′	34 α Lyræ M = + 1.7469 N = 352° 50′	Diff. for 10′	35 γ Aquilæ M = + 1.3695 N = 337° 16′	Diff. for 10′	36 α Aquilæ M = + 1.3609 N = 336° 15′	Diff. for 10′	Argument (⊙ + N)
0° 180°	+0″.0000−	40,7	+0″.0000	63,8	+0″.0000−	50,8	+0″.0000 −	39,8	+0″.0000−	39,6	180° 360°
3 177	0.0731	40,6	0.1149	63,7	0.0914	50,7	0.0717	39,7	0.0712	39,5	183 357
6 174	0.1462	40,3	0.2295	63,3	0.1826	50,4	0.1432	39,4	0.1423	39,2	186 354
9 171	0.2188	40,0	0.3435	62,8	0.2733	49,9	0.2142	39,2	0.2129	38,9	189 351
12 168	0.2908	39,6	0.4565	62,1	0.3632	49,4	0.2847	38,8	0.2830	38,4	192 348
15 165	0.3620	39,0	0.5683	61,2	0.4521	48,7	0.3545	38,2	0.3522	38,0	195 345
18 162	0.4322	38,3	0.6785	60,2	0.5398	47,9	0.4232	37,6	0.4206	37,3	198 342
21 159	0.5012	37,6	0.7869	59,0	0.6260	46,9	0.4908	36,8	0.4877	36,6	201 339
24 156	0.5689	36,7	0.8931	57,7	0.7105	45,9	0.5570	36,0	0.5535	35,8	204 336
27 153	0.6350	35,7	0.9969	56,1	0.7931	44,7	0.6218	35,0	0.6179	34,8	207 333
30 150	0.6993	34,6	1.0979	54,4	0.8735	43,3	0.6848	33,9	0.6805	33,7	210 330
33 147	0.7617	33,6	1.1959	52,7	0.9514	41,9	0.7459	32,9	0.7412	32,6	213 327
36 144	0.8221	32,3	1.2907	50,6	1.0268	40,3	0.8050	31,6	0.7999	31,4	216 324
39 141	0.8802	30,9	1.3818	48,6	1.0994	38,6	0.8619	30,3	0.8565	30,1	219 321
42 138	0.9359	29,5	1.4693	46,3	1.1689	36,8	0.9164	28,9	0.9106	28,7	222 318
45 135	0.9890	28,0	1.5526	44,0	1.2352	35,0	0.9684	27,4	0.9623	27,3	225 315
48 132	1.0394	26,4	1.6318	41,4	1.2982	33,0	1.0177	25,9	1.0114	25,7	228 312
51 129	1.0869	24,8	1.7064	38,9	1.3576	30,9	1.0643	24,3	1.0576	24,1	231 309
54 126	1.1315	23,1	1.7764	36,2	1.4133	28,8	1.1080	22,6	1.1010	22,4	234 306
57 123	1.1730	21,2	1.8415	33,4	1.4651	26,6	1.1486	20,8	1.1414	20,7	237 303
60 120	1.2112	19,4	1.9016	30,5	1.5129	24,2	1.1860	19,1	1.1786	18,9	240 300
63 117	1.2462	17,5	1.9565	27,4	1.5565	21,9	1.2203	17,1	1.2126	16,1	243 297
66 114	1.2777	15,6	2.0059	24,4	1.5959	19,4	1.2511	15,3	1.2433	15,1	246 294
69 111	1.3057	13,6	2.0499	21,3	1.6309	16,9	1.2786	13,3	1.2705	13,2	249 291
72 108	1.3302	11,6	2.0883	18,1	1.6614	14,4	1.3025	11,3	1.2943	11,3	252 288
75 105	1.3510	9,4	2.1209	14,9	1.6874	11,8	1.3228	9,3	1.3146	9,2	255 285
78 102	1.3680	7,4	2.1478	11,7	1.7087	9,3	1.3396	7,3	1.3312	7,2	258 282
81 99	1.3814	5,3	2.1688	8,3	1.7254	6,6	1.3527	5,2	1.3442	5,2	261 279
84 96	1.3909	3,2	2.1837	5,1	1.7373	4,0	1.3620	3,1	1.3535	3,2	264 276
87 93	1.3967	1,1	2.1928	1,7	1.7445	1,3	1.3676	1,1	1.3591	1,0	267 273
90 90	+1.3986−		+2.1958−		+1.7469−		+1.3695 −		+1.3609−		270 270
(⊙ + N) Argument	α Ophiuchi		γ Draconis		α Lyræ		γ Aquilæ		α Aquilæ		(⊙ + N) Argument

TABLE II. Aberration in Right Ascension, *in time.*

M. sin $(\odot + N)$

Argument $(\odot + N)$	37 β Aquilæ $M = +1.3519$ $N = 335° 12'$	Diff. for 10′	38, 39 α² Capricorni $M = +1.3711$ $N = 330° 2'$	Diff. for 10′	40 α Cygni $M = +1.8609$ $N = 323° 29'$	Diff. for 10′	41 α Cephei $M = +2.7660$ $N = 313° 50'$	Diff. for 10′	42 β Cephei $M = +3.7631$ $N = 310° 49'$	Diff. for 10′	Argument $(\odot + N)$
0° 180°	+0″.0000−	39,3	+0″.0000−	39,9	+0″.0000 −	54,1	+0″.0000−	80,4	+0″.0000−	109,4	180° 360°
3 177	0.0708	39,2	0.0718	39,7	0.0974	53,9	0.1448	80,2	0.1969	109,2	183 357
6 174	0.1413	39,0	0.1433	39,6	0.1945	53,7	0.2891	79,8	0.3934	108,5	186 354
9 171	0.2115	38,7	0.2145	39,2	0.2911	53,2	0.4327	79,1	0.5887	107,6	189 351
12 168	0.2811	38,2	0.2851	38,8	0.3869	52,6	0.5751	78,2	0.7824	106,4	192 348
15 165	0.3499	37,7	0.3549	38,2	0.4816	51,9	0.7159	77,7	0.9740	104,9	195 345
18 162	0.4178	37,1	0.4237	37,6	0.5750	51,1	0.8548	75,8	1.1629	103,2	198 342
21 159	0.4845	36,3	0.4914	36,8	0.6669	50,0	0.9913	74,3	1.3486	101,1	201 339
24 156	0.5499	35,5	0.5577	36,0	0.7569	48,8	1.1250	72,7	1.5306	98,8	204 336
27 153	0.6138	34,6	0.6225	35,1	0.8448	47,6	1.2558	70,7	1.7084	96,2	207 333
30 150	0.6760	33,5	0.6856	34,0	0.9304	46,2	1.3830	68,6	1.8816	93,3	210 330
33 147	0.7363	32,4	0.7468	32,8	1.0135	44,6	1.5065	66,3	2.0496	90,2	213 327
36 144	0.7947	31,2	0.8059	31,7	1.0938	42,9	1.6258	63,8	2.2119	86,8	216 324
39 141	0.8508	29,9	0.8629	30,3	1.1711	41,2	1.7407	61,2	2.3682	83,2	219 321
42 138	0.9046	28,6	0.9174	28,9	1.2452	39,2	1.8508	58,9	2.5180	79,4	222 318
45 135	0.9560	27,1	0.9695	27,4	1.3158	37,3	1.9559	55,4	2.6609	75,3	225 315
48 132	1.0047	25,6	1.0189	25,9	1.3829	35,2	2.0556	52,2	2.7965	71,1	228 312
51 129	1.0507	23,9	1.0655	24,3	1.4462	32,9	2.1496	49,0	2.9245	66,7	231 309
54 126	1.0938	22 2	1.1093	22,6	1.5055	30,7	2.2378	45,6	3.0445	61,9	234 306
57 123	1.1338	20,6	1.1499	20,8	1.5607	28,3	2.3198	42,1	3.1560	57,2	237 303
60 120	1.1708	18,8	1.1874	19,1	1.6116	25,8	2.3955	38,4	3.2590	52,2	240 300
63 117	1.2046	16,9	1.2217	17,2	1.6581	23,3	2.4646	34,6	3.3530	47,1	243 297
66 114	1.2351	15,0	1.2526	15,2	1.7000	20,7	2.5269	30,8	3.4378	41,9	246 294
69 111	1.2621	13,2	1.2800	13,3	1.7373	18,1	2.5823	26,9	3.5132	36,6	249 291
72 108	1.2858	11,2	1.3040	11,3	1.7698	15,4	2.6307	22,8	3.5790	31,1	252 288
75 105	1.3059	9,2	1.3244	9,3	1.7975	12,6	2.6718	18,8	3.6349	25,6	255 285
78 102	1.3224	7,2	1.3411	7,3	1.8202	9,9	2.7056	14,7	3.6809	19,9	258 282
81 99	1.3353	5,1	1.3542	5,2	1.8380	7,1	2.7320	10,5	3.7168	14,3	261 279
84 96	1.3445	3,1	1.3636	3,1	1.8507	4,2	2.7509	6,3	3.7425	8,6	264 276
87 93	1.3501	1,0	1.3692	1,1	1.8583	1,4	2.7622	2,1	3.7579	2,9	267 273
90 90	+1.3519−		+1.3711−		+1.8609 −		+2.7660−		+3.7631−		270 270
$(\odot + N)$ Argument	β Aquilæ		α² Capricorni		α Cygni		α Cephei		β Cephei		$(\odot + N)$ Argument

TABLE II. Aberration in Right Ascension, *in time.*

M. sin (☉ + N)

Argument (☉ + N)	43 α Aquarii M = + 1.2844 N = 302° 57′	Diff. for 10′	44 Fomalhaut M = + 1.4683 N = 289° 26′	Diff. for 10′	45 α Pegasi M = + 1.3029 N = 287° 17′	Diff. for 10′	46 α Andromedæ M = + 1.4219 N = 270° 6′	Diff. for 10′	Argument (☉ + N)
0° 180°	+0.0000″−	37,3	+0.0000″−	42,7	+0.0000″−	37,9	+0.0000″−	41,3	180° 360°
3 177	0.0672	37,3	0.0768	42,7	0.0682	37,8	0.0744	41,2	183 357
6 174	0.1343	37,0	0.1535	42,3	0.1362	37,6	0.1486	41,0	186 354
9 171	0.2009	36,7	0.2297	42,0	0.2038	37,3	0.2224	40,7	189 351
12 168	0.2670	36,3	0.3053	41,5	0.2709	36,8	0.2956	40,3	192 348
15 165	0.3324	35,8	0.3800	40,9	0.3372	36,3	0.3680	39,7	195 345
18 162	0.3969	35,2	0.4537	40,3	0.4026	35,7	0.4394	39,0	198 342
21 159	0.4603	34,5	0.5262	39,4	0.4669	35,0	0.5096	38,2	201 339
24 156	0.5224	33,7	0.5972	38,6	0.5299	34,2	0.5783	37,3	204 336
27 153	0.5831	32,8	0.6666	37,5	0.5915	33,3	0.6455	36,4	207 333
30 150	0.6422	31,8	0.7341	36,4	0.6514	32,3	0.7110	35,2	210 330
33 147	0.6995	30,8	0.7997	35,2	0.7096	31,2	0.7744	34,1	213 327
36 144	0.7550	29,6	0.8630	33,9	0.7658	30,1	0.8358	32,8	216 324
39 141	0.8083	28,4	0.9240	32,4	0.8199	28,8	0.8948	31,4	219 321
42 138	0.8594	27,1	0.9824	31,0	0.8718	27,5	0.9514	30,0	222 318
45 135	0.9082	25,7	1.0382	29,4	0.9213	26,1	1.0054	28,5	225 315
48 132	0.9545	24,3	1.0911	27,7	0.9682	24,6	1.0567	26,8	228 312
51 129	0.9982	22,7	1.1410	26,0	1.0125	23,1	1.1050	25,2	231 309
54 126	1.0391	21,2	1.1878	24,2	1.0540	21,5	1.1503	23,4	234 306
57 123	1.0772	19,5	1.2314	22,3	1.0927	19,8	1.1925	21,6	237 303
60 120	1.1123	17,8	1.2715	20,4	1.1283	18,1	1.2314	19,7	240 300
63 117	1.1444	16,1	1.3082	18,4	1.1609	16,3	1.2669	17,8	243 297
66 114	1.1734	14,3	1.3413	16,3	1.1902	14,5	1.2990	15,8	246 294
69 111	1.1991	12,4	1.3707	14,3	1.2163	12,7	1.3275	13,8	249 291
72 108	1.2215	10,6	1.3964	12,1	1.2391	10,8	1.3523	11,7	252 288
75 105	1.2406	8,7	1.4182	10,0	1.2585	8,8	1.3734	9,7	255 285
78 102	1.2563	6,8	1.4362	7,8	1.2744	6,9	1.3908	7,6	258 282
81 99	1.2686	4,9	1.4502	5,6	1.2868	4,9	1.4044	5,4	261 279
84 96	1.2774	2,9	1.4602	3,3	1.2957	3,0	1.4141	3,2	264 276
87 93	1.2826	1,0	1.4662	1,2	1.3011	1,0	1.4199	1,1	267 273
90 90	+1.2844−		+1.4683−		+1.3029−		+1.4219−		270 270
(☉ + N) Argument	α Aquarii		Fomalhaut		α Pegasi		α Andromedæ		(☉ + N) Argument

TABLE III. Lunar Nutation in Right Ascension, *in time.*

$M'. \sin(☊ + N')$

Argument (☊ + N')	1 γ Pegasi		2 α Cassiop.		3 Polaris		4 α Arietis		5 α Ceti		Argument (☊ + N')
	M′=−1.0398 N′=8° 24′	Diff. for 10′	M′=−1.4110 N′=37° 52′	Diff. for 10′	M′=−21.3643 N′=76° 7′	Diff. for 10′	M′=−1.1386 N′=11° 1′	Diff. for 10′	M′=−1.04498 N′=1° 26′	Diff. for 10′	
0° 180°	−0″.0000+	30,2	−0″.0000+	41,0	−0″.0000+	621,2	−0″.0000−	33,1	−0″.0000 +	30,4	180° 360°
3 177	0.0544	30,2	0.0738	40,9	1.1181	619,5	0.0596	33,0	0.0547	30,3	183 357
6 174	0.1087	30,0	0.1475	40,7	2.2332	616,1	0.1190	32,8	0.1092	30,1	186 354
9 171	0.1627	29,8	0.2207	40,4	3.3421	611,0	0.1781	32,6	0.1635	29,9	189 351
12 168	0.2162	29,4	0.2934	39,9	4.4419	604,3	0.2367	32,2	0.2173	29,5	192 348
15 165	0.2691	29,0	0.3652	39,3	5.5296	595,7	0.2947	31,7	0.2705	29,1	195 345
18 162	0.3213	28,5	0.4360	38,7	6.6019	585,8	0.3519	31,2	0.3229	28,6	198 342
21 159	0.3726	27,9	0.5057	37,9	7.6563	574,1	0.4081	30,6	0.3745	28,1	201 339
24 156	0.4229	27,3	0.5739	37,0	8.6896	560,9	0.4631	29,9	0.4250	27,4	204 336
27 153	0.4720	26,6	0.6406	36,1	9.6993	546,1	0.5169	29,1	0.4744	26,7	207 333
30 150	0.5199	25,8	0.7055	35,0	10.6822	529,8	0.5693	28,2	0.5225	25,9	210 330
33 147	0.5663	24,9	0.7685	33,8	11.6359	512,1	0.6202	27,3	0.5691	25,1	213 327
36 144	0.6112	23,9	0.8294	32,6	12.5577	492,9	0.6693	26,3	0.6142	24,1	216 324
39 141	0.6543	23,0	0.8880	31,2	13.4450	472,5	0.7166	25,2	0.6576	23,1	219 321
42 138	0.6957	21,9	0.9442	29,8	14.2955	450,7	0.7619	24,0	0.6992	22,0	222 318
45 135	0.7352	20,8	0.9977	28,3	15.1067	427,8	0.8051	22,8	0.7389	20,8	225 315
48 132	0.7727	19,6	1.0486	26,7	15.8767	403,6	0.8462	21,5	0.7763	19,9	228 312
51 129	0.8080	18,4	1.0966	25,0	16.6031	378,4	0.8849	20,2	0.8121	18,5	231 309
54 126	0.8412	17,1	1.1416	23,2	17.2842	351,9	0.9212	18,8	0.8454	17,2	234 306
57 123	0.8720	15,8	1.1834	21,4	17.9176	324,7	0.9549	17,3	0.8764	15,9	237 303
60 120	0.9005	14,4	1.2220	19,6	18.5021	296,5	0.9861	15,8	0.9050	14,5	240 300
63 117	0.9264	13,1	1.2572	17,7	19.0358	267,5	1.0145	14,3	0.9311	13,1	243 297
66 114	0.9499	11,6	1.2891	15,7	19.5173	237,8	1.0402	12,7	0.9546	11,6	246 294
69 111	0.9707	10,1	1.3173	13,7	19.9453	207,6	1.0630	11,1	0.9756	10,2	249 291
72 108	0.9889	8,6	1.3420	11,6	20.3189	176,3	1.0829	9,4	0.9938	8,6	252 288
75 105	1.0043	7,1	1.3630	9,6	20.6362	145,1	1.0998	7,7	1.0094	7,1	255 285
78 102	1.0170	5,5	1.3802	7,5	20.8973	113,3	1.1138	6,0	1.0221	5,5	258 282
81 99	1.0269	3,9	1.3937	5,3	21.1013	81,0	1.1246	4,3	1.0321	4,0	261 279
84 96	1.0340	2,4	1.4033	3,2	21.2471	48,8	1.1324	2,6	1.0393	2,3	264 276
87 93	1.0383	0,8	1.4091	1,1	21.3349	16,3	1.1371	0,9	1.0435	0,8	267 273
90 90	−1.0398+		−1.4110+		−21.3643+		−1.1386+		−1.0450+		270 270
(☊ + N′) Argument	γ Pegasi		α Cassiop.		Polaris		Arietis		α Ceti		(☊ + N′) Argument

TABLE III. Lunar Nutation in Right Ascension, *in time.*

$M'.\sin(☊ + N')$

Argument (☊ + N′)	6 α Persei M′ = −1.4853 N′ = 18° 13′	Diff. for 10′	7 Aldebaran M′ = −1.1469 N′ = 3° 27′	Diff. for 10′	8 Capella M′ = −1.4786 N′ = 5° 46′	Diff. for 10′	9 Rigel M′ = −0.9625 N′ = 358° 47′	Diff. for 10′	10 β Tauri M′ = −1.2648 N′ = 2° 50′	Diff. for 10′	Argument (☊ + N′)
0° 180	−0″.0000+	43,2	−0″.0000 +	33,3	−0″.0000 +	43,0	−0″.0000 +	28,0	−0″.0000+	36,8	180° 360°
3 177	0.0777	43,1	0.0600	33,3	0.0774	42,9	0.0504	27,9	0.0662	36,7	183 357
6 174	0.1553	42,8	0.1199	33,1	0.1546	42,6	0.1006	27,8	0.1322	36,5	186 354
9 171	0.2324	42,5	0.1794	32,8	0.2313	42,3	0.1506	27,5	0.1979	36,2	189 351
12 168	0.3088	42,0	0.2384	32,4	0.3074	41,8	0.2001	27,2	0.2630	35,8	192 348
15 165	0.3844	41,4	0.2968	32,0	0.3827	41,2	0.2491	26,8	0.3274	35,3	195 345
18 162	0.4590	40,7	0.3544	31,4	0.4569	40,5	0.2974	26,4	0.3908	34,7	198 342
21 159	0.5323	39,9	0.4110	30,8	0.5299	39,7	0.3449	25,9	0.4533	34,0	201 339
24 156	0.6041	39,0	0.4665	30,1	0.6014	38,8	0.3915	25,3	0.5144	33,2	204 336
27 153	0.6743	38,0	0.5207	29,3	0.6713	37,8	0.4370	24,6	0.5742	32,3	207 333
30 150	0.7427	36,8	0.5734	28,4	0.7393	36,7	0.4813	23,9	0.6324	31,4	210 330
33 147	0.8090	35,6	0.6246	27,5	0.8053	35,4	0.5242	23,1	0.6889	30,3	213 327
36 144	0.8731	34,3	0.6741	26,5	0.8691	34,1	0.5658	22,2	0.7434	29,2	216 324
39 141	0.9347	32,9	0.7217	25,4	0.9306	32,7	0.6057	21,3	0.7960	28,0	219 321
42 138	0.9939	31,3	0.7674	24,2	0.9894	31,2	0.6441	20,3	0.8463	26,7	222 318
45 135	1.0503	29,7	0.8109	23,0	1.0456	29,6	0.6806	19,3	0.8943	25,3	225 315
48 132	1.1038	28,1	0.8523	21,7	1.0989	27,9	0.7153	18,2	0.9399	23,9	228 312
51 129	1.1543	26,3	0.8913	20,3	1.1491	26,2	0.7480	17,1	0.9829	22,4	231 309
54 126	1.2017	24,5	0.9278	18,9	1.1963	24,4	0.7787	15,9	1.0232	20,8	234 306
57 123	1.2457	22,6	0.9618	17,4	1.2401	22,5	0.8072	14,6	1.0607	19,2	237 303
60 120	1.2863	20,6	0.9932	15,9	1.2806	20,5	0.8336	13,4	1.0953	17,6	240 300
63 117	1.3234	18,6	1.0219	14,4	1.3175	18,5	0.8576	12,1	1.1269	15,8	243 297
66 114	1.3569	16,5	1.0477	12,8	1.3508	16,5	0.8793	10,7	1.1554	14,1	246 294
69 111	1.3867	14,4	1.0707	11,1	1.3804	14,4	0.8986	9,4	1.1808	12,3	249 291
72 108	1.4126	12,3	1.0907	9,5	1.4063	12,2	0.9154	7,9	1.2029	10,4	252 288
75 105	1.4347	10,1	1.1078	7,8	1.4283	10,0	0.9297	6,5	1.2217	8,6	255 285
78 102	1.4528	7,9	1.1218	6,1	1.4463	7,8	0.9415	5,1	1.2371	6,7	258 282
81 99	1.4670	5,6	1.1327	4,3	1.4605	5,6	0.9507	3,6	1.2492	4,8	261 279
84 96	1.4772	3,4	1.1406	2,6	1.4706	3,4	0.9572	2,2	1.2579	2,9	264 276
87 93	1.4832	1,2	1.1453	0,9	1.4766	1,1	0.9612	0,7	1.2630	1,0	267 273
90 90	−1.4853 +		−1.1469 +		−1.4787 +		−0.9625 +		−1.2648 +		270 270
(☊ + N′) Argument	α Persei		Aldebaran		Capella		Rigel		β Tauri		(☊ + N′) Argument

TABLE III. Lunar Nutation in Right Ascension, *in time.*

M′. sin (☊ + N′)

Argument (☊ + N′)	11 α Orionis M′ = −1.0836 N′ = 0° 15′	Diff. for 10′	12 Sirius M′ = −0.8968 N′ = 1° 51′	Diff. for 10′	13 Castor M′ = −1.2960 N′ = 354° 1′	Diff. for 10′	14 Procyon M′ = −1.0673 N′ = 358° 47′	Diff. for 10′	15 Pollux M′ = −1.2540 N′ = 354° 2′	Diff. for 10′	Argument (☊ + N′)
0° 180°	−0″.0000+	31,5	−0″.0000+	26,1	−0″.0000+	37,7	−0″.0000+	31,0	−0″.0000+	36,4	180° 360°
3 177	0.0567	31,4	0.0469	26,0	0.0678	37,6	0.0558	31,0	0.0656	36,4	183 357
6 174	0.1133	31,3	0.0937	25,9	0.1355	37,3	0.1116	30,8	0.1311	36,2	186 354
9 171	0.1695	31,0	0.1403	25,7	0.2027	37,1	0.1670	30,5	0.1962	35,8	189 351
12 168	0.2253	30,6	0.1865	25,3	0.2695	36,6	0.2219	30,2	0.2607	35,5	192 348
15 165	0.2805	30,2	0.2321	25,0	0.3354	36,2	0.2762	29,8	0.3246	34,9	195 345
18 162	0.3349	29,7	0.2771	24,6	0.4005	35,5	0.3298	29,3	0.3875	34,4	198 342
21 159	0.3883	29,1	0.3214	24,1	0.4644	34,8	0.3825	28,7	0.4494	33,7	201 339
24 156	0.4407	28,5	0.3648	23,5	0.5271	34,1	0.4341	28,1	0.5100	32,9	204 336
27 153	0.4920	27,4	0.4071	22,9	0.5884	33,1	0.4846	27,3	0.5693	32,1	207 333
30 150	0.5418	26,9	0.4484	22,2	0.6480	32,1	0.5337	26,4	0.6270	31,1	210 330
33 147	0.5902	26,0	0.4884	21,5	0.7058	31,1	0.5813	25,6	0.6830	30,1	213 327
36 144	0.6369	25,0	0.5271	20,7	0.7618	29,9	0.6274	24,6	0.7371	28,9	216 324
39 141	0.6819	24,0	0.5644	19,8	0.8156	28,7	0.6717	23,6	0.7892	27,7	219 321
42 138	0.7251	22,9	0.6001	18,9	0.8672	27,3	0.7142	22,5	0.8391	26,4	222 318
45 135	0.7662	21,7	0.6341	17,9	0.9164	25,9	0.7547	21,4	0.8867	25,1	225 315
48 132	0.8053	20,5	0.6664	16,9	0.9631	24,5	0.7932	20,1	0.9319	23,7	228 312
51 129	0.8421	19,2	0.6969	15,9	1.0072	22,9	0.8294	18,9	0·9745	22,2	231 309
54 126	0.8767	17,8	0.7255	14,8	1.0485	21,3	0.8635	17,6	1.0145	20,7	234 306
57 123	0.9088	16,5	0.7521	13,6	1.0869	19,7	0.8951	16,2	1.0517	19,1	237 303
60 120	0.9384	15,0	0.7766	12,4	1.1224	17,9	0.9243	14,8	1.0860	17,4	240 300
63 117	0.9655	13,6	0.7990	11,1	1.1547	16,2	0.9510	13,3	1.1173	15,7	243 297
66 114	0.9899	12,1	0.8193	9,9	1.1839	14,4	0.9750	11,9	1.1456	13,9	246 294
69 111	1.0116	10,5	0.8372	8,7	1.2099	12,6	0.9964	10,4	1.1707	12,2	249 291
72 108	1.0306	8,9	0.8529	7,4	1.2326	10,7	1.0151	8,8	1.1926	10,3	252 288
75 105	1.0467	7,4	0.8662	6,1	1.2518	8,8	1.0309	7,3	1.2112	8,6	255 285
78 102	1.0599	5,8	0.8772	4,7	1.2677	6,8	1.0440	5,7	1.2266	6,6	258 282
81 99	1.0703	4,1	0.8857	3,4	1.2800	4,9	1.0542	4,1	1.2385	4,8	261 279
84 96	1.0777	2,5	0.8919	2,1	1.2889	2,9	1.0615	2,4	1.2471	2,8	264 276
87 93	1.0821	0,8	0.8956	0,7	1.2942	1,0	1.0658	0,8	1.2522	1,0	267 273
90 90	−1.0836 +		−0.8968 +		−1.2960 +		−1.0673 +		−1.2540 +		270 270
(☊ + N′) Argument	α Orionis		Sirius		Castor		Procyon		Pollux		(☊ + N′) Argument

TABLE III. Lunar Nutation in Right Ascension, *in time*.

M′. sin (☊ + N′)

Argument (☊ + N′)	16 α Hydræ M′ = −0.9884 N′ = 5° 41′	Diff. for 10′	17 Regulus M′ = −1.0836 N′ = 353° 46′	Diff. for 10′	18 α Ursæ Major. M′ = −1.6892 N′ = 318° 58′	Diff. for 10′	19 β Leonis M′ = −1.0501 N′ = 350° 56′	Diff. for 10′	20 β Virginis M′ = −1.0285 N′ = 358° 25′	Diff. for 10′	Argument (☊ + N′)
0° 180°	−0″.0000+	28,7	−0″.0000+	31,5	−0″.0000+	49,1	−0″.0000+	30,6	−0.0000+	29,9	180° 360°
3 177	0.0517	28,7	0.0567	31,4	0.0884	49,0	0.0550	30,4	0.0538	29,8	183 357
6 174	0.1033	28,5	0.1133	31,2	0.1766	48,7	0.1098	30,3	0.1075	29,7	186 354
9 171	0.1546	28,3	0.1695	31,0	0.2642	48,3	0.1643	30,0	0.1609	29,4	189 351
12 168	0.2055	27,9	0.2253	30,7	0.3512	47,8	0.2183	29,7	0.2138	29,1	192 348
15 165	0.2558	27,5	0.2805	30,2	0.4372	47,1	0.2718	29,3	0.2662	28,7	195 345
18 162	0.3054	27,1	0.3348	29,7	0.5220	46,3	0.3245	28,8	0.3178	28,2	198 342
21 159	0.3542	26,5	0.3883	29,1	0.6054	45,4	0.3763	28,2	0.3686	27,6	201 339
24 156	0.4020	25,9	0.4407	28,4	0.6871	44,3	0.4271	27,6	0.4183	27,0	204 336
27 153	0.4487	25,3	0.4919	27,7	0.7669	43,2	0.4767	26,9	0.4669	26,3	207 333
30 150	0.4942	24,5	0.5418	26,9	0.8446	41,9	0.5251	26,0	0.5142	25,5	210 330
33 147	0.5383	23,7	0.5902	25,9	0.9200	40,5	0.5719	25,2	0.5601	24,7	213 327
36 144	0.5810	22,8	0.6369	25,0	0.9929	38,9	0.6172	24,3	0.6045	23,7	216 324
39 141	0.6220	21,9	0.6819	24,0	1.0630	37,4	0.6609	23,2	0.6472	22,8	219 321
42 138	0.6614	20,8	0.7251	22,8	1.1303	35,6	0.7027	22,1	0.6882	21,7	222 318
45 135	0.6989	19,8	0.7662	21,7	1.1944	33,8	0.7425	21,1	0.7272	20,6	225 315
48 132	0.7345	18,7	0.8053	20,4	1.2553	31,9	0.7804	19,8	0.7643	19,4	228 312
51 129	0.7682	17,5	0.8421	19,2	1.3127	29,9	0.8161	18,6	0.7993	18,2	231 309
54 126	0.7997	16,3	0.8767	17,8	1.3666	27,8	0.8496	17,3	0.8321	16,9	234 306
57 123	0.8290	15,0	0.9088	16,4	1.4167	25,7	0.8807	15,9	0.8625	15,7	237 303
60 120	0.8560	13,7	0.9384	15,1	1.4629	23,4	0.9094	14,6	0.8907	14,3	240 300
63 117	0.8807	12,4	0.9655	13,6	1.5051	21,2	0.9357	13,1	0.9164	12,9	243 297
66 114	0.9030	11,0	0.9899	12,1	1.5432	18,8	0.9593	11,7	0.9396	11,4	246 294
69 111	0.9228	9,6	1.0116	10,6	1.5770	16,4	0.9804	10,2	0.9602	9,9	249 291
72 108	0.9401	8,2	1.0306	8,9	1.6065	13,9	0.9987	8,7	0.9781	8,5	252 288
75 105	0.9548	6,7	1.0467	7,3	1.6316	11,5	1.0143	7,2	0.9934	7,0	255 285
78 102	0.9668	5,3	1.0599	5,8	1.6523	8,9	1.0272	5,5	1.0060	5,4	258 282
81 99	0.9763	3,7	1.0703	4,1	1.6684	6,4	1.0372	4,0	1.0158	3,9	261 279
84 96	0.9830	2,3	1.0777	2,4	1.6799	3,9	1.0444	2,4	1.0228	2,3	264 276
87 93	0.9871	0,7	1.0821	0,8	1.6869	1,3	1.0487	0,8	1.0270	0,8	267 273
90 90	−0.9884 +		−1·0836 +		−1.6892 +		−1.0501 +		−1.0285 +		270 270
(☊ + N′) Argument	α Hydræ		Regulus		α Ursæ Major.		β Leonis		β Virginis		(☊ + N′) Argument

TABLE III. Lunar Nutation in Right Ascension, *in time*.

M′. sin (☊ + N′)

Argument	21 γ Ursæ Major.		22 Spica Virginis		23 η Ursæ Major.		24 Arcturus		25, 26 α² Libræ		Argument
(☊ + N′)	M′ = −1.3596 N′ = 321° 46′	Diff. for 10′	M′ = −1.0577 N′ = 5° 34′	Diff. for 10′	M′ = −1.0294 N′ = 320° 55′	Diff. for 10′	M′ = −0.9583 N′ = 348° 50′	Diff. for 10′	M′ = −1.1125 N′ = 6° 27′	Diff. for 10′	(☊ + N′)
0° 180°	−0″.0000+	39,6	−0″.0000+	30,8	−0″.0000+	29,9	−0″.0000+	27,9	−0″.0000+	32,3	180° 360°
3 177	0.0712	39,4	0.0554	30,7	0.0539	29,8	0.0502	27,8	0.0582	32,3	183 357
6 174	0.1421	39,2	0.1106	30,5	0.1076	29,7	0.1002	27,6	0.1163	32,1	186 354
9 171	0.2127	38,9	0.1655	30,2	0.1610	29,4	0.1499	27,4	0.1740	31,8	189 351
12 168	0.2827	38,4	0.2199	29,9	0.2140	29,1	0.1992	27,1	0.2313	31,4	192 348
15 165	0.3519	37,9	0.2738	29,5	0.2664	28,7	0.2480	26,7	0.2879	31,1	195 345
18 162	0.4201	37,3	0.3269	29,0	0.3181	28,2	0.2961	26,3	0.3438	30,5	198 342
21 159	0.4872	36,6	0.3791	28,4	0.3689	27,7	0.3434	25,8	0.3987	29,9	201 339
24 156	0.5530	35,7	0.4302	27,8	0.4187	27,0	0.3898	25,2	0.4525	29,2	204 336
27 153	0.6172	34,8	0.4802	27,1	0.4673	26,3	0.4351	24,4	0.5051	28,4	207 333
30 150	0.6798	33,7	0.5289	26,2	0.5147	25,5	0.4791	23,8	0.5562	27,6	210 330
33 147	0.7405	32,6	0.5761	25,3	0.5606	24,7	0.5219	23,0	0.6059	26,7	213 327
36 144	0.7991	31,4	0.6217	24,4	0.6051	24,3	0.5633	22,2	0.6539	25,7	216 324
39 141	0.8556	30,1	0.6657	23,4	0.6478	22,8	0.6031	21,2	0.7001	24,6	219 321
42 138	0.9097	28,7	0.7078	22,3	0.6888	21,8	0.6412	20,2	0.7444	23,4	222 318
45 135	0.9613	27,2	0.7479	21,2	0.7279	20,6	0.6776	19,2	0.7866	22,3	225 315
48 132	1.0103	25,7	0.7860	20,0	0.7650	19,4	0.7121	18,1	0.8267	21,1	228 312
51 129	1.0566	24,1	0.8220	18,7	0.8000	18,2	0.7447	17,0	0.8646	19,7	231 309
54 126	1.0999	22,4	0.8557	17,4	0.8328	16,9	0.7753	15,8	0.9000	18,3	234 306
57 123	1.1402	20,7	0.8871	16,1	0.8633	15,6	0.8037	14,6	0.9330	16,9	237 303
60 120	1.1774	18,9	0.9160	14,7	0.8913	14,4	0.8299	13,3	0.9635	15,4	240 300
63 117	1.2114	17,0	0.9425	13,2	0.9172	12,9	0.8538	12,0	0.9912	13,9	243 297
66 114	1.2420	15,2	0.9663	11,8	0.9404	11,4	0.8754	10,7	1.0163	12,4	246 294
69 111	1.2693	13,2	0.9875	10,3	0.9610	10,0	0.8946	9,3	1.0386	10,8	249 291
72 108	1.2930	11,2	1.0060	8,7	0.9790	8,5	0.9114	7,9	1.0581	9,2	252 288
75 105	1.3132	9,2	1.0217	7,2	0.9943	7,0	0.9256	6,5	1.0746	7,6	255 285
78 102	1.3298	7,2	1.0346	5,6	1.0069	5,4	0.9373	5,1	1.0882	5,9	258 282
81 99	1.3428	5,2	1.0447	4,0	1.0167	3,9	0.9465	3,6	1.0988	4,2	261 279
84 96	1.3521	3,1	1.0519	2,4	1.0237	1,8	0.9530	2,2	1.1064	2,6	264 276
87 93	1.3577	1,1	1.0563	0,8	1.0280	0,8	0.9570	0,7	1.1110	0,8	267 273
90 90	−1.3596+		−1.0577+		−1.0294+		−0.9583+		−1.1125+		270 270
(☊ + N′) Argument	γ Ursæ Major.		Spica Virginis		η Ursæ Major.		Arcturus		α² Libræ		(☊ + N′) Argument

TABLE III. Lunar Nutation in Right Ascension, *in time.*

$$M'.\sin(\Omega + N')$$

Argument (Ω + N′)	27 β Ursæ Minor. M′ = + 1.6239 N′ = 86° 45′	Diff. for 10′	28 α Cor. Bor. M′ = −0.8668 N′ = 347° 18′	Diff. for 10′	29 α Serpentis M′ = −0.9833 N′ = 351° 30′	Diff. for 10′	30 Antares M′ = − 1.2296 N′ = 5° 49	Diff. for 10′	31 α Herculis M′ = −0.9142 N′ = 357° 45′	Diff. for 10′	Argument (Ω + N′)
0° 180°	+0″.0000−	47,2	−0″.0000+	25,2	−0″.0000+	28,6	−0″.0000+	35,8	−0″.0000+	26,6	180° 360°
3 177	0.0850	47,1	0.0454	25,1	0.0515	28,5	0.0644	35,6	0.0478	26,6	183 357
6 174	0.1697	46,8	0.0906	25,0	0.1028	28,3	0.1285	35,5	0.0956	26,3	186 354
9 171	0.2540	46,4	0.1356	24,8	0.1538	28,1	0.1924	35,2	0.1430	26,2	189 351
12 168	0.3376	45,9	0.1802	24,6	0.2044	27,8	0.2557	34,8	0.1901	25,8	192 348
15 165	0.4203	45,3	0.2244	24,2	0.2545	27,4	0.3183	34,3	0.2366	25,5	195 345
18 162	0.5018	44,6	0.2679	23,7	0.3039	26,9	0.3800	33,7	0.2825	25,1	198 342
21 159	0.5820	43,6	0.3106	23,3	0.3524	26,4	0.4407	33,0	0.3276	24,6	201 339
24 156	0.6605	42,6	0.3526	22,7	0.3999	25,8	0.5001	32,3	0.3718	24,0	204 336
27 153	0.7372	41,6	0.3935	22,2	0.4464	25,1	0.5582	31,4	0.4150	23,4	207 333
30 150	0.8120	40,2	0.4334	21,5	0.4916	24,4	0.6148	30,5	0.4571	22,7	210 330
33 147	0.8844	38,9	0.4721	20,8	0.5355	23,6	0.6697	29,5	0.4979	21,9	213 327
36 144	0.9545	37,5	0.5095	20,0	0.5780	22,6	0.7228	28,3	0.5374	21,1	216 324
39 141	1.0220	35,9	0.5455	19,2	0.6188	21,7	0.7738	27,2	0.5753	20,2	219 321
42 138	1.0866	34,3	0.5800	18,3	0.6579	20,8	0.8228	25,9	0.6117	19,3	222 318
45 135	1.1483	32,5	0.6129	17,4	0.6953	19,7	0.8695	24,6	0.6464	18,3	225 315
48 132	1.2068	30,7	0.6442	16,3	0.7307	18,6	0.9138	23,2	0.6794	17,3	228 312
51 129	1.2620	28,8	0.6736	15,4	0.7642	17,4	0.9556	21,8	0.7105	16,3	231 309
54 126	1.3138	26,7	0.7013	14,3	0.7955	16,2	0.9948	20,2	0.7396	15,1	234 306
57 123	1.3619	24,7	0.7270	13,2	0.8247	14,9	1.0312	18,7	0.7667	13,9	237 303
60 120	1.4063	22,6	0.7507	12,0	0.8515	13,7	1.0649	17,1	0.7917	12,7	240 300
63 117	1.4469	20,3	0.7723	10,9	0.8761	12,3	1.0956	15,4	0.8146	11,4	243 297
66 114	1.4835	18,1	0.7919	9,6	0.8983	10,9	1.1233	13,7	0.8352	10,2	246 294
69 111	1.5160	15,8	0.8092	8,4	0.9180	9,6	1.1479	11,9	0.8535	8,9	249 291
72 108	1.5444	13,4	0.8244	7,2	0.9352	8,1	1.1694	10,2	0.8695	7,6	252 288
75 105	1.5686	11,0	0.8373	5,9	0.9498	6,7	1.1877	8,3	0.8831	6,2	255 285
78 102	1.5884	8,6	0.8479	4,6	0.9618	5,2	1.2027	6,6	0.8942	4,9	258 282
81 99	1.6039	6,2	0.8562	3,3	0.9712	3,7	1.2145	4,7	0.9030	3,4	261 279
84 96	1.6150	3,7	0.8621	1,9	0.9779	2,2	1.2229	2,8	0.9092	2,1	264 276
87 93	1.6217	1,2	0.8656	0,7	0.9819	0,8	1.2279	0,9	0.9130	0,7	267 273
90 90	+1.6239−		−0.8668+		−0.9833+		−1.2296+		−0.9142+		270 270
(Ω + N′) Argument	β Ursæ Minor.		α Cor. Bor.		α Serpentis		Antares		α Herculis		(Ω + N′) Argument

TABLE III. Lunar Nutation in Right Ascension, *in time.*

M′. sin (☊ + N′)

Argument (☊ + N′)	32 α Ophiuchi M′ = −0.9272 N′ = 258° 48′	Diff. for 10′	33 γ Draconis M′ = −0.4671 N′ = 357° 3′	Diff. for 10′	34 α Lyræ M′ = −0.6768 N′ = 5° 30′	Diff. for 10′	35 γ Aquilæ M′ = −0.9545 N′ = 2° 41′	Diff. for 10′	36 α Aquilæ M′ = −0.9677 N′ = 2° 16′	Diff. for 10′	Argument (☊ + N′)
0° 180°	−0″.0000+	26,9	−0″.0000+	13,6	−0″.0000+	19,7	−0″.0000+	27,8	−0″.0000+	28,1	180° 360°
3 177	0.0485	26,9	0.0244	13,6	0.0354	19,6	0.0500	27,7	0.0506	28,1	183 357
6 174	0.0969	26,8	0.0488	13,5	0.0707	19,6	0.0998	27,5	0.1011	27,9	186 354
9 171	0.1451	26,5	0.0731	13,3	0.1059	19,3	0.1493	27,3	0.1514	27,7	189 351
12 168	0.1928	26,3	0.0971	13,2	0.1407	19,2	0.1984	27,0	0.2012	27,3	192 348
15 165	0.2400	25,8	0.1209	13,0	0.1752	18,8	0.2470	26,8	0.2504	27,0	195 345
18 162	0.2865	25,4	0.1443	12,8	0.2091	18,6	0.2950	26,2	0.2990	26,6	198 342
21 159	0.3323	24,9	0.1674	12,6	0.2425	18,2	0.3421	25,6	0.3468	26,0	201 339
24 156	0.3771	24,4	0.1900	12,2	0.2753	17,8	0.3882	25,1	0.3936	25,4	204 336
27 153	0.4210	23,7	0.2120	11,9	0.3073	17,3	0.4333	24,4	0.4393	24,7	207 333
30 150	0.4636	23,0	0.2335	11,6	0.3384	16,8	0.4772	23,7	0.4838	24,0	210 330
33 147	0.5050	22,2	0.2544	11,2	0.3686	16,2	0.5199	22,8	0.5270	23,2	213 327
36 144	0.5450	21,4	0.2745	10,8	0.3978	15,6	0.5610	22,1	0.5688	22,3	216 324
39 141	0.5835	20,5	0.2939	10,3	0.4259	15,0	0.6007	21,1	0.6090	21,4	219 321
42 138	0.6204	19,6	0.3125	9,8	0.4529	14,3	0.6387	20,1	0.6475	20,4	222 318
45 135	0.6556	18,6	0.3302	9,4	0.4786	13,6	0.6749	19,1	0.6842	19,4	225 315
48 132	0.6891	17,5	0.3471	8,8	0.5030	12,8	0.7093	18,1	0.7191	18,3	228 312
51 129	0.7206	16,4	0.3630	8,3	0.5260	11,9	0.7418	16,9	0.7520	17,2	231 309
54 126	0.7502	15,2	0.3779	7,7	0.5475	11,2	0.7722	15,7	0.7829	15,9	234 306
57 123	0.7776	14,1	0.3917	7,1	0.5676	10,3	0.8005	14,5	0.8115	14,7	237 303
60 120	0.8030	12,9	0.4045	6,4	0.5861	9,4	0.8266	13,3	0.8380	13,4	240 300
63 117	0.8262	11,6	0.4161	5,9	0.6030	8,5	0.8505	11,9	0.8622	12,1	243 297
66 114	0.8471	10,3	0.4267	5,2	0.6183	7,5	0.8720	10,6	0.8840	10,8	246 294
69 111	0.8656	9,1	0.4360	4,6	0.6318	6,6	0.8911	9,3	0.9034	9,4	249 291
72 108	0.8819	7,6	0.4442	3,8	0.6437	5,6	0.9078	7,9	0.9203	8,0	252 288
75 105	0.8956	6,3	0.4511	3,2	0.6537	4,6	0.9220	6,4	0.9347	6,6	255 285
78 102	0.9070	4,9	0.4568	2,5	0.6620	3,6	0.9336	5,1	0.9465	5,1	258 282
81 99	0.9158	3,5	0.4613	1,8	0.6685	2,6	0.9427	3,6	0.9557	3,7	261 279
84 96	0.9221	2,2	0.4645	1,1	0.6731	1,6	0.9492	2,2	0.9623	2,2	264 276
87 93	0.9260	0,7	0.4664	0,4	0.6759	0,5	0.9532	0,7	0.9663	0,8	267 273
90 90	−0.9272+		−0.4671+		−0.6768+		−0.9545+		−0.9677+		270 270
(☊ + N′) Argument	α Ophiuchi		γ Draconis		α Lyræ		γ Aquilæ		α Aquilæ		(☊ + N′) Argument

TABLE III. Lunar Nutation in Right Ascension, *in time.*

M′. sin (☊ + N′)

Argument (☊ + N′)	37 β Aquilæ M′ = −0.9850 N′ = 1° 39′	Diff. for 10′	38, 39 α² Capricorni M′ = −1.1163 N′ = 356° 12′	Diff. for 10′	40 α Cygni M′ = −0.7782 N′ = 28° 32′	Diff. for 10′	41 α Cephei M′ = −0.9653 N′ = 60° 28′	Diff. for 10′	42 β Cephei M′ = −1.3051 N′ = 71° 53′	Diff. for 10′	Argument (☊ + N′)
0° 180°	−0″.0000+	28,7	−0″.0000+	32,4	−0″.0000+	22,6	−0″.0000+	28,1	−0″.0000+	37,9	180° 360°
3 177	0.0516	28,6	0.0584	32,4	0.0407	22,6	0.0505	28,0	0.0683	37,8	183 357
6 174	0.1030	28,4	0.1167	32,2	0.0813	22,4	0.1009	27,8	0.1364	37,7	186 354
9 171	0.1541	28,2	0.1746	31,9	0.1217	22,3	0.1510	27,6	0.2042	37,3	189 351
12 168	0.2048	27,8	0.2321	31,6	0.1618	22,0	0.2007	27,3	0.2713	36,9	192 348
15 165	0.2549	27,5	0.2889	31,2	0.2014	21,7	0.2499	26,9	0.3378	36,4	195 345
18 162	0.3044	27,0	0.3450	30,6	0.2405	21,3	0.2983	26,4	0.4033	35,8	198 342
21 159	0.3530	26,4	0.4001	30,0	0.2789	20,9	0.3459	25,9	0.4677	35,1	201 339
24 156	0.4006	25,9	0.4541	29,3	0.3165	20,4	0.3926	25,4	0.5308	34,3	204 336
27 153	0.4472	25,2	0.5068	28,6	0.3533	19,9	0.4383	24,7	0.5925	33,3	207 333
30 150	0.4925	24,4	0.5582	27,7	0.3891	19,3	0.4827	23,9	0.6525	32,4	210 330
33 147	0.5365	23,6	0.6080	26,8	0.4238	18,7	0.5258	23,1	0.7108	31,3	213 327
36 144	0.5790	22,7	0.6562	25,8	0.4574	17,9	0.5674	22,3	0.7671	30,1	216 324
39 141	0.6199	21,8	0.7026	24,7	0.4897	17,2	0.6075	21,3	0.8213	28,9	219 321
42 138	0.6591	20,8	0.7470	23,6	0.5207	16,4	0.6459	20,4	0.8733	27,5	222 318
45 135	0.6965	19,7	0.7894	22,3	0.5502	15,6	0.6826	19,3	0.9228	26,1	225 315
48 132	0.7320	18,6	0.8296	21,1	0.5783	14,7	0.7174	18,2	0.9698	24,7	228 312
51 129	0.7655	17,4	0.8676	19,8	0.6047	13,8	0.7502	17,1	1.0142	23,1	231 309
54 126	0.7969	16,2	0.9032	18,4	0.6295	12,8	0.7810	15,9	1.0558	21,5	234 306
57 123	0.8261	15,0	0.9363	16,9	0.6526	11,8	0.8096	14,7	1.0945	19,8	237 303
60 120	0.8531	13,7	0.9668	15,5	0.6739	10,8	0.8360	13,4	1.1302	18,1	240 300
63 117	0.8777	12,3	0.9947	14,0	0.6933	9,8	0.8601	12,1	1.1628	16,3	243 297
66 114	0.8999	10,9	1.0199	12,4	0.7109	8,7	0.8819	10,7	1.1922	14,6	246 294
69 111	0.9196	9,6	1.0422	10,8	0.7265	7,6	0.9012	9,4	1.2184	12,7	249 291
72 108	0.9368	8,2	1.0617	9,2	0.7401	6,4	0.9181	7,9	1.2412	10,8	252 288
75 105	0.9515	6,8	1.0783	7,6	0.7516	5,3	0.9324	6,6	1.2606	8,8	255 285
78 102	0.9635	5,2	1.0920	5,9	0.7611	4,2	0.9442	5,2	1.2765	6,9	258 282
81 99	0.9729	3,7	1.1026	4,2	0.7686	2,9	0.9535	3,6	1.2890	4,9	261 279
84 96	0.9796	2,3	1.1102	2,6	0.7739	1,8	0.9600	2,2	1.2979	3,0	264 276
87 93	0.9837	0,7	1.1148	0,8	0.7771	0,6	0.9640	0,7	1.3033	1,0	267 273
90 90	−0.9850+		−1.1163+		−0.7782+		−0.9653+		−1.3051+		270 270
(☊ + N′) Argument	β Aquilæ		α² Capricorni		α Cygni		α Cephei		β Cephei		(☊ + N′) Argument

TABLE III. Lunar Nutation in Right Ascension, *in time.*

$$M'.\sin(☊ + N')$$

Argument (☊ + N')	43 α Aquarii M′=−1.0308 N′=359° 26′	Diff. for 10′	44 Fomalhaut M′=−1.1570 N′=343° 8′	Diff. for 10′	45 α Pegasi M′=−1.0060 N′=6° 23′	Diff. for 10′	46 α Andromedæ M′=−1.0748 N′=17° 20′	Diff. for 10′	Argument (☊ + N′)
0° 180°	−0″.0000+	29,9	−0″.0000+	33,7	−0″.0000+	29,2	−0″.0000 +	31,3	180° 360°
3 177	0.0539	29,9	0.0606	33,5	0.0526	29,2	0.0563	31,1	183 357
6 174	0.1077	29,8	0.1209	33,4	0.1052	29,0	0.1123	31,0	186 354
9 171	0.1613	29,4	0.1810	33,1	0.1574	28,8	0.1681	30,8	189 351
12 168	0.2143	29,2	0.2406	32,7	0.2092	28,4	0.2235	30,4	192 348
15 165	0.2668	28,7	0.2995	32,2	0.2604	28,1	0.2782	29,9	195 345
18 162	0.3185	28,3	0.3575	31,7	0.3109	27,6	0.3321	29,5	198 342
21 159	0.3694	27,7	0.4146	31,1	0.3605	27,1	0.3852	28,9	201 339
24 156	0.4193	27,1	0.4706	30,4	0.4092	26,4	0.4372	28,2	204 336
27 153	0.4680	26,3	0.5253	29,6	0.4567	25,7	0.4880	27,4	207 333
30 150	0.5154	25,6	0.5785	28,7	0.5030	24,9	0.5374	26,7	210 330
33 147	0.5614	24,7	0.6302	27,7	0.5479	24,1	0.5854	25,8	213 327
36 144	0.6059	23,8	0.6801	26,7	0.5913	23,2	0.6318	24,8	216 324
39 141	0.6487	22,8	0.7281	25,6	0.6331	22,2	0.6764	23,8	219 321
42 138	0.6897	21,8	0.7742	24,4	0.6731	21,2	0.7192	22,7	222 318
45 135	0.7289	20,6	0.8181	23,2	0.7113	20,2	0.7600	21,5	225 315
48 132	0.7660	19,5	0.8598	21,9	0.7476	19,0	0.7987	20,3	228 312
51 129	0.8011	18,2	0.8992	20,5	0.7818	17,8	0.8353	19,1	231 309
54 126	0.8339	17,0	0.9361	19,1	0.8139	16,6	0.8696	17,7	234 306
57 123	0.8645	15,7	0.9704	17,6	0.8437	15,3	0.9014	16,3	237 303
60 120	0.8927	14,3	1.0020	16,1	0.8712	13,9	0.9308	14,9	240 300
63 117	0.9184	12,9	1.0309	14,5	0.8963	12,6	0.9577	13,4	243 297
66 114	0.9417	11,4	1.0570	12,9	0.9190	11,2	0.9819	11,9	246 294
69 111	0.9623	10,0	1.0802	11,2	0.9392	9,8	1.0034	10,4	249 291
72 108	0.9803	8,6	1.1004	9,6	0.9568	8,3	1.0222	8,9	252 288
75 105	0.9957	7,0	1.1176	7,8	0.9717	6,8	1.0382	7,4	255 285
78 102	1.0083	5,4	1.1317	6,2	0.9840	5,3	1.0513	5,7	258 282
81 99	1.0181	3,9	1.1428	4,4	0.9936	3,8	1.0616	4,1	261 279
84 96	1.0251	2,4	1.1507	2,6	1.0005	2,3	1.0689	2,3	264 276
87 93	1.0294	0,8	1.1554	0,9	1.0046	0,8	1.0733	0,8	267 273
90 90	−1.0308 +		−1.1570+		−1.0060+		−1.0748+		270 270
(☊ + N′) Argument	α Aquarii		Fomalhaut		α Pegasi		α Andromedæ		(☊ + N′) Argument

TABLE IV. Solar Nutation in Right Ascension, *in time*.

M″. (2 ☉ + N″)

	Names of the Stars.	Values of N″	Argument (2 ☉ + N″) 0° 180	10° 170	20° 160	30° 150	40° 140	50° 130	60° 120	70° 110	80° 100	90° 90
1	γ Pegasi	6° 50′	∓0.0000	0.0143	0.0282	0.0412	0.0530	0.0632	0.0714	0.0775	0.0812	0.0825
2	α Cassiop.	32 15	.0000	.0182	.0359	.0524	.0674	.0803	.0908	.0985	.1032	.1048
3	Polaris	76 9	.0000	.2466	.4856	.7101	.9129	1.0879	1.2298	1.3345	1.3985	1.4201
4	α Arietis	8 59	.0000	.0156	.0308	.0451	.0579	0.0690	0.0780	0.0846	0.0887	0.0901
5	α Ceti	1 9	.0000	.0144	.0284	.0416	.0535	.0637	.0720	·0782	.0819	.0832
6	α Persei	14 57	.0000	.0202	.0398	.0581	.0747	.0890	.1006	.1092	.1145	.1162
7	Aldebaran	2 48	.0000	.0158	.0312	.0456	.0587	.0699	.0790	.0857	.0898	.0912
8	Capella	4 41	.0000	.0204	.0402	.0587	.0755	.0900	.1018	.1104	.1157	.1175
9	Rigel	359 1	.0000	.0133	.0262	.0383	.0492	.0587	.0663	.0720	.0754	.0766
10	β Tauri	2 18	.0000	.0175	.0344	.0503	.0647	.0771	.0872	.0946	.0991	.1006
11	α Orionis	0 12	.0000	.0150	.0295	.0431	.0555	.0661	.0747	.0811	.0850	.0863
12	Sirius	1 30	.0000	.0124	.0244	.0357	.0459	.0547	.0618	.0671	.0703	.0714
13	Castor	355 9	.0000	.0179	.0352	.0515	.0662	.0789	.0892	.0968	.1014	.1030
14	Procyon	359 0	.0000	.0148	.0291	.0425	.0546	.0651	.0736	.0798	.0837	.0850
15	Pollux	355 9	.0000	.0173	.0341	.0498	.0640	.0763	.0863	.0936	.0981	.0996
16	α Hydræ	3 0	.0000	.0136	.0269	.0393	.0505	.0602	.0681	.0739	.0774	.0786
17	Regulus	354 56	.0000	.0149	.0294	.0430	.0553	.0659	.0745	.0809	.0848	.0861
18	α Ursæ Major.	324 46	.0000	.0216	.0425	.0621	.0798	.0951	.1075	.1167	.1223	.1242
19	β Leonis	352 37	.0000	.0145	.0285	.0416	.0535	.0638	.0721	.0782	.0820	.0832
20	β Virginis	358 43	.0000	.0142	.0280	.0409	.0526	.0627	.0709	.0769	.0806	.0818
21	γ Ursæ Major.	327 24	.0000	.0175	.0345	.0504	.0649	.0773	.0874	.0948	.0994	.1009
22	Spica Virginis	4 31	.0000	.0146	.0287	.0420	.0540	.0644	.0728	.0790	.0828	.0841
23	η Ursæ Major.	326 36	.0000	.0132	.0261	.0381	.0490	.0582	.0660	.0716	.0750	.0762
24	Arcturus	350 54	.0000	.0132	.0259	.0379	.0487	.0580	.0656	.0712	.0746	.0758
25, 26	} α² Libræ	5 15	∓ .0000	.0153	.0302	.0442	0568	.0677	.0765	.0830	.0870	.0884
27	β Ursæ Minor.	86 0	± .0000	.0182	.0359	.0525	.0675	.0804	.0909	.0987	.1034	.1050
28	α Cor. Bor.	349 38	∓ .0000	.0119	.0234	.0342	.0440	.0524	.0593	.0643	.0674	.0684
29	α Serpentis	357 58	.0000	.0136	.0267	.0391	.0503	.0599	.0677	.0735	.0770	.0782
30	Antares	4 44	.0000	.0170	.0334	.0489	.0628	.0748	.0846	.0918	.0962	.0977
31	α Herculis	358 11	.0000	.0126	.0249	.0364	.0468	.0557	.0630	.0684	.0716	.0727
32	α Ophiuchi	359 2	.0000	.0128	.0252	.0369	.0474	.0565	.0639	.0694	.0727	.0738
33	γ Draconis	357 36	.0000	.0064	.0127	.0186	.0239	.0284	.0322	.0349	.0366	.0372
34	α Lyræ	4 28	.0000	.0094	.0184	.0269	.0345	.0412	.0466	.0505	.0530	.0538
35	γ Aquilæ	2 11	.0000	.0132	.0260	.0380	.0488	.0582	.0658	.0714	.0748	.0759
36	α Aquilæ	1 51	.0000	.0134	.0264	.0385	.0495	.0590	.0667	.0724	.0758	.0770
37	β Aquilæ	1 20	.0000	.0136	.0268	.0392	.0504	.0600	.0697	.0737	.0072	.0784
38, 39	} α² Capricor.	356 55	.0000	.0154	.0304	.0444	.0571	.0680	.0769	.0835	.0874	.0888
40	α Cygni	23 48	.0000	.0103	.0203	.0298	.0382	.0456	.0515	.0559	.0586	.0595
41	α Cephei	55 5	.0000	.0115	.0226	.0331	.0425	.0507	.0573	.0622	.0652	.0662
42	β Cephei	75 11	.0000	.0148	.0291	.0426	.0548	.0653	.0738	.0801	.0840	.0853
43	α Aquarii	359 32	.0000	.0143	.0281	.0410	.0528	.0628	.0711	.0771	.0808	.0820
44	Fomalhaut	348 11	.0000	.0158	.0310	.0454	.0583	.0695	.0786	.0853	.0894	[illegible]
45	α Pegasi	6 49	.0000	.0139	.0273	.0399	0513	.0611	.0691	.0750	.0786	[illegible]
46	α Andromedæ	14 13	∓0.0000	0.0146	0.0288	0.0421	0.0542	0.0645	0.0729	0.0792	0.0830	0.0843
	Names of the Stars.	N″	180° 360	190° 350	200° 340	210° 330	220° 320	230° 310	240° 300	250° 290	260° 280	270° 270

TABLE V. Lunar Inequality in Right Ascension, *in time.*

$$M'''. \ (2\,☾ + N''')$$

	Names of the Stars.	Values of N'''	Argument (2 ☾ + N''') 0° 180	10° 170	20° 160	30° 150	40° 140	50° 130	60° 120	70° 110	80° 100	90° 90
1	γ Pegasi	6° 50′	∓0.0000	0.0022	0.0042	0.0062	0.0080	0.0095	0.0107	0.0117	0.0122	0.0124
2	α Cassiop.	32 15	.0000	.0027	.0054	.0079	.0101	.0121	.0137	.0148	.0156	.0158
3	Polaris	73 4	.0000	.0367	.0723	.1055	.1356	.1619	.1830	.1983	.2078	.2110
4	α Arietis	8 59	.0000	.0023	.0046	.0068	.0087	.0104	.0117	.0128	.0134	.0136
5	α Ceti	1 9	.0000	.0022	.0042	.0062	.0081	.0096	.0108	.0118	.0123	.0125
6	α Persei	14 57	.0000	.0030	.0060	.0088	.0112	.0134	.0152	.0165	.0172	.0175
7	Aldebaran	2 48	.0000	.0024	.0047	.0069	.0088	.0105	.0119	.0129	.0135	.0138
8	Capella	4 41	.0000	.0031	.0060	.0089	.0113	.0136	.0154	.0166	.0174	.0177
9	Rigel	359 1	.0000	.0020	.0039	.0058	.0074	.0089	.0100	.0108	.0114	.0115
10	β Tauri	2 18	.0000	.0026	.0052	.0076	.0097	.0116	.0131	.0142	.0149	.0152
11	α Orionis	0 12	.0000	.0023	.0044	.0065	.0083	.0100	.0113	.0122	.0128	.0130
12	Sirius	1 30	.0000	.0019	.0037	.0054	.0069	.0082	.0093	.0101	.0106	.0107
13	Castor	355 9	.0000	.0027	.0053	.0078	.0100	.0119	.0134	.0146	.0153	.0155
14	Procyon	359 0	.0000	.0022	.0044	.0064	.0082	.0098	.0111	.0120	.0126	.0128
15	Pollux	355 9	.0000	.0026	.0051	.0075	.0096	.0115	.0130	.0141	.0148	.0151
16	α Hydræ	3 0	.0000	.0021	.0041	.0059	.0076	.0091	.0103	.0111	.0117	.0118
17	Regulus	354 56	.0000	.0022	.0044	.0065	.0083	.0099	.0112	.0122	.0128	.0130
18	α Ursæ Major.	324 46	.0000	.0032	.0064	.0094	.0120	.0143	.0162	.0176	.0184	.0187
19	β Leonis	352 37	.0000	.0022	.0042	.0063	.0080	.0097	.0108	.0118	.0123	.0125
20	β Virginis	358 43	.0000	.0022	.0042	.0062	.0079	.0095	.0107	.0116	.0121	.0123
21	γ Ursæ Major.	327 24	.0000	.0026	.0052	.0076	.0098	.0116	.0132	.0143	.0150	.0152
22	Spica Virginis	4 31	.0000	.0022	.0042	.0064	.0080	.0099	.0111	.0118	.0125	.0127
23	η Ursæ Major.	326 36	.0000	.0020	.0039	.0057	.0074	.0088	.0099	.0108	.0113	.0115
24	Arcturus	350 54	.0000	.0020	.0039	.0057	.0073	.0087	.0099	.0107	.0112	.0114
25, 26	} α² Libræ	5 15	∓ .0000	.0023	.0045	.0067	.0086	.0102	.0115	.0125	.0131	.0133
27	β Ursæ Minor.	86 0	± .0000	.0027	.0054	.0079	.0102	.0121	.0137	.0149	.0156	.0158
28	α Cor. Bor.	349 38	∓ .0000	.0017	.0035	.0051	.0066	.0079	.0090	.0097	.0101	.0103
29	α Serpentis	357 58	.0000	.0020	.0040	.0059	.0076	.0090	.0102	.0111	.0116	.0118
30	Antares	4 44	.0000	.0025	.0050	.0074	.0094	.0114	.0128	.0138	.0144	.0147
31	α Herculis	358 11	.0000	.0019	.0037	.0054	.0070	.0085	.0096	.0103	.0108	.0119
32	α Ophiuchi	359 2	.0000	.0019	.0037	.0056	.0071	.0086	.0097	.0104	.0109	.0111
33	γ Draconis	357 36	.0000	.0010	.0019	.0028	.0036	.0043	.0048	.0053	.0055	.0056
34	α Lyræ	4 28	.0000	.0014	.0028	.0040	.0052	.0062	.0070	.0076	.0080	.0081
35	γ Aquilæ	2 11	.0000	.0020	.0038	.0057	.0073	.0088	.0099	.0107	.0112	.0114
36	α Aquilæ	1 51	.0000	.0020	.0039	.0058	.0075	.0089	.0100	.0109	.0114	.0116
37	β Aquilæ	1 20	.0000	.0021	.0039	.0059	.0075	.0092	.0103	.0110	.0116	.0118
38, 39	} α² Capricor.	356 55	.0000	.0024	.0044	.0067	.0085	.0104	.0118	.0125	.0132	.0134
40	α Cygni	23 48	.0000	.0016	.0031	.0045	.0058	.0069	.0078	.0084	.0088	.0090
41	α Cephei	55 5	.0000	.0017	.0034	.0050	.0064	.0077	.0087	.0094	.0198	.0100
42	β Cephei	75 11	.0000	.0023	.0042	.0064	.0081	.0099	.0112	.0119	.0126	.0128
43	α Aquarii	359 32	.0000	.0022	.0042	.0062	.0080	.0095	.0107	.0117	.0122	.0124
44	Fomalhaut	346 11	.0000	.0025	.0045	.0069	.0086	.0107	.0120	.0127	.0135	.0137
45	α Pegasi	6 49	.0000	.0021	.0040	.0060	.0076	.0093	.0105	.0112	.0118	.0120
46	α Andromedæ	14 13	∓0.0000	0.0023	0.0042	0.0064	0.0080	0.0099	0.0111	0.0118	0.0125	0.0127
	Names of the Stars.	N'''	180° 360	190° 350	200° 340	210° 330	220° 320	230° 310	240° 300	250° 290	260° 280	270° 270

TABLE VI. Aberration in Declination.

$m.\sin(\odot + n)$

Argument $(\odot + n)$	1 γ Pegasi $m=-9.3044$ $n=57°\ 11'$	Diff. for 10′	2 α Cassiop. $m=-16.9512$ $n=8°\ 34'$	Diff. for 10′	3 Polaris $m=-20.3320$ $n=346°\ 56'$	Diff. for 10′	4 α Arietis $m=-7.9453$ $n=30°\ 2'$	Diff. for 10′	5 α Ceti $m=-7.4304$ $n=83°\ 8'$	Diff. for 10′	Argument $(\odot + n)$
0° 180°	−0″.0000+	270,6	−0″.0000+	492,9	−0″.0000 +	591,2	−0″.0000+	231,0	−0″.0000+	216,1	180° 360°
3 177	0.4870	269,8	0.8872	491,5	1.0641	589,6	0.4158	230,4	0.3889	215,4	183 357
6 174	0.9726	267,8	1.7719	488,8	2.1253	586,3	0.8305	229,1	0.7767	214,3	186 354
9 171	1.4555	266,1	2.6517	484,8	3.1806	581,5	1.2429	227,2	1.1624	212,5	189 351
12 168	1.9345	263,2	3.5244	479,4	4.2273	575,1	1.6519	224,7	1.5449	210,1	192 348
15 165	2.4082	259,3	4.3873	472,7	5.2624	566,9	2.0564	221,6	1.9231	207,2	195 345
18 162	2.8752	255,1	5.2382	464,8	6.2829	557,5	2.4552	217,8	2.2961	203,7	198 342
21 159	3.3344	250,0	6.0748	455,4	7.2864	546,3	2.8473	213,5	2.6628	199,7	201 339
24 156	3.7844	244,3	6.8946	445,1	8.2697	533,8	3.2316	208,6	3.0222	195,1	204 336
27 153	4.2242	237,8	7.6957	433,3	9.2306	519,7	3.6071	203,1	3.3733	189,9	207 333
30 150	4.6522	230,8	8.4756	420,4	10.1660	504,2	3.9726	197,1	3.7152	184,3	210 330
33 147	5.0676	223,0	9.2323	406,3	11.0736	487,4	4.3273	190,4	4.0469	178,1	213 327
36 144	5.4690	214,7	9.9637	391,1	11.9509	466,7	4.6701	183,3	4.3675	171,4	216 324
39 141	5.8554	205,8	10.6677	374,9	12.7953	449,7	5.0001	175,7	4.6761	164,3	219 321
42 138	6.2259	196,3	11.3425	357,6	13.6047	428,9	5.3164	167,6	4.9719	156,7	222 318
45 135	6.5792	186,3	11.9862	339,4	14.3767	407,1	5.6181	159,1	5.2540	148,8	225 315
48 132	6.9145	175,8	12.5971	320,2	15.1095	384,1	5.9045	150,1	5.5218	140,3	228 312
51 129	7.2309	164,8	13.1735	300,2	15.8008	360,1	6.1746	140,7	5.7744	131,6	231 309
54 126	7.5275	153,2	13.7139	279,2	16.4490	334,9	6.4279	130,9	6,0113	122,4	234 306
57 123	7.8033	141,4	14.2164	257,6	17.0518	309,0	6.6635	120,7	6.2316	112,9	237 303
60 120	8.0579	129,1	14.6801	235,3	17.6060	282,2	6.8808	110,3	6.4349	103,1	240 300
63 117	8.2903	116,5	15.1036	212,3	18.1159	254,6	7.0793	99,5	6.6205	93,1	243 297
66 114	8.5000	103,6	15.4857	188,6	18.5742	226,3	7.2584	88,4	6.7880	82,7	246 294
69 111	8.6864	90,4	15.8252	164,7	18.9815	197,5	7.4175	77,2	6.9368	72,2	249 291
72 108	8.8491	76,8	16.1217	139,8	19.3370	167,8	7.5565	65,6	7.0668	61,3	252 288
75 105	8.9873	63,2	16.3734	115,1	19.6390	138,1	7.6745	53,9	7.1771	50,4	255 285
78 102	9.1010	49,4	16.5806	89,9	19.8875	107,9	7.7716	42,2	7.2679	39,4	258 282
81 99	9.1899	35,3	16.7425	64,3	20.0817	77,1	7.8475	30,1	7.3389	28,2	261 279
84 96	9.2534	21,2	16.8582	38,7	20.2204	46,4	7.9017	18,1	7.3896	16,9	264 276
87 93	9.2916	7,1	16.9278	13,0	20.3039	15,6	7.9343	6,1	7.4201	5,7	267 273
90 90	−9.3044 +		−16.9512+		−20.3320+		−7.9453+		−7.4304 +		270 270
$(\odot + n)$ Argument	γ Pegasi		α Cassiop.		Polaris		α Arietis		α Ceti		$(\odot + n)$ Argument

TABLE VI. Aberration in Declination.

$m. \sin (\odot + n)$

Argument $(\odot + n)$	6 α Persei $m = -11.6405$, $n = 335° 5'$	Diff. for 10′	7 Aldebaran $m = -3.7929$, $n = 55° 13'$	Diff. for 10′	8 Capella $m = -8.2046$, $n = 295° 37'$	Diff. for 10′	9 Rigel $m = +10.7694$, $n = 273° 42'$	Diff. for 10′	10 β Tauri $m = -2.4814$, $n = 319° 21'$	Diff. for 10′	Argument $(\odot + n)$
0° 180°	−0″.0000+	338,4	−0″.0000+	110,3	−0″.0000+	238,6	+0″.0000−	313,1	−0″.0000+	72,2	180° 360°
3 177	0.6092	337,5	0.1985	110,0	0.4294	237,9	0.5636	312,3	0.1299	71,9	183 357
6 174	1.2167	335,7	0.3965	109,3	0.8576	236,6	1.1257	310,6	0.2594	71,6	186 354
9 171	1.8210	332,9	0.5933	108,5	1.2835	234,6	1.6847	308,0	0.3882	70,9	189 351
12 168	2.4202	329,2	0.7886	107,3	1.7058	232,1	2.2391	304,6	0.5159	70,2	192 348
15 165	3.0128	324,6	0.9817	105,8	2.1235	228,8	2.7873	300,3	0.6422	69,2	195 345
18 162	3.5971	319,2	1.1721	104,0	2.5354	224,9	3.3279	295,3	0.7668	68,0	198 342
21 159	4.1716	312,8	1.3593	101,9	2.9403	220,4	3.8594	289,4	0.8892	66,7	201 339
24 156	4.7346	305,6	1.5427	99,6	3.3371	215,4	4.3803	282,7	1.0093	65,1	204 336
27 153	5.2847	297,9	1.7220	96,9	3.7249	209,7	4.8892	275,3	1.1265	63,4	207 333
30 150	5.8202	288,7	1.8964	94,1	4.1023	203,5	5.3847	267,1	1.2407	61,5	210 330
33 147	6.3399	279,0	2.0658	90,9	4.4686	196,7	5.8654	258,2	1.3514	59,5	213 327
36 144	6.8421	268,6	2.2294	87,5	4.8226	189,3	6.3301	248,5	1.4585	57,3	216 324
39 141	7.3255	257,5	2.3869	83,9	5.1633	181,5	6.7774	238,2	1.5616	54,9	219 321
42 138	7.7890	245,6	2.5379	80,0	5.4900	173,1	7.2061	227,2	1.6604	52,3	222 318
45 135	8.2310	233,1	2.6819	75,9	5.8015	164,3	7.6150	215,6	1.7546	49,7	225 315
48 132	8.6505	219,9	2.8186	71,7	6.0972	155,0	8.0031	203,4	1.8440	46,9	228 312
51 129	9.0463	206,2	2.9476	67,2	6.3762	145,3	8.3693	190,7	1.9284	43,9	231 309
54 126	9.4174	191,7	3.0685	62,5	6.6377	135,2	8.7126	177,4	2.0075	40,8	234 306
57 123	9.7625	176,9	3.1810	57,6	6.8810	124,7	9.0319	163,7	2.0810	37,7	237 303
60 120	10.0809	162,1	3.2847	52,7	7.1054	113,9	9.3265	149,5	2.1489	34,4	240 300
63 117	10.3717	145,8	3.3795	47,5	7.3104	102,7	9.5956	134,8	2.2109	31,1	243 297
66 114	10.6341	129,6	3.4650	42,2	7.4953	91,3	9.8383	119,8	2.2668	27,6	246 294
69 111	10.8673	113,1	3.5410	36,8	7.6597	79,7	10.0540	104,7	2.3165	24,1	249 291
72 108	11.0708	96,1	3.6073	31,3	7.8031	67,7	10.2424	88,8	2.3599	20,5	252 288
75 105	11.2437	79,1	3.6636	25,8	7.9250	55,7	10.4023	73,1	2.3968	16,8	255 285
78 102	11.3860	61,7	3.7100	20,1	8.0253	43,5	10.5339	57,2	2.4271	13,2	258 282
81 99	11.4971	44,2	3.7462	14,4	8.1036	31,1	10.6368	40,8	2.4508	9,4	261 279
84 96	11.5766	26,6	3.7721	8,6	8.1596	18,7	10.7103	24,6	2.4677	5,7	264 276
87 93	11.6244	8,9	3.7876	2,9	8.1933	6,3	10.7545	8,3	2.4779	1,9	267 273
90 90	−11.6405+		−3.7929+		−8.2046+		+10.7694−		−2.4814+		270 270
$(\odot + n)$ Argument	α Persei		Aldebaran		Capella		Rigel		β Tauri		$(\odot + n)$ Argument

TABLE VI. Aberration in Declination.

$m.\sin(\odot + n)$

Argument $(\odot + n)$	11 α Orionis $m=-5.6889$ $n=88°\ 23'$	Diff. for 10′	12 Sirius $m=-13.1259$ $n=85°\ 51'$	Diff. for 10′	13 Castor $m=+4.6244$ $n=32°\ 18'$	Diff. for 10′	14 Procyon $m=+6.4580$ $n=276°\ 54'$	Diff. for 10′	15 Pollux $m=+4.0569$ $n=14°\ 32'$	Diff. for 10′	Argument $(\odot + n)$
° 0 180 °	″ −0.0000+	165,4	″ −0.0000+	381,7	″ +0.0000−	134,4	″ +0.0000−	187,8	+0.0000−	117,9	° 180 360 °
3 177	0.2977	164,9	0.6870	380,6	0.2420	134,1	0.3380	187,2	0.2123	117,7	183 357
6 174	0.5946	164,1	1.3720	378,5	0.4834	133,3	0.6750	186,3	0.4241	116,9	186 354
9 171	0.8899	162,7	2.0533	375,4	0.7234	132,3	1.0103	184,7	0.6346	116,1	189 351
12 168	1.1828	160,9	2.7290	371,3	0.9615	130,8	1.3427	182,7	0.8435	114,7	192 348
15 165	1.4724	158,7	3.3973	366,0	1.1969	128,9	1.6715	180,1	1.0500	113,1	195 345
18 162	1.7580	155,9	4.0561	359,9	1.4290	126,8	1.9956	177,1	1.2536	111,8	198 342
21 159	2.0387	152,9	4.7039	352,7	1.6573	124,2	2.3144	173,5	1.4539	109,0	201 339
24 156	2.3139	149,3	5.3388	344,6	1.8809	121,4	2.6267	169,6	1.6501	106,5	204 336
27 153	2.5827	145,4	5.9591	335,5	2.0995	118,2	2.9319	165,6	1.8418	103,7	207 333
30 150	2.8445	141,1	6.5630	325,5	2.3122	114,7	3.2290	160,2	2.0284	100,6	210 330
33 147	3.0984	136,4	7.1489	314,7	2.5187	110,8	3.5173	154,8	2.2095	97,3	213 327
36 144	3.3439	131,2	7.7153	302,8	2.7182	106,7	3.7959	149,1	2.3846	93,6	216 324
39 141	3.5801	125,8	8.2604	290,3	2.9102	102,3	4.0642	142,8	2.5531	89,7	219 321
42 138	3.8066	120,0	8.7829	276,9	3.0944	97,5	4.3213	136,2	2.7146	85,6	222 318
45 135	4.0226	113,9	9.2813	262,8	3.2699	92,6	4.5665	129,3	2.8686	81,2	225 315
48 132	4.2277	107,4	9.7544	247,9	3.4366	87,3	4.7992	122 0	3.0148	76,7	228 312
51 129	4.4211	100,8	10.2007	232,9	3.5938	81,9	5.0188	114,4	3.1528	71,8	231 309
54 126	4.6025	93,7	10.6192	216,2	3.7413	76,4	5.2247	106,3	3.2821	66,8	234 306
57 123	4.7711	86,4	11.0083	199,5	3.8784	70,3	5.4161	98,2	3.4024	61,6	237 303
60 120	4.9267	79,0	11.3674	182,2	4.0049	64,2	5.5928	89,6	3.5133	56,3	240 300
63 117	5.0689	71,2	11.6953	164,3	4.1204	57,9	5.7541	80,9	3.6147	50,8	243 297
66 114	5.1971	63,3	11.9911	146,1	4.2246	51,5	5.8997	71,9	3.7061	45,2	246 294
69 111	5.3110	55,3	12.2541	127,9	4.3173	44,9	6.0291	62,7	3.7874	39,4	249 291
72 108	5.4105	46,9	12.4836	108,3	4.3981	38,2	6.1420	53,3	3.8583	33,5	252 288
75 105	5.4950	38,6	12.6786	89,1	4.4668	31,4	6.2379	43,8	3.9186	27,6	255 285
78 102	5.5645	30,2	12.8390	69,6	4.5233	24,6	6.3168	34,3	3.9682	21,5	258 282
81 99	5.6189	21,6	12.9643	49,8	4.5675	17,6	6.3785	24,5	4.0069	15,4	261 279
84 96	5.6577	13,0	13.0539	29,9	4.5991	10,6	6.4226	14,7	4.0346	9,3	264 276
87 93	5.6811	4,3	13.1078	10,1	4.6181	3,5	6.4491	4,9	4.0513	3,1	267 273
90 90	−5.6889+		−13.1259+		+4.6244−		+6.4580−		+4.0569−		270 270
$(\odot + n)$ Argument	α Orionis		Sirius		Castor		Procyon		Pollux		$(\odot + n)$ Argument

TABLE VI. Aberration in Declination.

$m. \sin(\odot + n)$

Argument	16 α Hydræ		17 Regulus		18 α Ursæ Major.		19 β Leonis		20 β Virginis		Argument
(☉ + n)	m = −9.9926 n = 77° 31′	Diff. for 10′	m = +7.0578 n = 303° 47′	Diff. for 10′	m = +17.4743 n = 3° 28′	Diff. for 10′	m = +9.2270 n = 306° 20′	Diff. for 10′	m = +8.1397 n = 276° 51′	Diff. for 10′	(☉ + n)
0° 180°	−0″.0000+	290,6	+0″.0000−	205,2	+0″.0000−	508,1	+0″.0000−	268,3	+0″.0000−	236,7	180° 360°
3 177	0.5230	289,7	0.3694	204,5	0.9145	506,7	0.4829	267,6	0.4260	236,0	183 357
6 174	1.0445	288,2	0.7377	203,6	1.8265	503,9	0.9645	266,1	0.8508	234,7	186 354
9 171	1.5632	285,8	1.1041	201,8	2.7336	499,7	1.4434	263,9	1.2733	232,8	189 351
12 168	2.0776	282,6	1.4674	199,6	3.6331	494,2	1.9184	260,9	1.6923	230,2	192 348
15 165	2.5863	278,7	1.8267	196,8	4.5227	487,3	2.3881	257,3	2.1067	227,0	195 345
18 162	3.0879	273,9	2.1810	193,5	5.3998	479,1	2.8513	253,0	2.5153	223,2	198 342
21 159	3.5810	268,5	2.5293	189,6	6.2622	469,6	3.3067	247,9	2.9170	218,7	201 339
24 156	4.0643	262,4	2.8706	185,3	7.1074	458,8	3.7529	242,3	3.3107	213,7	204 336
27 153	4.5366	255,4	3.2042	180,4	7.9332	446,7	4.1890	235,8	3.6954	208,1	207 333
30 150	4.9963	247,8	3.5289	175,1	8.7372	433,3	4.6135	228,8	4.0699	201,8	210 330
33 147	5.4424	239,6	3.8440	169,2	9.5172	418,9	5.0254	221,2	4.4332	195,1	213 327
36 144	5.8736	230,5	4.1485	162,8	10.2712	403,2	5.4235	212,9	4.7844	187,8	216 324
39 141	6.2885	221,1	4.4416	156,1	10.9969	386,5	5.8067	204,1	5.1225	180,0	219 321
42 138	6.6864	210,8	4.7226	148,9	11.6926	368,6	6.1740	194,7	5.4465	171,7	222 318
45 135	7.0658	200,1	4.9906	141,3	12.3561	349,8	6.5244	184,7	5.7556	162,9	225 315
48 132	7.4259	188,8	5.2449	133,3	12.9858	330,1	6.8569	174,3	6.0489	153,8	228 312
51 129	7.7657	177,0	5.4849	125,0	13.5800	309,5	7.1707	163,4	6.3257	144,2	231 309
54 126	8.0843	164,6	5.7099	116,3	14.1371	287,8	7.4648	152,0	6.5852	134,1	234 306
57 123	8.3805	151,9	5.9192	107,2	14.6551	265,6	7.7384	140,2	6.8265	123,7	237 303
60 120	8.6539	138,7	6.1122	97,9	15.1332	242,5	7.9908	128,1	7.0492	112,9	240 300
63 117	8.9035	125,1	6.2885	88,4	15.5697	218,8	8.2213	115,6	7.2525	101,9	243 297
66 114	9.1287	111,2	6.4476	78,6	15.9636	194,4	8.4293	102,7	7.4360	90,6	246 294
69 111	9.3289	97,1	6.5890	68,6	16.3136	169,8	8.6141	89,7	7.5991	79,1	249 291
72 108	9.5036	82,5	6.7124	58,3	16.6192	144,2	8.7755	76,1	7.7414	67,2	252 288
75 105	9.6521	67,8	6.8173	47,9	16.8787	118,7	8.9125	62,7	7.8623	55,3	255 285
78 102	9.7742	53,0	6.9035	37,4	17.0923	92,7	9.0253	48,9	7.9618	43,2	258 282
81 99	9.8696	37,9	6.9709	26,8	17.2592	66,0	9.1134	35,0	8.0395	30,9	261 279
84 96	9.9378	22,8	7.0191	16,1	17.3780	40,1	9.1764	21,1	8.0951	18,6	264 276
87 93	9.9788	7,7	7.0481	5,4	17.4502	13,4	9.2143	7,1	8.1285	6,2	267 273
90 90	−9.9926+		+7.0578−		+17.4743−		+9.2270−		+8.1397−		270 270
(☉ + n) Argument	α Hydræ		Regulus		α Ursæ Major.		β Leonis		β Virginis		(☉ + n) Argument

TABLE VI. Aberration in Declination.

$m. \sin(\odot + n)$

Argument $(\odot + n)$	21 γ Ursæ Major. $m = +7.0891$ $n = 347° 28'$	Diff. for 10′	22 Spica Virginis $m = -7.7485$ $n = 63° 31'$	Diff. for 10′	23 η Ursæ Major. $m = +18.2297$ $n = 321° 23'$	Diff. for 10′	24 Arcturus $m = +12.5884$ $n = 298° 18'$	Diff. for 10′	25, 26 α² Libræ $m = -6.2193$ $n = 48° 24'$	Diff. for 10′	Argument $(\odot + n)$
0° 180°	+0″.0000−	496,9	−0″.0000+	225,3	+0″.0000−	530,1	+0″.0000−	366,0	−0″.0000+	180,8	180° 360°
3 177	0.8944	495,5	0.4055	224,7	0.9541	528,6	0.6588	365,0	0.3255	180,3	183 357
6 174	1.7863	492,8	0.8099	223,4	1.9055	525,7	1.3158	363,0	0.6501	179,3	186 354
9 171	2.6733	488,7	1.2121	221,6	2.8517	521,4	1.9692	360,1	0.9729	177,9	189 351
12 168	3.5530	483,3	1.6110	219,2	3.7902	515,6	2.6173	356,0	1.2931	175,9	192 348
15 165	4.4230	476,6	2.0055	216,1	4.7182	508,4	3.2581	351,1	1.6097	173,4	195 345
18 162	5.2808	468,6	2.3944	212,4	5.6333	499,8	3.8900	345,2	1.9219	170,5	198 342
21 159	6.1242	459,2	2.7768	208,2	6.5330	489,8	4.5113	338,2	2.2288	167,1	201 339
24 156	6.9507	448,7	3.1516	203,4	7.4146	478,7	5.1201	330,5	2.5296	163,3	204 336
27 153	7.7584	436,8	3.5178	198,1	8.2762	465,9	5.7150	321,8	2.8235	158,9	207 333
30 150	8.5446	423,8	3.8743	192,2	9.1149	452,1	6.2942	312,2	3.1096	154,3	210 330
33 147	9.3074	409,7	4.2202	185,7	9.9286	437,0	6.8561	301,8	3.3873	149,1	213 327
36 144	10.0448	394,3	4.5545	178,8	10.7152	420,6	7.3993	290,4	3.6556	143,5	216 324
39 141	10.7545	377,9	4.8763	171,4	11.4723	403,2	7.9221	278,4	3.9139	137,6	219 321
42 138	11.4348	360,5	5.1848	161,8	12.1980	384,6	8.4233	265,5	4.1615	131,2	222 318
45 135	12.0837	342,2	5.4790	155,1	12.8902	365,0	8.9012	252,1	4.3976	124,6	225 315
48 132	12.6996	322,8	5.7582	146,4	13.5472	344,4	9.3549	237,8	4.6218	117,5	228 312
51 129	13.2807	302,7	6.0217	137,2	14.1671	322,8	9.7830	222,9	4.8333	110,1	231 309
54 126	13.8255	281,4	6.2687	127,7	14.7482	300,3	10.1843	207,3	5.0315	102,4	234 306
57 123	14.3321	259,7	6.4985	117,7	15.2887	277,1	10.5575	191,3	5.2159	94,5	237 303
60 120	14.7996	237,2	6.7104	107,6	15.7874	253,0	10.9018	174,7	5.3860	86,3	240 300
63 117	15.2265	214,0	6.9040	97,0	16.2428	228,3	11.2163	157,7	5.5414	77,9	243 297
66 114	15.6117	190,2	7.0786	86,3	16.6537	202,8	11.5001	140,1	5.6816	69,2	246 294
69 111	15.9540	166,1	7.2339	75,3	17.0188	177,1	11.7522	122,3	5.8062	60,4	249 291
72 108	16.2529	141,0	7.3694	63,9	17.3376	150,4	11.9724	103,8	5.9149	51,3	252 288
75 105	16.5067	116,0	7.4845	52,6	17.6084	123,8	12.1593	85,5	6.0073	42,2	255 285
78 102	16.7155	90,7	7.5791	41,1	17.8312	96,7	12.3132	66,8	6.0833	33,0	258 282
81 99	16.8787	64,8	7.6531	29,4	18.0053	69,2	12.4334	47,7	6.1427	23,6	261 279
84 96	16.9953	39,0	7.7060	17,7	18.1297	41,6	12.5193	28,7	6.1851	14,2	264 276
87 93	17.0655	13,1	7.7378	5,9	18.2045	14,0	12.5710	9,7	6.2107	4,8	267 273
90 90	+17.0891−		−7.7485+		+18.2297−		+12.5884−		−6.2193+		270 270
$(\odot + n)$ Argument	γ Ursæ Major.		Spica Virginis		η Ursæ Major.		Arcturus		α² Libræ		$(\odot + n)$ Argument

TABLE VI. Aberration in Declination.

$m.\sin(\odot + n)$

Argument $(\odot + n)$	27 β Ursæ Minor. $m = +20''.4951$, $n = 315^\circ 5'$	Diff. for 10′	28 α Cor. Bor. $m = +15''.1887$, $n = 292^\circ 28'$	Diff. for 10′	29 α Serpentis $m = +10''.0529$, $n = 278^\circ 22'$	Diff. for 10′	30 Antares $m = -3''.8420$, $n = 357^\circ 59'$	Diff. for 10′	31 α Herculis $m = +12''.5652$, $n = 275^\circ 25'$	Diff. for 10′	Argument $(\odot + n)$
0° 180°	+0″.0000−	595,9	+0″.0000−	441,6	+0″.0000−	292,3	−0″.0000+	111,7	+0″.0000−	365,3	180° 360°
3 177	1.0726	594,3	0.7949	440,4	0.5261	291,5	0.2011	111,4	0.6576	364,3	183 357
6 174	2.1423	591,0	1.5876	438,0	1.0508	289,9	0.4016	110,8	1.3134	362,3	186 354
9 171	3.2061	586,2	2.3760	434,4	1.5726	287,5	0.6010	109,9	1.9656	359,4	189 351
12 168	4.2612	579,7	3.1579	429,6	2.0901	284,3	0.7988	108,7	2.6125	355,3	192 348
15 165	5.3046	571,5	3.9312	423,5	2.6019	280,3	0.9944	107,1	3.2521	350,4	195 345
18 162	6.3333	561,9	4.6935	416,4	3.1065	275,6	1.1872	105,4	3.8828	344,6	198 342
21 159	7.3448	550,7	5.4431	408,1	3.6026	270,1	1.3769	103,2	4.5030	337,6	201 339
24 156	8.3360	538,1	6.1777	398,8	4.0888	263,9	1.5627	100,9	5.1107	329,9	204 336
27 153	9.3046	523,3	6.8956	388,2	4.5639	256,9	1.7443	98,2	5.7045	321,2	207 333
30 150	10.2466	508,8	7.5943	376,7	5.0264	249,3	1.9210	95,3	6.2826	311,6	210 330
33 147	11.1625	491,3	8.2724	364,1	5.4752	240,9	2.0925	92,1	6.8435	301,2	213 327
36 144	12.0468	472,8	8.9277	350,4	5.9089	231,9	2.2583	88,7	7.3857	289,9	216 324
39 141	12.8979	453,3	9.5585	335,9	6.3264	222,4	2.4179	84,9	7.9075	277,9	219 321
42 138	13.7139	432,3	10.1632	320,4	6.7267	212,1	2.5708	81,1	8.4078	265,0	222 318
45 135	14.4921	410,3	10.7399	304,1	7.1084	201,3	2.7167	76,9	8.8848	251,6	225 315
48 132	15.2307	387,2	11.2873	286,9	7.4707	189,9	2.8552	72,6	9.3377	237,3	228 312
51 129	15.9276	363,0	11.8037	269,1	7.8125	178,1	2.9858	68,1	9.7649	222,6	231 309
54 126	16.5810	337,6	12.2880	250,2	8.1330	165,6	3.1083	63,3	10.1655	206,9	234 306
57 123	17.1886	311,4	12.7383	230,8	8.4310	152,8	3.2222	58,4	10.5380	191,0	237 303
60 120	17.7492	284,4	13.1538	210,8	8.7060	139,6	3.3273	53,3	10.8818	174,4	240 300
63 117	18.2612	256,7	13.5332	190,2	8.9572	125,8	3.4233	48,1	11.1957	157,3	243 297
66 114	18.7232	228,1	13.8756	169,0	9.1837	111,9	3.5099	42,7	11.4789	139,8	246 294
69 111	19.1337	199,2	14.1798	147,6	9.3851	97,7	3.5868	37,3	11.7306	122,1	249 291
72 108	19.4922	169,1	14.4454	125,3	9.5609	82,9	3.6540	31,7	11.9503	103,7	252 288
75 105	19.7966	139,1	14.6710	103,1	9.7102	68,3	3.7111	26,1	12.1370	85,3	255 285
78 102	20.0470	108,8	14.8566	80,6	9.8331	53,3	3.7580	20,4	12.2905	66,7	258 282
81 99	20.2428	77,7	15.0017	57,6	9.9291	38,1	3.7947	14,6	12.4105	47,7	261 279
84 96	20.3826	46,8	15.1053	34,7	9.9977	22,9	3.8209	8,8	12.4963	28,7	264 276
87 93	20.4668	15,7	15.1677	11,7	10.0390	7,7	3.8367	2,9	12.5479	9,6	267 273
90 90	+20.4951−		+15.1887−		+10.0529−		−3.8420+		+12.5652−		270 270
$(\odot + n)$ Argument	β Ursæ Minor.		α Cor. Bor.		α Serpentis		Antares		α Herculis		$(\odot + n)$ Argument

TABLE VI. Aberration in Declination.

$m. \sin(\odot + n)$

Argument ($\odot + n$)	32 α Ophiuchi $m = +12.0679$ $n = 273° 4'$	Diff. for 10′	33 γ Draconis $m = +19.7984$ $n = 271° 29'$	Diff. for 10′	34 α Lyræ $m = -18.0912$ $n = 84° 30'$	Diff. for 10′	35 γ Aquilæ $m = -11.1658$ $n = 82° 14'$	Diff. for 10′	36 α Aquilæ $m = -10.6378$ $n = 82° 59'$	Diff. for 10′	Argument ($\odot + n$)
0° 180°	+0.0000″−	350,9	+0.0000″−	575,7	−0.0000″+	526,0	−0.0000″+	324,7	−0.0000″+	309,3	180° 360°
3 177	0.6316	349,9	1.0362	574,1	0.9468	524,6	0.5844	323,7	0.5567	308,4	183 357
6 174	1.2614	348,0	2.0695	570,9	1.8910	521,7	1.1671	322,0	1.1119	306,8	186 354
9 171	1.8878	345,2	3.0971	566,2	2.8301	517,4	1.7467	319,3	1.6641	304,2	189 351
12 168	2.5091	341,3	4.1163	559,9	3.7614	511,7	2.3215	315,8	2.2117	300,9	192 348
15 165	3.1234	336,6	5.1242	552,1	4.6824	504,5	2.8899	311,4	2.7533	296,6	195 345
18 162	3.7292	330,8	6.1180	542,8	5.5905	496,1	3.4504	306,2	3.2872	291,7	198 342
21 159	4.3247	324,3	7.0951	532,0	6.4834	486,1	4.0015	300,0	3.8122	285,8	201 339
24 156	4.9084	316,8	8.0527	519,8	7.3583	475,0	4.5415	293,2	4.3267	279,3	204 336
27 153	5.4787	308,4	8.9884	506,0	8.2133	462,4	5.0692	285,4	4.8295	271,9	207 333
30 150	6.0339	299,3	9.8992	491,0	9.0457	448,7	5.5829	276,9	5.3189	263,8	210 330
33 147	6.5726	289,3	10.7830	474,6	9.8533	433,6	6.0814	267,6	5.7938	254,9	213 327
36 144	7.0933	278,4	11.6372	456,8	10.6338	417,4	6.5631	257,6	6.2527	245,4	216 324
39 141	7.5945	266,9	12.4595	437,9	11.3852	400,1	7.0268	247,0	6.6945	235,3	219 321
42 138	8.0750	254,6	13.2477	417,6	12.1054	381,6	7.4714	235,5	7.1180	224,4	222 318
45 135	8.5332	241,6	13.9994	396,4	12.7923	362,3	7.8953	223,6	7.5219	213,0	225 315
48 132	8.9681	227,9	14.7130	374,0	13.4444	341,7	8.2977	210,9	7.9053	200,9	228 312
51 129	9.3784	213,8	15.3862	350,6	14.0595	320,4	8.6774	197,8	8.2670	188,4	231 309
54 126	9.7632	198,7	16.0173	326,1	14.6363	297,9	9.0334	183,9	8.6062	175,2	234 306
57 123	10.1209	183,4	16.6043	300,9	15.1726	274,9	9.3644	169,7	8.9215	161,7	237 303
60 120	10.4511	167,4	17.1459	274,8	15.6675	251,1	9.6699	154,9	9.2126	147,6	240 300
63 117	10.7525	151,1	17.6405	247,9	16.1194	226,6	9.9488	139,8	9.4783	133,2	243 297
66 114	11.0245	134,3	18.0867	220,3	16.5272	201,3	10.2005	124,2	9.7181	118,4	246 294
69 111	11.2663	117,2	18.4833	192,4	16.8896	175,8	10.4241	108,5	9.9312	103,3	249 291
72 108	11.4773	99,6	18.8296	163,3	17.2060	149,3	10.6194	92,1	10.1172	87,8	252 288
75 105	11.6565	81,9	19.1236	134,4	17.4747	122,8	10.7852	75,8	10.2752	72,2	255 285
78 102	11.8040	64,1	19.3656	105,1	17.6958	96,0	10.9217	59,2	10.4052	56,4	258 282
81 99	11.9193	45,6	19.5547	75,0	17.8686	68,6	11.0283	42,3	10.5068	40,3	261 279
84 96	12.0013	27,6	19.6897	45,2	17.9920	41,3	11.1045	25,5	10.5794	24,3	264 276
87 93	12.0512	9,3	19.7711	15,2	18.0663	13,9	11.1504	8,6	10.6231	8,2	267 273
90 90	+12.0679−		+19.7984−		−18.0913+		−11.1658+		−10.6378+		270 270
($\odot + n$) Argument	α Ophiuchi		γ Draconis		α Lyræ		γ Aquilæ		α Aquilæ		$\odot$ Argument

TABLE VI. Aberration in Declination.

$m. \sin(\odot + n)$

Argument $(\odot + n)$	37 β Aquilæ $m = -9.9163$ $n = 84° 26'$	Diff. for 10′	38, 39 α² Capricorni $m = +5.0018$ $n = 299° 32'$	Diff. for 10′	40 α Cygni $m = -18.4639$ $n = 60° 38'$	Diff. for 10′	41 α Cephei $m = -20.0835$ $n = 47° 30'$	Diff. for 10′	42 β Cephei $m = -20.4268$ $n = 42° 25'$	Diff. for 10′	Argument $(\odot + n)$
0° 180°	−0″.0000+	288,3	+0″.0000−	145,4	−0″.0000+	536,8	−0″.0000+	583,9	−0″.0000+	593,9	180° 360°
3 177	0.5190	287,5	0.2618	145,0	0.9663	535,4	1.0511	582,3	1.0691	592,3	183 357
6 174	1.0365	285,9	0.5228	144,3	1.9300	532,4	2.0993	579,1	2.1352	589,0	186 354
9 171	1.5512	283,6	0.7825	143,0	2.8884	528,0	3.1417	574,4	3.1954	584,2	189 351
12 168	2.0617	280,5	1.0399	141,5	3.8388	522,2	4.1756	568,0	4.2470	577,7	192 348
15 165	2.5666	276,5	1.2946	139,3	4.7788	514,9	5.1980	560,1	5.2869	569,6	195 345
18 162	3.0643	271,9	1.5456	137,2	5.7056	506,2	6.2061	550,7	6.3122	560,1	198 342
21 159	3.5537	266,4	1.7925	134,4	6.6168	496,1	7.1973	539,6	7.3203	548,9	201 339
24 156	4.0333	260,3	2.0344	131,3	7.5098	484,8	8.1686	527,3	8.3083	536,3	204 336
27 153	4.5019	253,5	2.2708	127,8	8.3824	471,9	9.1178	513,3	9.2736	522,1	207 333
30 150	4.9582	245,9	2.5009	124,1	9.2319	457,9	10.0418	498,1	10.2134	506,6	210 330
33 147	5.4008	237,7	2.7242	119,9	10.0561	442,6	10.9383	481,4	11.1253	489,6	213 327
36 144	5.8287	228,8	2.9400	115,4	10.8528	426,0	11.8048	463,4	12.0066	471,3	216 324
39 141	6.2405	219,3	3.1477	110,7	11.6196	408,3	12.6389	444,2	12.8549	451,8	219 321
42 138	6.6353	209,2	3.3469	105,5	12.3546	389,5	13.4385	423,6	13.6682	430,9	222 318
45 135	7.0118	198,6	3.5368	100,2	13.0557	369,1	14.2010	402,2	14.4438	408,9	225 315
48 132	7.3692	187,3	3.7171	94,4	13.7211	348,8	14.9249	379,3	15.1799	385,9	228 312
51 129	7.7064	175,6	3.8871	88,6	14.3489	327,1	15.6077	355,7	15.8745	356,2	231 309
54 126	8.0225	163,3	4.0466	82,4	14.9376	304,1	16.2480	330,8	16.5257	336,4	234 306
57 123	8.3165	150,7	4.1949	75,0	15.4850	380,6	16.8434	305,3	17.1313	310,4	237 303
60 120	8.5878	137,6	4.3317	69,4	15.9901	256,2	17.3929	278,7	17.6901	283,5	240 300
63 117	8.8355	124,2	4.4567	62,6	16.4513	231,2	17.8945	251,5	18.2004	255,8	243 297
66 114	9.0590	110,3	4.5694	55,7	16.8675	205,4	18.3472	223,5	18.6608	227,3	246 294
69 111	9.2576	96,4	4.6696	48,6	17.2373	179,4	18.7495	195,1	19.0700	198,4	249 291
72 108	9.4311	81,8	4.7571	41,3	17.5602	152,4	19.1007	165,7	19.4272	168,6	252 288
75 105	9.5783	67,3	4.8314	33,9	17.8345	125,3	19.3990	136,4	19.7306	138,7	255 285
78 102	9.6995	52,6	4.8925	26,6	18.0601	97,9	19.6445	101,0	19.9802	108,4	258 282
81 99	9.7942	37,6	4.9403	18,9	18.2364	70,0	19.8363	76,1	20.1753	77,4	261 279
84 96	9.8619	22,6	4.9744	11,4	18.3624	42,2	19.9733	45,8	20.3147	46,6	264 276
87 93	9.9026	7,6	4.9949	3,8	18.4383	14,2	20.0558	15,4	20.3986	15,7	267 273
90 90	−9.9163+		+5.0018−		−18.4638+		−20.0835+		−20.4268+		270 270
$(\odot + n)$ Argument	β Aquilæ		α² Capricorni		α Cygni		α Cephei		β Cephei		$(\odot + n)$ Argument

TABLE VI. Aberration in Declination.

$m. \sin(\odot + n)$

Argument ($\odot + n$)	43 α Aquarii $m = +7.9759$ $n = 272° 31'$	Diff. for 10′	44 Fomalhaut $m = +10.7157$ $n = 337° 34'$	Diff. for 10′	45 α Pegasi $m = -10.3927$ $n = 62° 5'$	Diff. for 10′	46 α Andromedæ $m = -12.0642$ $n = 36° 42'$	Diff. for 10′	Argument ($\odot + n$)
0° 180°	+0″.0000−	231,9	+0″.0000−	311,6	−0″.0000+	302,2	−0″.0000+	350,8	180° 360°
3 177	0.4174	231,3	0.5608	310,7	0.5439	301,3	0.6314	349,8	183 357
6 174	0.8337	230,0	1.1201	309,0	1.0863	299,7	1.2610	347,9	186 354
9 171	1.2477	228,1	1.6763	306,4	1.6258	297,2	1.8873	345,0	189 351
12 168	1.6583	225,6	2.2279	303,1	2.1608	293,9	2.5083	341,2	192 348
15 165	2.0643	222,4	2.7734	298,8	2.6899	289,8	3.1225	336,4	195 345
18 162	2.4647	218,7	3.3113	293,8	3.2115	284,9	3.7280	330,8	198 342
21 159	2.8583	214,3	3.8402	287,9	3.7244	279,3	4.3234	324,2	201 339
24 156	3.2441	209,4	4.3584	281,4	4.2271	272,8	4.9069	316,8	204 336
27 153	3.6210	203,9	4.8649	273,8	4.7182	265,7	5.4771	308,3	207 333
30 150	3.9880	197,8	5.3578	265,8	5.1964	257,7	6.0321	299,2	210 330
33 147	4.3440	191,2	5.8362	256,8	5.6603	249,1	6.5707	289,2	213 327
36 144	4.6881	184,1	6.2985	246,7	6.1087	239,8	7.0912	278,3	216 324
39 141	5.0194	181,9	6.7436	237,0	6.5403	229,9	7.5922	266,8	219 321
42 138	5.3369	168,3	7.1702	226,0	6.9541	219,2	8.0725	254,5	222 318
45 135	5.6398	159,7	7.5770	214,6	7.3487	208,1	8.5306	241,6	225 315
48 132	5.9272	150,7	7.9632	202,4	7.7232	196,3	8.9654	227,9	228 312
51 129	6.1984	141,3	8.3276	189,8	8.0766	184,1	9.3756	213,7	231 309
54 126	6.4527	131,3	8.6692	176,5	8.4079	171,2	9.7602	198,7	234 306
57 123	6.6891	121,2	8.9869	162,8	8.7161	157,9	10.1179	183,3	237 303
60 120	6.9073	110,7	9.2800	148,7	9.0004	144,2	10.4479	167,4	240 300
63 117	7.1066	99,9	9.5477	134,2	9.2600	130,1	10.7493	151,1	243 297
66 114	7.2864	88,7	9.7893	119,2	9.4942	115,7	11.0212	134,3	246 294
69 111	7.4461	77,5	10.0039	104,1	9.7024	101,0	11.2629	117,2	249 291
72 108	7.5856	65,8	10.1913	88,4	9.8842	85,7	11.4739	99,6	252 288
75 105	7.7041	54,1	10.3505	72,7	10.0385	70,6	11.6531	82,4	255 285
78 102	7.8015	42,3	10.4814	56,9	10.1655	55,2	11.8005	64,0	258 282
81 99	7.8777	30,2	10.5838	40,6	10.2648	39,4	11.9157	45,7	261 279
84 96	7.9321	18,2	10.6569	24,4	10.3357	23,7	11.9980	27,6	264 276
87 93	7.9649	6,1	10.7009	8,2	10.3784	7,9	12.0476	9,2	267 273
90 90	+7.9759−		+10.7157−		−10.3927+		−12.0642+		270 270
($\odot + n$) Argument	α Aquarii		Fomalhaut		α Pegasi		α Andromedæ		($\odot + n$) Argument

TABLE VII. Lunar Nutation in Declination.

$m. \sin(☊ + n)$

Argument (☊ + n)	1 γ Pegasi $m=-6.6834$ $n=358°\ 29'$	Diff. for 10′	2 α Cassiop. $m=-6.7309$ $n=349°\ 40'$	Diff. for 10′	3 Polaris $m=-6.8574$ $n=340°\ 42'$	Diff. for 10′	4 α Arietis $m=-7.3018$ $n=322°\ 52'$	Diff. for 10′	5 α Ceti $m=-7.8478$ $n=308°\ 15'$	Diff. for 10′	Argument (☊ + n)
° 0 ° 180	″ −0.0000+	194,3	″ −0.0000+	195,7	″ −0.0000+	199,4	″ −0.0000+	212,3	″ −0.0000+	228,2	° 180 ° 360
3 177	0.3498	193,8	0.3523	195,2	0.3589	198,8	0.3821	211,7	0.4107	227,6	183 357
6 174	0.6986	192,7	0.7036	194,1	0.7168	197,7	0.7632	210,6	0.8203	226,3	186 354
9 171	1.0455	191,2	1.0529	192,5	1.0727	196,1	1.1422	208,8	1.2277	224,4	189 351
12 168	1.3896	189,0	1.3994	190,4	1.4257	193,9	1.5181	206,6	1.6317	221,9	192 348
15 165	1.7298	186,4	1.7421	187,7	1.7748	191,2	1.8899	203,6	2.0312	218,8	195 345
18 162	2.0653	183,2	2.0799	184,6	2.1190	188,1	2.2564	200,2	2.4251	215,2	198 342
21 159	2.3951	179,6	2.4121	180,9	2.4575	184,2	2.6167	196,2	2.8124	210,9	201 339
24 156	2.7184	175,4	2.7377	176,7	2.7891	180,1	2.9699	191,7	3.1920	206,1	204 336
27 153	3.0342	170,8	3.0558	172,0	3.1132	175,3	3.3150	186,6	3.5629	200,6	207 333
30 150	3.3417	165,8	3.3654	166,9	3.4287	170,1	3.6509	181,1	3.9239	194,7	210 330
33 147	3.6401	160,2	3.6659	161,3	3.7348	164,4	3.9769	175,0	4.2743	188,1	213 327
36 144	3.9284	154,2	3.9563	155,3	4.0307	158,2	4.2919	168,5	4.6129	181,1	216 324
39 141	4.2060	147,8	4.2358	148,9	4.3155	151,7	4.5952	161,4	4.9388	173,6	219 321
42 138	4.4721	141,0	4.5038	142,0	4.5885	144,7	4.8858	154,1	5.2512	165,6	222 318
45 135	4.7259	133,8	4.7594	134,8	4.8489	137,3	5.1631	146,2	5.5492	157,1	225 315
48 132	4.9667	126,3	5.0020	127,1	5.0960	129,6	5.4263	137,9	5.8320	148,3	228 312
51 129	5.1940	118,3	5.2308	119,2	5.3292	121,4	5.6745	129,3	6.0989	139,0	231 309
54 126	5.4070	110,1	5.4454	110,8	5.5478	112,9	5.9073	120,3	6.3491	129,2	234 306
57 123	5.6052	101,6	5.6449	102,3	5.7511	104,2	6.1238	110,9	6.5817	119,3	237 303
60 120	5.7880	92,8	5.8291	93,4	5.9387	95,2	6.3235	101,3	6.7964	108,9	240 300
63 117	5.9550	83,7	5.9972	84,3	6.1100	85,9	6.5059	91,4	6.9925	98,3	243 297
66 114	6.1056	74,4	6.1489	74,9	6.2646	76,3	6.6705	81,3	7.1694	87,3	246 294
69 111	6.2395	64,9	6.2838	65,4	6.4019	66,6	6.8168	70,9	7.3266	76,2	249 291
72 108	6.3564	55,2	6.4015	55,5	6.5218	56,6	6.9445	60,2	7.4638	64,8	252 288
75 105	6.4557	45,3	6.5014	45,7	6.6237	46,6	7.0529	49,6	7.5804	53,3	255 285
78 102	6.5373	35,5	6.5837	35,7	6.7075	36,4	7.1422	38,7	7.6763	41,6	258 282
81 99	6.6012	25,3	6.6480	25,5	6.7730	26,0	7.2119	27,7	7.7512	29,8	261 279
84 96	6.6468	15,2	6.6939	15,4	6.8198	15,6	7.2617	16,7	7.8048	17,9	264 276
87 93	6.6742	5,1	6.7216	5,2	6.8479	5,3	7.2917	5,6	7.8370	6,0	267 273
90 90	−6.6834+		−6.7309+		−6.8574+		−7.3018+		−7.8478+		270 270
(☊ + n) Argument	γ Pegasi		α Cassiop.		Polaris		α Arietis		α Ceti		(☊ + n) Argument

TABLE VII. Lunar Nutation in Declination.

$m. \sin(☊ + n)$

Argument (☊ + n)	6 α Persei $m = -8.0334$ $n = 303° 47'$	Diff. for 10′	7 Aldebaran $m = -8.6541$ $n = 287° 54'$	Diff. for 10′	8 Capella $m = -8.8597$ $n = 280° 29'$	Diff. for 10′	9 Rigel $m = 8.8689$ $n = 280° 4'$	Diff. for 10′	10 β Tauri $m = -8.9025$ $n = 278° 19'$	Diff. for 10′	Argument (☊ + n)
0° 180°	−0″.0000+	233,6	−0″.0000+	251,6	−0″.0000+	257,6	−0″.0000+	257,9	−0″.0000+	258,8	180° 360°
3 177	0.4204	232,9	0.4529	250,9	0.4637	256,9	0.4642	257,1	0.4659	258,2	183 357
6 174	0.8397	231,7	0.9046	249,6	0.9261	255,5	0.9270	255,8	0.9306	256,7	186 354
9 171	1.2567	229,8	1.3538	247,5	1.3860	253,3	1.3874	253,7	1.3926	254,6	189 351
12 168	1.6703	227,2	1.7993	244,8	1.8420	250,6	1.8440	250,8	1.8509	251,8	192 348
15 165	2.0792	224,0	2.2399	241,3	2.2931	247,1	2.2955	247,3	2.3041	248,3	195 345
18 162	2.4824	220,3	2.6742	237,3	2.7378	242,9	2.7406	243,2	2.7510	244,1	198 342
21 159	2.8789	215,9	3.1013	232,6	3.1750	238,1	3.1783	238,3	3.1904	239,2	201 339
24 156	3.2675	210,9	3.5199	227,2	3.6035	232,7	3.6073	232,8	3.6209	233,8	204 336
27 153	3.6471	205,3	3.9289	221,2	4.0223	226,4	4.0264	226,7	4.0417	227,5	207 333
30 150	4.0167	199,2	4.3270	214,7	4.4299	219,7	4.4345	219,9	4.4512	220,8	210 330
33 147	4.3753	192,6	4.7134	207,4	4.8254	212,3	4.8304	212,6	4.8486	213,4	213 327
36 144	4.7219	185,4	5.0867	199,7	5.2076	204,4	5.2130	204,7	5.2328	205,4	216 324
39 141	5.0556	177,7	5.4462	191,4	5.5756	195,9	5.5814	196,1	5.6025	196,9	219 321
42 138	5.3754	169,4	5.7907	182,6	5.9283	186,9	5.9344	187,1	5.9569	187,8	222 318
45 135	5.6804	160,3	6.1193	173,3	6.2647	177,4	6.2712	177,0	6.2949	178,3	225 315
48 132	5.9699	151,8	6.4312	163,4	6.5840	167,4	6.5908	167,6	6.6158	168,2	228 312
51 129	6.2431	142,3	6.7254	153,3	6.8853	156,9	6.8924	157,1	6.9185	157,7	231 309
54 126	6.4992	132,3	7.0013	142,6	7.1677	145,9	7.1751	146,1	7.2023	146,6	234 306
57 123	6.7374	122,1	7.2579	131,5	7.4304	134,6	7.4381	134,8	7.4662	135,3	237 303
60 120	6.9571	111,5	7.4946	120,1	7.6727	123,0	7.6807	123,1	7.7097	123,6	240 300
63 117	7.1578	100,6	7.7108	108,4	7.8941	110,9	7.9022	111,1	7.9321	111,5	243 297
66 114	7.3389	89,4	7.9059	96,3	8.0938	98,6	8.1021	98,7	8.1328	99,1	246 294
69 111	7.4998	78,1	8.0792	84,1	8.2712	86,1	8.2798	86,2	8.3111	86,5	249 291
72 108	7.6403	66,3	8.2306	71,4	8.4262	73,1	8.4349	73,2	8.4668	73,4	252 288
75 105	7.7596	54,6	8.3591	58,8	8.5578	60,1	8.5666	60,2	8.5990	60,4	255 285
78 102	7.8578	42,6	8.4649	45,9	8.6660	47,0	8.6750	47,1	8.7078	47,3	258 282
81 99	7.9345	30,4	8.5475	32,8	8.7506	33,6	8.7597	33,6	8.7929	33,7	261 279
84 96	7.9893	18,3	8.6066	19,7	8.8111	20,2	8.8202	20,3	8.8536	20,3	264 276
87 93	8.0223	6,2	8.6421	6,7	8.8475	6,8	8.8567	6,8	8.8902	6,8	267 273
90 90	−8.0334+		−8.6541+		−8.8597+		−8.8689+		−8.9025+		270 270
(☊ + n) Argument	α Persei		Aldebaran		Capella		Rigel		β Tauri		(☊ + n) Argument

TABLE VII. Lunar Nutation in Declination.

$m. \sin(\Omega + n)$

Argument ($\Omega + n$)	11 α Orionis $m=-8.9695$ $n=272° 37'$	Diff. for 10′	12 Sirius $m=+8.9234$ $n=82° 58'$	Diff. for 10′	13 Castor $m=+8.7179$ $n=74° 6'$	Diff. for 10′	14 Procyon $m=+8.6764$ $n=72° 47'$	Diff. for 10′	15 Pollux $m=+8.6469$ $n=71° 53'$	Diff. for 10′	Argument ($\Omega + n$)
0° 180°	−0″.0000+	260,8	+0″.0000−	259,4	+0″.0000−	253,5	+0″.0000−	252,3	+0″.0000−	251,4	180° 360°
3 177	0.4694	260,1	0.4670	258,7	0.4563	252,8	0.4541	251,6	0.4525	250,7	183 357
6 174	0.9376	258,6	0.9327	257,3	0.9113	251,4	0.9069	250,2	0.9038	249,4	186 354
9 171	1.4031	256,6	1.3959	255,2	1.3638	249,3	1.3573	248,1	1.3527	247,3	189 351
12 168	1.8649	253,7	1.8553	252,4	1.8126	246,6	1.8039	245,4	1.7978	244,6	192 348
15 165	2.3215	250,1	2.3096	248,8	2.2564	243,1	2.2456	241,9	2.2380	241,1	195 345
18 162	2.7717	245,9	2.7575	244,7	2.6940	239,0	2.6811	237,9	2.6720	237,1	198 342
21 159	3.2144	241,0	3.1979	239,7	3.1242	234,2	3.1094	233,1	3.0988	232,3	201 339
24 156	3.6482	235,5	3.6294	234,3	3.5458	228,9	3.5290	227,8	3.5170	227,0	204 336
27 153	4.0721	229,3	4.0512	228,1	3.9579	222,8	3.9390	221,8	3.9256	221,0	207 333
30 150	4.4848	222,4	4.4617	221,3	4.3589	216,2	4.3382	215,2	4.3234	214,4	210 330
33 147	4.8852	215,0	4.8600	213,9	4.7481	208,9	4.7255	208,0	4.7094	207,3	213 327
36 144	5.2722	206,9	5.2451	205,8	5.1242	201,2	5.0999	200,2	5.0825	199,5	216 324
39 141	5.6447	198,4	5.6156	197,4	5.4863	192,8	5.4602	191,9	5.4416	191,3	219 321
42 138	6.0018	189,2	5.9709	188,2	5.8334	183,9	5.8056	183,1	5.7859	182,4	222 318
45 135	6.3423	179,6	6.3097	178,7	6.1644	174,6	6.1351	173,7	6.1142	173,1	225 315
48 132	6.6656	169,4	6.6313	168,6	6.4786	164,7	6.4478	163,9	6.4258	163,4	228 312
51 129	6.9706	158,8	6.9347	158,1	6.7750	154,4	6.7428	153,7	6.7199	153,1	231 309
54 126	7.2565	147,8	7.2192	147,0	7.0529	143,6	7.0194	142,9	6.9955	142,4	234 306
57 123	7.5225	136,3	7.4838	135,6	7.3114	132,5	7.2766	131,9	7.2519	131,4	237 303
60 120	7.7678	124,5	7.7279	123,8	7.5499	121,0	7.5140	120,4	7.4884	120,0	240 300
63 117	7.9919	112,3	7.9508	111,7	7.7677	109,2	7.7307	108,7	7.7044	108,3	243 297
66 114	8.1941	99,8	8.1519	99,3	7.9642	97,0	7.9263	96,6	7.8993	96,2	246 294
69 111	8.3738	87,1	8.3307	86,8	8.1388	84,7	8.1001	84,3	8.0725	84,1	249 291
72 108	8.5306	74,0	8.4867	73,7	8.2913	71,9	8.2518	71,6	8.2238	71,3	252 288
75 105	8.6638	60,9	8.6193	60,6	8.4207	59,2	8.3807	58,9	8.3522	58,7	255 285
78 102	8.7734	47,6	8.7283	47,3	8.5273	46,2	8.4867	46,1	8.4579	45,8	258 282
81 99	8.8591	34,0	8.8135	33,8	8.6105	33,1	8.5696	32,9	8.5404	32,8	261 279
84 96	8.9203	20,5	8.8744	20,4	8.6700	19,9	8.6288	19,8	8.5994	19,8	264 276
87 93	8.9572	6,8	8.9111	6,8	8.7058	6,7	8.6644	6,7	8.6350	6,6	267 273
90 90	−8.9695+		+8.9234−		+8.7179−		+8.6764−		+8.6469−		270 270
($\Omega + n$) Argument	α Orionis		Sirius		Castor		Procyon		Pollux		($\Omega + n$) Argument

TABLE VII. Lunar Nutation in Declination.

$m. \sin(\Omega + n)$

Argument ($\Omega + n$)	16 α Hydræ $m = +7.7215$ $n = 48° 37'$	Diff. for 10′	17 Regulus $m = +7.3303$ $n = 37° 59'$	Diff. for 10′	18 α Ursæ Major. $m = +6.9011$ $n = 21° 58'$	Diff. for 10′	19 β Leonis $m = +6.7021$ $n = 6° 34'$	Diff. for 10′	20 β Virginis $m = +6.6993$ $n = 6° 5'$	Diff. for 10′	Argument ($\Omega + n$)
0° 180°	+0″.0000−	224,5	+0″.0000−	213,1	+0″.0000−	200,7	+0″.0000−	194,9	+0″.0000−	194,8	180° 360°
3 177	0.4041	223,9	0.3836	212,6	0.3612	200,1	0.3508	194,3	0.3506	194,3	183 357
6 174	0.8071	222,7	0.7662	211,4	0.7214	199,0	0.7006	193,3	0.7003	193,2	186 354
9 171	1.2079	220,8	1.1467	209,7	1.0796	197,3	1.0485	191,6	1.0480	191,6	189 351
12 168	1.6054	218,4	1.5241	207,3	1.4348	195,2	1.3934	189,6	1.3929	189,4	192 348
15 165	1.9985	215,3	1.8972	204,4	1.7862	192,4	1.7346	186,9	1.7339	186,8	195 345
18 162	2.3861	211,7	2.2652	200,9	2.1326	189,2	2.0710	183,8	2.0702	183,7	198 342
21 159	2.7671	207,5	2.6269	197,0	2.4731	185,4	2.4018	180,1	2.4008	180,0	201 339
24 156	3.1406	202,7	2.9815	192,4	2.8069	181,2	2.7260	175,9	2.7248	175,9	204 336
27 153	3.5055	197,3	3.3279	187,3	3.1331	176,4	3.0427	171,3	3.0414	171,3	207 333
30 150	3.8607	191,5	3.6651	181,8	3.4506	171,1	3.3510	166,2	3.3497	165,6	210 330
33 147	4.2054	185,1	3.9924	175,7	3.7586	165,4	3.6502	160,7	3.6487	160,6	213 327
36 144	4.5386	178,2	4.3086	169,2	4.0564	159,2	3.9394	154,6	3.9378	154,6	216 324
39 141	4.8593	170,8	4.6131	162,1	4.3430	152,6	4.2177	148,3	4.2160	148,2	219 321
42 138	5.1667	162,8	4.9049	154,6	4.6177	145,6	4.4846	141,3	4.4827	141,3	222 318
45 135	5.4598	154,6	5.1832	146,8	4.8798	138,2	4.7390	134,2	4.7371	134,1	225 315
48 132	5.7381	145,9	5.4474	138,5	5.1285	130,4	4.9806	126,6	4.9785	126,6	228 312
51 129	6.0007	136,7	5.6967	129,8	5.3632	122,2	5.2085	118,7	5.2063	118,7	231 309
54 126	6.2468	127,2	5.9303	120,8	5.5832	113,7	5.4221	110,4	5.4199	110,3	234 306
57 123	6.4757	117,4	6.1477	111,4	5.7878	104,8	5.6208	101,9	5.6185	101,8	237 303
60 120	6.6870	107,2	6.3482	101,7	5.9765	95,8	5.8042	93,0	5.8018	92,9	240 300
63 117	6.8799	96,7	6.5313	91,8	6.1489	86,4	5.9716	83,9	5.9691	83,9	243 297
66 114	7.0539	85,9	6.6965	81,6	6.3045	76,8	6.1227	74,6	6.1201	74,6	246 294
69 111	7.2086	75,0	6.8434	71,2	6.4427	67,1	6.2569	65,1	6.2543	65,1	249 291
72 108	7.3436	63,7	6.9716	60,4	6.5634	56,9	6.3741	55,3	6.3715	55,3	252 288
75 105	7.4583	52,4	7.0804	49,8	6.6659	46,9	6.4737	45,5	6.4710	45,4	255 285
78 102	7.5527	40,9	7.1700	38,9	6.7503	36,6	6.5556	35,6	6.5528	35,6	258 282
81 99	7.6264	29,3	7.2400	27,8	6.8162	26,2	6.6196	25,4	6.6168	25,4	261 279
84 96	7.6791	17,6	7.2900	16,8	6.8633	15,7	6.6653	15,3	6.6625	15,3	264 276
87 93	7.7108	5,9	7.3202	5,6	6.8916	5,3	6.6928	5,2	6.6901	5,1	267 273
90 90	+7.7215−		+7.3303−		+6.9011−		+6.7021−		+6.6993−		270 270
($\Omega + n$) Argument	α Hydræ		Regulus		α Ursæ Major.		β Leonis		Virginis		($\Omega + n$) Argument

TABLE VII. Lunar Nutation in Declination.

$m. \sin(\Omega + n)$

Argument ($\Omega + n$)	21 γ Ursæ Major. $m = +6.6942$ $n = 5^\circ\ 5'$	Diff. for 10′	22 Spica Virginis $m = +6.9634$ $n = 335^\circ\ 6'$	Diff. for 10′	23 η Ursæ Major. $m = +7.1535$ $n = 327^\circ\ 41'$	Diff. for 10′	24 Arcturus $m = +7.3983$ $n = 320^\circ\ 1'$	Diff. for 10′	25, 26 α² Libræ $m = +7.7286$ $n = 311^\circ\ 12'$	Diff. for 10′	Argument ($\Omega + n$)
0° 180°	+0″.0000—	194,6	+0″.0000 —	202,4	+0″.0000—	208,0	+0″.0000—	215,1	+0″.0000—	224,7	180° 360°
3 177	0.3503	194,1	0.3644	201,9	0.3744	207,4	0.3872	214,5	0.4045	224,1	183 357
6 174	0.6997	193,1	0.7279	200,8	0.7477	206,3	0.7733	213,2	0.8078	222,9	186 354
9 171	1.0472	191,4	1.0893	199,2	1.1191	204,6	1.1571	211,7	1.2090	221,1	189 351
12 168	1.3918	189,3	1.4478	196,9	1.4873	202,3	1.5382	209,2	1.6069	218,6	192 348
15 165	1.7326	186,7	1.8023	194,2	1.8515	199,4	1.9148	206,3	2.0003	215,6	195 345
18 162	2.0686	183,6	2.1518	190,9	2.2105	196,2	2.2862	202,8	2.3883	211,9	198 342
21 159	2.3990	179,9	2.4955	187,1	2.5636	192,2	2.6513	198,8	2.7697	207,7	201 339
24 156	2.7228	175,7	2.8322	182,8	2.9096	187,8	3.0091	194,3	3.1435	202,9	204 336
27 153	3.0391	171,1	3.1613	178,0	3.2477	182,8	3.3588	189,1	3.5087	197,6	207 333
30 150	3.3471	166,0	3.4817	172,7	3.5768	177,4	3.6991	183,5	3.8643	191,7	210 330
33 147	3.6459	160,5	3.7925	166,9	3.8961	171,4	4.0294	177,3	4.2093	185,3	213 327
36 144	3.9348	154,4	4.0930	160,7	4.2047	165,1	4.3486	170,7	4.5428	178,3	216 324
39 141	4.2128	148,1	4.3822	154,0	4.5018	158,2	4.6559	163,6	4.8637	170,9	219 321
42 138	4.4793	141,2	4.6594	146,9	4.7866	150,9	4.9504	156,1	5.1714	163,1	222 318
45 135	4.7335	134,0	4.9238	139,4	5.0582	143,3	5.2313	148,1	5.4649	154,7	225 315
48 132	4.9747	126,5	5.1748	131,5	5.3161	135,1	5.4979	139,8	5.7434	146,0	228 312
51 129	5.2024	118,6	5.4115	123,3	5.5593	126,7	5.7495	131,1	6.0062	136,9	231 309
54 126	5.4158	110,2	5.6335	114,7	5.7874	117,8	5.9854	121,8	6.2526	127,3	234 306
57 123	5.6142	101,8	5.8400	105,8	5.9994	108,7	6.2047	112,4	6.4817	117,4	237 303
60 120	5.7974	92,8	6.0305	96,6	6.1951	99,3	6.4071	102,7	6.6931	107,3	240 300
63 117	5.9646	83,8	6.2044	87,2	6.3738	89,6	6.5919	92,7	6.8862	96,8	243 297
66 114	6.1155	74,5	6.3614	77,4	6.5351	79,6	6.7587	82,3	7.0604	86,0	246 294
69 111	6.2496	65,0	6.5008	67,7	6.6784	69,5	6.9068	71,9	7.2152	75,1	249 291
72 108	6.3666	55,3	6.6226	57,4	6.8035	59,0	7.0362	61,1	7.3504	63,8	252 288
75 105	6.4661	45,4	6.7260	47,3	6.9097	48,6	7.1461	50,2	7.4652	52,4	255 285
78 102	6.5479	35,5	6.8111	36,8	6.9971	38,0	7.2365	39,3	7.5596	41,0	258 282
81 99	6.6118	25,4	6.8773	26,6	7.0655	27,1	7.3072	28,1	7.6334	29,3	261 279
84 96	6.6575	15,3	6.9252	15,9	7.1143	16,3	7.3577	16,9	7.6862	17,6	264 276
87 93	6.6850	5,1	6.9538	5,3	7.1436	5,5	7.3881	5,7	7.7179	5,9	267 273
90 90	+6.6942—		+6.9634—		+7.1535—		+7.3983—		+7.7286—		270 270
($\Omega + n$) Argument	γ Ursæ Major.		Spica Virginis		η Ursæ Major.		Arcturus		α² Libræ		($\Omega + n$) Argument

TABLE VII. Lunar Nutation in Declination.

$m. \sin(\Omega + n)$

Argument ($\Omega + n$)	27 β Ursæ Minor. $m=+7.8267$ $n=308°\ 46'$	Diff. for 10′	28 α Cor. Bor. $m=+8.1786$ $n=300°\ 18'$	Diff. for 10′	29 α Serpentis $m=+8.2558$ $n=298°\ 26'$	Diff. for 10′	30 Antares $m=+8.6052$ $n=289°\ 21'$	Diff. for 10′	31 α Herculis $m=+8.8709$ $n=279°\ 58'$	Diff. for 10′	Argument ($\Omega + n$)
0° 180°	+0″.0000−	227,6	+0″.0000−	237,8	+0″.0000−	240,1	+0″.0000−	250,2	+0″.0000−	257,9	180° 360°
3 177	0.4096	226,9	0.4280	237,2	0.4321	239,4	0.4504	249,5	0.4643	257,2	183 357
6 174	0.8181	225,7	0.8549	235,8	0.8630	238,1	0.8995	248,1	0.9273	255,8	186 354
9 171	1.2244	223,8	1.2794	233,9	1.2915	236,1	1.3461	246,1	1.3877	253,7	189 351
12 168	1.6273	221,3	1.7004	231,3	1.7165	233,5	1.7891	243,4	1.8444	250,9	192 348
15 165	2.0257	218,3	2.1168	228,1	2.1368	230,2	2.2272	239,9	2.2960	247,4	195 345
18 162	2.4186	214,6	2.5273	224,3	2.5512	226,3	2.6591	235,9	2.7413	243,2	198 342
21 159	2.8049	210,3	2.9310	219,7	2.9586	221,8	3.0838	231,2	3.1791	238,3	201 339
24 156	3.1834	205,5	3.3265	214,7	3.3579	216,8	3.5000	225,9	3.6081	232,9	204 336
27 153	3.5533	200,1	3.7130	209,1	3.7481	211,0	3.9067	219,9	4.0274	226,7	207 333
30 150	3.9134	194,1	4.0893	202,8	4.1279	204,8	4.3026	213,4	4.4355	220,0	210 330
33 147	4.2628	187,6	4.4544	196,1	4.4965	197,9	4.6867	206,3	4.8315	212,6	213 327
36 144	4.6004	180,6	4.8073	188,7	4.8527	190,4	5.0580	198,6	5.2142	204,7	216 324
39 141	4.9255	173,1	5.1470	180,9	5.1955	182,6	5.4154	190,3	5.5826	196,2	219 321
42 138	5.2371	165,1	5.4726	172,5	5.5242	174,2	5.7580	181,5	5.9358	187,1	222 318
45 135	5.5343	156,7	5.7831	163,8	5.8377	165,3	6.0847	172,3	6.2726	177,6	225 315
48 132	5.8163	147,9	6.0779	154,4	6.1352	155,9	6.3948	162,6	6.5923	167,6	228 312
51 129	6.0825	138,6	6.3559	144,9	6.4159	146,2	6.6874	152,4	6.8940	157,1	231 309
54 126	6.3320	128,9	6.6167	134,7	6.6791	136,0	6.9618	141,7	7.1768	146,1	234 306
57 123	6.5640	118,9	6.8591	124,3	6.9239	125,4	7.2169	130,8	7.4398	134,8	237 303
60 120	6.7781	108,6	7.0829	113,5	7.1497	114,6	7.4523	119,4	7.6825	123,1	240 300
63 117	6.9736	98,1	7.2872	102,4	7.3560	103,4	7.6673	107,7	7.9041	111,1	243 297
66 114	7.1501	87,1	7.4715	91,1	7.5421	91,8	7.8612	95,8	8.1040	98,7	246 294
69 111	7.3068	76,1	7.6354	79,4	7.7074	80,2	8.0336	83,6	8.2817	86,2	249 291
72 108	7.4437	64,6	7.7784	67,5	7.8518	68,1	8.1841	71,0	8.4368	73,2	252 288
75 105	7.5600	53,1	7.8999	55,5	7.9744	56,1	8.3119	58,4	8.5686	60,2	255 285
78 102	7.6556	41,6	7.9998	43,4	8.0753	43,8	8.4171	45,6	8.6770	47,1	258 282
81 99	7.7304	29,7	8.0779	31,0	8.1542	31,3	8.4992	32,7	8.7617	33,7	261 279
84 96	7.7838	17,8	8.1337	18,7	8.2105	18,8	8.5580	19,6	8.8223	20,2	264 276
87 93	7.8159	6,0	8.1673	6,3	8.2444	6,3	8.5933	6,6	8.8587	6,8	267 273
90 90	+7.8267−		+8.1786−		+8.2558−		+8.6052−		+8.8709−		270 270
($\Omega + n$) Argument	β Ursæ Minor.		α Cor. Bor.		α Serpentis		Antares		α Herculis		($\Omega + n$) Argument

TABLE VII. Lunar Nutation in Declination.

$m. \sin(☊ + n)$

Argument (☊ + n)	32 α Ophiuchi $m = +8.9359$ $n = 276° 9'$	Diff. for 10′	33 γ Draconis $m = +8.9751$ $n = 271° 22'$	Diff. for 10′	34 α Lyræ $m = -8.9403$ $n = 84° 11'$	Diff. for 10′	35 γ Aquilæ $m = -8.6248$ $n = 71° 13'$	Diff. for 10′	36 α Aquilæ $m = -8.5947$ $n = 70° 21'$	Diff. for 10′	Argument (☊ + n)
0° 180°	+0″.0000−	259,8	+0″.0000−	260,9	−0″.0000+	259,9	−0″.0000+	250,8	−0″.0000+	249,9	180° 360°
3 177	0.4677	259,1	0.4697	260,2	0.4679	259,2	0.4514	250,1	0.4498	249,2	183 357
6 174	0.9340	257,7	0.9381	258,8	0.9345	257,8	0.9015	248,7	0.8984	247,8	186 354
9 171	1.3979	255,6	1.4040	256,7	1.3986	255,7	1.3492	246,7	1.3445	245,8	189 351
12 168	1.8579	252,7	1.8660	253,9	1.8588	252,8	1.7932	243,9	1.7869	243,1	192 348
15 165	2.3128	249,2	2.3230	250,2	2.3139	249,3	2.2323	240,5	2.2245	239,7	195 345
18 162	2.7613	245,1	2.7734	246,1	2.7627	245,1	2.6652	236,5	2.6559	235,7	198 342
21 159	3.2024	240,1	3.2164	241,2	3.2039	240,2	3.0909	231,7	3.0801	230,9	201 339
24 156	3.6345	234,7	3.6505	235,6	3.6363	234,7	3.5080	226,4	3.4957	225,7	204 336
27 153	4.0569	228,4	4.0746	229,4	4.0588	228,5	3.9156	220,4	3.9019	219,7	207 333
30 150	4.4680	221,6	4.4876	222,6	4.4701	221,7	4.3124	213,9	4.2973	213,2	210 330
33 147	4.8669	214,2	4.8882	215,2	4.8692	214,3	4.6974	206,8	4.6810	206,0	213 327
36 144	5.2524	206,2	5.2755	207,1	5.2550	206,3	5.0696	199,0	5.0518	198,3	216 324
39 141	5.6235	197,7	5.6482	198,5	5.6263	197,7	5.4278	190,7	5.4088	190,1	219 321
42 138	5.9793	188,5	6.0055	189,3	5.9822	188,6	5.7711	181,9	5.7510	181,3	222 318
45 135	6.3186	178,9	6.3463	179,7	6.3216	179,1	6.0986	172,7	6.0773	172,1	225 315
48 132	6.6406	168,8	6.6698	169,5	6.6439	168,8	6.4094	162,9	6.3870	162,4	228 312
51 129	6.9445	158,3	6.9749	159,0	6.9478	158,4	6.7027	152,8	6.6793	152,2	231 309
54 126	7.2294	147,2	7.2611	147,8	7.2329	147,2	6.9777	142,1	6.9533	141,6	234 306
57 123	7.4943	135,8	7.5271	136,4	7.4979	135,9	7.2334	131,1	7.2081	130,6	237 303
60 120	7.7387	124,1	7.7727	124,6	7.7425	124,1	7.4693	119,7	7.4432	119,3	240 300
63 117	7.9620	111,9	7.9969	112,4	7.9658	111,9	7.6848	108,0	7.6579	107,6	243 297
66 114	8.1634	99,4	8.1992	99,9	8.1673	99,5	7.8792	95,9	7.8516	95,7	246 294
69 111	8.3424	86,8	8.3790	87,2	8.3464	86,9	8.0519	83,8	8.0238	83,5	249 291
72 108	8.4987	73,7	8.5359	74,1	8.5028	73,8	8.2028	71,2	8.1741	70,9	252 288
75 105	8.6314	60,7	8.6692	60,9	8.6356	60,7	8.3309	58,6	8.3018	58,3	255 285
78 102	8.7406	47,4	8.7789	47,6	8.7448	47,4	8.4363	45,7	8.4068	45,6	258 282
81 99	8.8259	33,9	8.8646	34,1	8.8302	33,9	8.5186	32,7	8.4889	32,6	261 279
84 96	8.8869	20,4	8.9259	20,4	8.8912	20,4	8.5775	19,7	8.5475	19,6	264 276
87 93	8.9236	6,8	8.9627	6,9	8.9279	6,9	8.6129	6,6	8.5828	6,6	267 273
90 90	+8.9359−		+8.9751−		−8.9403+		−8.6248+		−8.5947+		270 270
(☊ + n) Argument	α Ophiuchi		γ Draconis		α Lyræ		γ Aquilæ		α Aquilæ		(☊ + n) Argument

TABLE VII. Lunar Nutation in Declination.

$m. \sin(\Omega + n)$

Argument ($\Omega + n$)	37 β Aquilæ $m = -8.5623$ $n = 69°\ 26'$	Diff. for 10′	38, 39 α^2 Capricorni $m = -8.3911$ $n = 64°\ 55'$	Diff. for 10′	40 α Cygni $m = -8.1493$ $n = 59°\ 0'$	Diff. for 10′	41 α Cephei $m = -7.7687$ $n = 49°\ 48'$	Diff. for 10′	42 β Cephei $m = -7.6494$ $n = 46°\ 47'$	Diff. for 10′	Argument ($\Omega + n$)
0° 180°	−0″.0000+	248,9	−0″.0000+	244,0	−0″.0000+	236,9	−0″.0000+	225,9	−0″.0000+	222,4	180° 360°
3 177	0.4481	248,3	0.4392	243,3	0.4265	236,3	0.4066	225,2	0.4003	221,8	183 357
6 174	0.8950	246,9	0.8771	242,0	0.8518	235,0	0.8120	224,1	0.7996	220,6	186 354
9 171	1.3394	244,9	1.3127	239,9	1.2748	233,1	1.2153	222,2	1.1966	218,8	189 351
12 168	1.7802	242,2	1.7446	237,3	1.6943	230,5	1.6152	219,7	1.5904	216,3	192 348
15 165	2.2161	238,8	2.1718	234,0	2.1092	227,3	2.0107	216,7	1.9798	213,3	195 345
18 162	2.6459	234,8	2.5930	230,1	2.5183	223,4	2.4007	213,0	2.3638	209,7	198 342
21 159	3.0685	230,1	3.0071	225,5	2.9205	218,9	2.7841	208,7	2.7413	205,6	201 339
24 156	3.4826	224,8	3.4130	220,3	3.3146	213,9	3.1598	204,0	3.1113	200,8	204 336
27 153	3.8872	218,8	3.8095	214,5	3.6997	208,3	3.5270	198,6	3.4728	195,5	207 333
30 150	4.2811	212,4	4.1956	208,1	4.0746	202,1	3.8844	192,7	3.8247	189,9	210 330
33 147	4.6634	205,2	4.5701	201,2	4.4384	195,3	4.2312	186,2	4.1662	183,3	213 327
36 144	5.0328	197,6	4.9322	193,6	4.7900	188,1	4.5664	179,2	4.4962	176,5	216 324
39 141	5.3884	189,4	5.2807	185,6	5.1285	180,2	4.8890	171,8	4.8139	169,2	219 321
42 138	5.7293	180,6	5.6147	177,1	5.4529	171,9	5.1983	163,9	5.1185	161,3	222 318
45 135	6.0544	171,4	5.9334	168,0	5.7624	163,2	5.4933	155,5	5.4089	153,2	225 315
48 132	6.3630	161,7	6.2358	158,5	6.0561	153,9	5.7732	146,8	5.6846	144,5	228 312
51 129	6.6541	151,7	6.5211	148,6	6.3332	144,3	6.0374	137,2	5.9447	135,5	231 309
54 126	6.9271	141,0	6.7886	138,2	6.5930	134,2	6.2851	127,9	6.1886	125,9	234 306
57 123	7.1809	130,2	7.0374	127,5	6.8346	123,8	6.5154	118,1	6.4153	116,3	237 303
60 120	7.4152	118,8	7.2669	116,4	7.0575	113,1	6.7279	107,8	6.6246	106,2	240 300
63 117	7.6290	107,2	7.4765	105,1	7.2611	102,0	6.9220	97,3	6.8157	95,8	243 297
66 114	7.8220	95,3	7.6657	93,4	7.4447	90,7	7.0971	86,4	6.9881	85,1	246 294
69 111	7.9936	83,2	7.8338	81,5	7.6080	79,2	7.2527	75,5	7.1413	74,3	249 291
72 108	8.1433	70,7	7.9805	69,2	7.7505	67,2	7.3886	64,1	7.2751	63,1	252 288
75 105	8.2705	58,1	8.1051	57,0	7.8715	55,3	7.5040	52,7	7.3887	51,9	255 285
78 102	8.3751	45,4	8.2077	44,5	7.9711	43,3	7.5989	41,2	7.4822	40,6	258 282
81 99	8.4569	32,4	8.2878	31,8	8.0490	30,9	7.6731	29,4	7.5553	29,0	261 279
84 96	8.5153	19,6	8.3451	19,1	8.1046	18,6	7.7261	17,7	7.6075	17,4	264 276
87 93	8.5505	6,6	8.3795	6,4	8.1380	6,3	7.7580	5,9	7.6389	5,8	267 273
90 90	−8.5623 +		−8.3911+		−8.1493 +		−7.7687 +		−7.6494+		270 270
($\Omega + n$) Argument	β Aquilæ		α^2 Capricorni		α Cygni		α Cephei		β Cephei		($\Omega + n$) Argument

TABLE VII. Lunar Nutation in Declination.

$m.\sin(☊ + n)$

Argument (☊ + n)	43 α Aquarii $m=-7.3512$ $n=38°\ 31'$	Diff. for 10′	44 Fomalhaut $m=-6.9328$ $n=23°\ 30'$	Diff. for 10′	45 α Pegasi $m=-6.8819$ $n=20°\ 58'$	Diff. for 10′	46 α Andromedæ $m=-6.6825$ $n=0°\ 8'$	Diff. for 10′	Argument (☊ + n)
0° 180°	−0″.0000+	213,7	−0″.0000+	201,6	−0″.0000+	200,1	−0″.0000+	194,3	180° 360°
3 177	0.3847	213,2	0.3628	201,1	0.3602	199,5	0.3498	193,8	183 357
6 174	0.7684	212,0	0.7247	199,9	0.7193	198,5	0.6985	192,7	186 354
9 171	1.1500	210,2	1.0845	198,3	1.0766	196,8	1.0454	191,1	189 351
12 168	1.5284	207,9	1.4414	196,1	1.4308	194,7	1.3894	189,0	192 348
15 165	1.9027	204,9	1.7944	193,3	1.7812	191,9	1.7296	186,3	195 345
18 162	2.2716	201,6	2.1423	190,1	2.1266	188,7	2.0650	183,2	198 342
21 159	2.6345	197,5	2.4845	186,3	2.4663	184,9	2.3948	179,6	201 339
24 156	2.9900	193,0	2.8198	182,1	2.7991	180,7	2.7180	175,4	204 336
27 153	3.3374	187,9	3.1475	177,2	3.1244	175,9	3.0338	170,8	207 333
30 150	3.6756	182,3	3.4664	171,9	3.4410	170,7	3.3413	165,7	210 330
33 147	4.0038	176,2	3.7759	166,2	3.7482	164,9	3.6396	160,2	213 327
36 144	4.3210	169,6	4.0750	159,9	4.0451	158,8	3.9279	154,2	216 324
39 141	4.6263	162,6	4.3629	153,4	4.3309	152,2	4.2054	147,8	219 321
42 138	4.9189	155,1	4.6390	145,1	4.6049	145,2	4.4715	140,9	222 318
45 135	5.1980	147,2	4.9022	138,8	4.8662	137,8	4.7252	133,8	225 315
48 132	5.4630	138,8	5.1520	131,0	5.1142	130,0	4.9660	126,3	228 312
51 129	5.7129	130,2	5.3878	122,8	5.3482	121,9	5.1933	118,3	231 309
54 126	5.9473	121,1	5.6088	114,2	5.5676	113,4	5.4063	110,1	234 306
57 123	6.1652	111,7	5.8143	105,4	5.7717	104,6	5.6044	101,6	237 303
60 120	6.3663	102,1	6.0040	96,2	5.9599	95,5	5.7872	92,8	240 300
63 117	6.5500	92,1	6.1772	86,8	6.1318	86,2	5.9542	83,7	243 297
66 114	6.7157	81,8	6.3334	77,2	6.2869	76,6	6.1048	74,3	246 294
69 111	6.8629	71,4	6.4723	67,4	6.4248	66,9	6.2386	64,9	249 291
72 108	6.9915	60,7	6.5936	57,2	6.5452	56,8	6.3555	55,2	252 288
75 105	7.1007	49,9	6.6965	47,1	6.6474	46,7	6.4548	45,3	255 285
78 102	7.1905	39,0	6.7813	36,8	6.7315	36,5	6.5364	35,4	258 282
81 99	7.2607	27,9	6.8475	26,3	6.7972	26,1	6.6002	25,3	261 279
84 96	7.3109	16,8	6.8948	15,8	6.8442	15,7	6.6458	15,3	264 276
87 93	7.3411	5,6	6.9232	5,3	6.8724	5,3	6.6733	5,1	267 273
90 90	−7.3512+		−6.9328+		−6.8819+		−6.6825+		270 270
(☊ + n) Argument	α Aquarii		Fomalhaut		α Pegasi		α Andromedæ		(☊ + n) Argument

TABLE VIII.

ARGUMENT. ($2\odot + n''$)

			Diff. for 1°		
0°	180°	−0.0000+	97	180°	360°
5	175	.0485	96	185	355
10	170	.0965	95	190	350
15	165	.1439	92	195	345
20	160	.1901	90	200	340
25	155	.2349	86	205	335
30	150	.2779	82	210	330
35	145	.3188	77	215	325
40	140	.3573	72	220	320
45	135	.3931	65	225	315
50	130	.4258	59	230	310
55	125	.4553	52	235	305
60	120	.4814	45	240	300
65	115	.5038	37	245	295
70	110	.5224	29	250	290
75	105	.5369	21	255	285
80	100	.5474	13	260	280
85	95	.5538	4	265	275
90	90	−0.5559+		270	270

For the value of n'' see Table X.

TABLE IX.

ARGUMENT. ($2☾ + n'''$)

			Diff. for 1°		
0°	180°	+0.0000−	15	180°	360°
10	170	.0146	14	190	350
20	160	.0288	13	200	340
30	150	.0420	12	210	330
40	140	.0540	10	220	320
50	130	.0644	8	230	310
60	120	.0728	6	240	300
70	110	.0790	4	250	290
80	100	.0828	1	260	280
90	90	+0.0840−		270	270

For the value of n''' see Table X.

TABLE X.

	Names of the Stars.	Values of n'' and n'''
1	γ Pegasi	358° 47′
2	α Cassiop.	351 35
3	Polaris	345 25
4	α Arietis	328 26
5	α Ceti	314 11
6	α Persei	309 30
7	Aldebaran	291 42
8	Capella	282 51
9	Rigel	282 20
10	β Tauri	280 13
11	α Orionis	273 13
12	Sirius	261 21
13	Castor	250 40
14	Procyon	249 6
15	Pollux	248 3
16	α Hydræ	222 39
17	Regulus	212 22
18	α Ursæ Major.	198 7
19	β Leonis	185 21
20	β Virginis	184 57
21	γ Ursæ Major.	184 8
22	Spica Virginis	159 21
23	η Ursæ Major.	152 50
24	Arcturus	145 45
25, 26	α^2 Libræ	137 10
27	β Ursæ Minor.	134 42
28	α Cor. Bor.	125 45
29	α Serpentis	123 42
30	Antares	113 24
31	α Herculis	102 13
32	α Ophiuchi	97 34
33	γ Draconis	91 41
34	α Lyræ	82 51
35	γ Aquilæ	67 16
36	α Aquilæ	66 15
37	β Aquilæ	65 12
38, 39	α^2 Capricor.	60 2
40	α Cygni	53 29
41	α Cephei	43 50
42	β Cephei	40 49
43	α Aquarii	32 57
44	Fomalhaut	19 27
45	α Pegasi	17 17
46	α Andromedæ	0 6
	Names of the Stars.	Values of n'' and n'''

REPORT OF THE COUNCIL OF THE SOCIETY

TO THE

THIRD ANNUAL GENERAL MEETING

FEBRUARY 14, 1823.

THE Council of the Astronomical Society have great pleasure in being able to make a Report of the most favourable nature upon the affairs and prospects of the Society, which is in a most flourishing condition, and such a one as convinces them of the high estimation in which it is held by the public, and of the necessity which existed for its formation, as pointed out in their original Address.

When they last had the honour of addressing you, the total number of Members and Associates elected amounted to 188, since which time 22 names have been added to the list, and seven are now waiting for election; making a total of 217 proposed and elected; among whom will be recognised the names of almost all the most eminent Astronomers of the continent, who have not only joined the Society as Associates, but, from the number and value of their respective communications, have proved their zeal for its welfare, and their readiness to advance its objects.

While they thus exult in the number of valuable acquisitions the Society has gained, they cannot help, on the other hand, adverting with the deepest regret and sorrow to the severe losses which the Society has sustained since its

last General Meeting, in the death of their much esteemed and never-to-be-forgotten President, as well as several of its most valued Members, by which the number of effective Members and Associates has been reduced to 187. Though deprived of the personal exertions of Sir William Herschel by his age, and the distance of his residence from London, his name was so intimately connected with the progress of modern Astronomy, and the science had been so eminently enriched by his discoveries and researches, that it could not fail to shed a lustre upon the Society over which he presided; and when to this name is added those of Sir Henry Englefield, Dr. Hutton, Delambre, and Professor Tralles (an Associate elect, who was cut off but a few days previous to his election), they feel confident that the Society will sympathize with them in the loss of such justly esteemed and highly valuable Members.

Since the last General Meeting, the Society has sent forth the first volume of its Memoirs to its Members and the Public; and the Council trust that the matter it contains, as well as the manner in which it has been printed and sold, must give general satisfaction to the Members. They now possess sufficient valuable and interesting papers to commence the publication of a second volume, which will appear with all convenient expedition.

In their last Report, the Council stated that a Committee had been appointed for the purpose of procuring the calculation of tables for determining the apparent places of the 46 principal fixed stars.

These computations, having since been made and completed, will form part of their next publication *; and in the mean time they have printed a set of tables of the apparent places of 36 stars for the first four months of the year, which it was the wish of the Council should have appeared with the commencement of the present year; but some unavoidable delays have occurred which have prevented their laying a copy of them before you until the present time, and they will shortly be ready for distribution among such Members of the Society as may apply for them.

* These tables will be found at page 421 of the present volume.

The library of the Society has been very considerably augmented by the liberal donations of its Members and others; and the Council are happy to state that the purchase of some of the periodical works mentioned in their last Report has been discontinued, from those works having been regularly presented by their respective authors or publishers; so that they hope shortly to open an Astronomical Library of no small interest or value to the Members, and are sorry that this has hitherto been prevented by the inconvenience of their present premises for such a purpose; and although a Committee has been appointed to search for other more suitable rooms, yet in the present infant state of the Society it has been thought better to submit to a temporary inconvenience than increase the expenditure of the Society's funds.

The Council are however happy in being able to state, that the finances of the Society are in a flourishing condition, as will appear by the Report of the Auditors upon the accounts of the Treasurer, which will be read. The printing of the Memoirs of the Society has been one of its heaviest expenses; but from the number of volumes already sold, there is every reason to believe that this expense will be greatly, if not wholly, returned.

A list of the papers that have been read since the last General Meeting is also subjoined: many of these are of great importance, and will be published in the next volume of Memoirs *; and the Council take this opportunity of calling generally upon its Members and Associates to fulfil the objects of the Institution by every means in their power. Isolated facts or observations can be of no use to any one; but if transmitted to a Society, as to a focus where they will be registered and compared, they may become important. The entire examination of the heavens is beyond the power of any individual: but on dividing the labour, the difficulty vanishes; and while our continental neighbours are each taking charge of the examination of a small portion of the visible heavens, so as to produce a general scrutiny by their conjoint labours, the Council are convinced that the Astronomical Society of London will not suffer itself to be called upon in vain to partake in this highly useful labour.

---

* The major part of these Papers will be found in the present volume.

The Council cannot conclude this Report without expressing to the Society their regret at the retirement of their late Secretary, Mr. Baily, who by his unwearied exertions in conducting the affairs of the Society, as well during its infancy as by his valuable contributions to its Memoirs, has essentially assisted in bringing it to its present prosperity.

The Society will doubtless approve of a resolution of the Council, by which they directed their Secretary to write to Mr. Baily, requesting him to attend their meetings during the remainder of the year, as well as adequately appreciate the promptness with which he complied with their request.

REPORT OF THE COUNCIL OF THE SOCIETY

TO THE

FOURTH ANNUAL GENERAL MEETING

FEBRUARY 13, 1824.

IN meeting you at this, the fourth annual general assembly of the Astronomical Society of London, the Council have great pleasure in announcing the prosperity of the Society's affairs, and the importance into which it has risen in the estimation of men of science. No pains have been spared by the Council to promote its utility; and the measures that have been adopted with this view, are such as, they have no doubt, will meet your cordial support and approbation.

Short, comparatively, as has been the duration of the Society, it has called forth many ingenious and highly valuable papers from its Members and others, which, but for its existence, might never have been given to the world; these have been read at the several Meetings, and a list of them is subjoined. (See the note in page 499.)

A spirit of research has in this way been excited, from which the greatest benefits may be expected to arise to the science of Astronomy; and as the Society has kept the practical, as well as the theoretical branches of its object constantly in view, several important improvements in instruments, as well as in their application, have been developed and explained.

Desirous of contributing to the extension of Astronomical science, the Society not only suggested many objects of research and investigation in its first Address to the public, but it likewise proposed a distinct prize question in physical Astronomy, as an honorary object of reward. The same question has been continued until the present time; but not having been answered, will now be replaced by others of like importance to the advancement of the science *.

The Council being pledged, by the terms of their first Address, to take notice of important discoveries and investigations, connected either mediately or immediately with Astronomy, felt no inclination to overlook such inventions as that of their own distinguished member, Mr. Babbage, or such investigations and discoveries as those of Professor Encke, or Dr. Rumker, and M. Pons. It has therefore been unanimously determined, that the Honorary Gold Medal of this Society shall be presented to Charles Babbage, Esq., as a token of the high estimation in which the Society holds his invention of an engine for calculating Mathematical and Astronomical tables:—that a similar Gold Medal shall also be presented to Professor J. F. Encke, for his investigations relative to the comet which bears his name, and which led to the re-discovery of it in 1822:—and, further, that the Silver Honorary Medal of the Society shall be presented to Dr. Karl Rumker, for the re-discovery of this comet in the year mentioned; and that the Society's Silver Medal shall be presented to M. Jean Louis Pons, for the discovery of two comets on the 31st of May, and 13th of July, 1822, as well as for his indefatigable assiduity in that department of Astronomy; and it having been previously determined, that all Honorary Medals, or rewards, shall be presented to the parties in person, or to their proxies, at the Anniversary of the Society, next following the time when they shall be respectively awarded, such Medals will be so presented by the Chairman on this day.

The Council beg further to state, that application was made through one of their members, by Commodore Krusenstern, to furnish Astronomical instructions for Captain Kotzebue, in his intended voyage of exploration, first towards the south, and then towards the north pole, by order of His

* The new prize questions will be found at the end of the present Report.

Majesty the Emperor of RUSSIA. This requisition was cheerfully complied with, and instructions were furnished to Captain KOTZEBUE, who touched upon England, in the progress of his voyage, in the month of September last.

The Council have instituted some experiments on the properties of glass, for the formation of the object lenses of refracting telescopes. With this view a 30-inch telescope was constructed of some foreign glass for a trial of it, by Mr. TULLEY. The result was satisfactory, and the telescope has since been sold to one of the Members. Three other pieces of glass, presented to the Society by M. GUINEAND of Neufchatel, (one of which is of very large dimensions, and promising aspect,) are now undergoing trial in the hands of a Committee, to whom the examination of their merits has been referred.

The Council also announce, that they have, at considerable expense, caused a set of original tables to be computed, for the purpose of deducing the apparent places of the 46 Greenwich stars, from the year 1820 to 1840, or to a later period, by means of subsidiary reductions. These tables, which are eight in number, will be printed in the ensuing part of the volume of the Society's Memoirs; and will, the Council have no doubt, be found of high importance to the practical Astronomer. To these tables are prefixed adequate explanations and precepts, as given, with his accustomed precision and taste, by your Foreign Secretary *.

The Council regret that it is not in their power to lay the concluding part of their first volume before you on this day; but some unavoidable circumstances have occurred to render it impracticable.

On this account they have determined upon extending the first volume to the period of the present Anniversary, instead of closing it with the end of the last session in June 1823, as was first intended. It will therefore contain all the papers and communications, which the Council have deemed it expedient to publish, up to the date of this Anniversary. From the

* This paper will be found at page 421.

present forward state of the printing, the Council hope that the second fasciculus, now adverted to, will be ready for delivery to the Members and the Public within one month from this time; and they doubt not that its contents will be found highly interesting and useful. To the close of it will be subjoined a correct list of the Members and Associates, together with the Officers and Council for the ensuing year, as elected by you this day; likewise a list of the presents made to the Society up to this time.

The Council take this opportunity of announcing to the Members and Associates in general, that from the period of the present anniversary, they have resolved upon a new method of publishing the communications that may be made to them; that is to say, instead of withholding them, as heretofore, till the end of the year or season, to form a complete volume or part, each individual paper or communication will be printed and published separately, without reference to the others, as soon as it has been read, and its publication determined on. An uniformity of size, type and paper, and the appropriate succession of pages will be preserved; while at the close of the year, or whenever the Council may find it expedient, a title-page, preface and index, will be printed, for the purpose of binding the papers into volumes. The Council feel confident that this arrangement will meet with the sanction and approbation of the Members, and that it will prove alike beneficial to the public and to the interests of the Society: it will evidently afford the means of speedy distribution to much important astronomical information and discussion, which in many cases is of so temporary a nature as to lose its utility and interest unless it obtain immediate circulation. Independently of which, the Members and the public will have an opportunity of availing themselves of such papers only as may be interesting to them, without the purchase of the whole: at the same time that a sufficient number of entire sets will be preserved for those who may wish to possess the Memoirs in that form.

With respect to the number of Members and Associates of the Society, a gratifying progressive change has taken place since the last anniversary. There are at this time 179 Members of the Society and 22 Associates, exclusive of the loss which the Society has to lament of 3 of its esteemed Members and one Associate by death, and 5 Members who have resigned; making the total number of Members greater by 12, and of Associates greater by 2, than it was at the preceding anniversary Meeting.

On adverting to the deprivations which the Society has suffered by death, the name of Lieut.-Colonel WILLIAM LAMBTON of Madras, as a Member, and of Doctor HENRY JOHN WALBECK of Abö, as an Associate, cannot be passed over in silence. The former of these gentlemen, Lieut.-Colonel WILLIAM LAMBTON, was more than twenty years engaged on a trigonometric survey and measurement of an arc of the meridian extending northward from the southern extremity of India, and which had been completed by him from the eighth to the eighteenth degree of N. latitude, in the able and scientific manner described by Colonel LAMBTON himself in his successive papers in the Transactions of the Asiatic Society.

The important labour had been further prosecuted beyond the twentieth degree, and was still in progress at the time of the Colonel's decease. This great work, embracing the largest continuous arc yet measured on the surface of the earth, was undertaken by the Government at his express suggestion and recommendation, and was executed by him with indefatigable industry and eminent ability, amidst all the difficulties incident to an enterprise of this nature in a country so remote from the ordinary resources of instrumental assistance. His communications on the subject of this great performance, which was the occupation of his life, fill a distinguished place in the Transactions of the Asiatic Society of Calcutta. They reflect great credit on his scientific attainments, and entitle him to the respectful remembrance of astronomers.

This Society will derive some consolation from knowing that the further prosecution of this important work will not be relinquished. It has devolved on Captain EVEREST, one of our Members, from whom a communication on a geodetical subject has been received, and is inserted in the forthcoming volume of the Memoirs of this Society *. It may be confidently expected that the measurement of a meridional arc will be prolonged as far northward as the British authority in India extends; and it may be hoped that it will be completed to an uninterrupted portion of more than twenty degrees.

Doctor HENRY JOHN WALBECK was at the head of the astronomical observatory at Abö. The distinguished abilities and industry which he had mani-

* This communication will be found at page 255 of the present volume.

fested, afforded the promise of valuable results of his diligence exerted in the most northern observatory of Europe: and his loss must be deplored in proportion to the expectation justly formed of his talents and assiduity.

There is a progressive improvement in the funds of the Society, as will appear from the statement of accounts this day delivered by the Treasurer, and the report of the Auditors appointed to examine the same, which the Council have directed shall be laid before you and read at the close of this Report. The library has likewise received many valuable acquisitions, through the liberality of numerous donors, of whom several are Members of the Society, and others, lovers of astronomical science, who take an interest in its welfare.

The Council cannot conclude their Report without expressing their regret at the loss of the valuable assistance of Mr. Babbage as one of their Secretaries; but they feel assured that you will cordially join them in offering him the best thanks of the Society for his exertions and the many services he has rendered in that capacity, at the same time that you will approve the request the Council made to him of attending their meetings and lending his assistance as often as convenient; a request, with which he has most obligingly complied.

Enrolling amongst its Members and Associates, as the Astronomical Society now does, a considerable proportion of the most able astronomers in Great Britain and its Colonies, and several of the most distinguished astronomers in Europe; receiving frequent communications from its Foreign Associates, on the most interesting topics connected with the theory and practice of the science; holding out inducements, as well to sedulous observers, as to profound investigators, by its prize questions; and aiming to promote a liberal interchange of correspondence with institutions formed for kindred purposes in every part of the world, your Council cannot but entertain the hope that each successive year in the history of the Society will witness the extension of its utility as well as of its prosperity; and prove that it has been decidedly instrumental in increasing the facilities of the observer, enlarging the boundaries of physical theory, or adding to the catalogue, already rich and splendid, of astronomical facts.

PRIZE QUESTIONS

ALLUDED TO IN THE PRECEDING REPORT.

1st.—The Silver Medal.

To any person, who shall contrive and have executed an Instrument, by which the relative magnitudes of the stars may be measured or determined, and the utility of which for this object shall be sufficiently established by numerous observations and comparisons of known stars.

2nd.—The Gold Medal.

For approved Formulæ for determining the true places of either of the four newly discovered planets, Ceres, Juno, Vesta and Pallas, within such limits as the Council may think sufficiently correct for the present state of Astronomy. Such formulæ in each case to be accompanied with comparisons of the observed places, at various periods.

3rd.—The Gold Medal.

For a new mode of developing the differential equation for expressing the Problem of the three bodies, by which *a smaller number* of tables shall be required, in order to compute the Moon's place to the same degree of accuracy as by any existing tables, and with greater facility.

To be entitled to a competition for the Three Prizes, or either of them, the answers must be received as follows, viz.:

To the 1st	Prize, before February 1,	1826.
2nd	. . . .	1827.
3rd	. . . .	1828.

ADDRESSES

OF

HENRY THOMAS COLEBROOKE, Esq. F.R.S.

President of the Astronomical Society of London,

ON PRESENTING THE HONORARY MEDALS OF THE SOCIETY

TO THE SEVERAL PERSONS TO WHOM THEY HAD BEEN AWARDED.

On presenting the Gold Medal to CHARLES BABBAGE, *Esq. F.R.S.*

THIS country and the present age have been pre-eminently distinguished for ingenuity in the contrivance, or in the improvement, of machinery. In none has that been more singularly evinced, than in the instance to which I have the gratification of now calling the attention of the Society. The invention is as novel, as the ingenuity manifested by it is extraordinary.

In other cases, mechanical devices have substituted machines for simpler tools or for bodily labour. The artist has been furnished with command of power beyond human strength, joined with precision surpassing any ordinary attainment of dexterity. He is enabled to perform singly the work of a multitude, with the accuracy of a select few, by mechanism which takes the place of manual labour or assists its efforts. But the invention, to which I am adverting, comes in place of mental exertion: it substitutes mechanical performance for an intellectual process: and that performance is effected with celerity and exactness unattainable in ordinary methods, even by incessant practice and undiverted attention.

The invention is in scope, as in execution, unlike any thing before accomplished to assist operose computations. I pass by, as what is obviously quite different, the Shwanpan, or Chinese abacus, the tangible arithmetic of FREND, NAPIER's rods, with the ruder devices of antiquity, the tallies, the checque,

and the counters. They are unconnected with it in purpose, as in form. Mechanical aid of calculation has in truth been before proposed by very eminent persons. PASCAL invented a very complicated instrument for the simplest arithmetical processes, addition and subtraction, and reaching by very tedious repetition to multiplication and division. LEIBNITZ proposed another, of which the power extends no further. DELEPINE's and BOITISSENDEAU's contrivances, which a century ago were applauded by the Academy of Sciences at Paris, are upon the model of PASCAL's, and may no doubt be improvements of it, but do not vary or enlarge its objects. MORELAND's instruments, described in an early volume of the *Philosophical Transactions* (the 8th), are confined, the one to addition and subtraction, the other to multiplication. The *Rotula Arithmetica* of BROWN, simpler in construction, reaches not beyond the four arithmetical operations.

The principle, which essentially distinguishes Mr. BABBAGE's invention from all these, is, that it proposes to calculate a series of numbers following any law by the aid of *differences;* and that, by setting a few figures at the outset, a long series of numbers is readily produced by a mechanical operation. The method of *differences*, in a very wide sense, is the mathematical principle of the contrivance. A machine to add a number of arbitrary figures together is no economy of time or trouble; since each individual figure must be placed in the machine. But it is otherwise when those figures follow some law. The insertion of a few at first determines the magnitude of the next; and these of the succeeding. It is this constant repetition of similar operations, which renders the computation of tables a fit subject for the application of machinery. Mr. BABBAGE's invention puts an engine in the place of the computer. The question is set to the instrument; or the instrument is set to the question; and, by simply giving it motion, the solution is wrought and a string of answers is exhibited.

Nor is this all; for the machine may be rendered capable of recording its answer, and even multiplying copies of it. The usefulness of the instrument is thus more than doubled: for it not only saves time and trouble in transcribing results into tabular form, and setting types for the printing of the table constructed with them, but it likewise accomplishes the yet more important object of ensuring accuracy, obviating numerous sources of error through the careless hands of transcribers and compositors.

On this part of the invention, which is yet a subject of experiment for the selection of the most eligible among divers modes of accomplishing it, I shall not dwell longer; as it is not for that superaddition, but for the machine in the finished form of a calculating instrument, that I am to make an acknowledgement, in the name of this Society, to Mr. Babbage for his very useful invention.

I speak of it as complete with reference to a model which satisfactorily exhibited the machine's performance, and am apprised that a more finished engine, which is in progress of preparation, may not yet for some time be in a forward state to be put in activity and receive its practical application.

In no department of science or of the arts, does this discovery promise to be so eminently useful as in that of astronomy and its kindred sciences, with the various arts dependent on them. In none are computations more operose than those which astronomy in particular requires: in none are preparatory facilities more needful: in none is error more detrimental. The practical astronomer is interrupted in his pursuit, is diverted from his task of observation, by the irksome labour of computation; or his diligence in observing becomes ineffectual for want of yet greater industry of calculation. Let the aid, which tables previously computed afford, be furnished to the utmost extent which mechanism has made attainable through Mr. Babbage's invention, the most irksome portion of the astronomer's task is alleviated, and a fresh impulse is given to astronomical research.

Nor is it among the least curious results of the ingenious device, of which I am speaking, that it affords a new opening for discovery; since it is applicable, as has been shown by its inventor, to surmount novel difficulties of analysis. Not confined to *constant differences*, it is available in every case of differences that follow a definite law, reducible therefore to an equation. An engine, adjusted to the purpose, being set to work, will produce any distant term, or succession of terms, required: thus presenting the numerical solution of a problem, even though the analytical solution of it be yet undetermined.

It may not therefore be deemed too sanguine an anticipation, when I express the hope, that an instrument, which in its simpler form attains to the

extraction of the roots of numbers and approximates to the roots of equations, may in a more advanced state of improvement rise to the approximate solution of algebraic equations of elevated degrees. I refer to solutions of such equations proposed by LA GRANGE, and more recently by other analysts, which involve operations too tedious and intricate for use, and which must remain without efficacy, unless some mode be devised of abridging the labour or facilitating the means of its performance. In any case this engine tends to lighten the excessive and accumulating burden of arithmetical application of mathematical formulæ, and to relieve the progress of science from what is justly termed by the author of this invention, the overwhelming incumbrance of numerical detail.

For this singular and pregnant discovery, I have the authority of the Astronomical Society of London to present to Mr. BABBAGE, its Gold medal, which accordingly I now do, as a token of the high estimation in which it holds his invention of an engine for calculating mathematical and astronomical tables.

On presenting the Gold Medal to Professor ENCKE, and the Silver Medal to Dr. P. K. RUMKER.

The greatest step which has been made in the astronomy of comets, since the verification of HALLEY'S comet, which reappeared in 1759, has been the identifying of ENCKE'S comet, at once determined by the evidence of its frequent appearance within short periods, and already confirmed by its rediscovery in a distant hemisphere.

It is scarcely to be doubted, that other, many other, like bodies, moving through very eccentric orbits, in short periods, belong to our solar system. Though LEXELL'S comet has not been again observed since 1770, it is not therefore to be despaired of. More extended, more diffused diligence may yet detect it, if in truth it has not ceased to be capable of becoming luminous. Nor is it an over-sanguine expectation, which counts upon more discoveries of the like nature. It is not *likely* that ENCKE'S should be solitary of its kind; the only one revolving in a short period; or the only visible one.

The Astronomical Society is desirous of drawing the attention of observers, in an especial manner, to this department of research; with the confidence that increased vigilance cannot fail of being rewarded by abundant discovery; and I may here take leave to remark, that multiplied observations at very remote stations, determining a greater portion of a comet's orbit, will tend to the earlier and more precise ascertainment of its period.

In this view, as in so many others, the establishment of observatories at the Cape of Good Hope, and Australasia, has been matter of congratulation with astronomers: and I have peculiar satisfaction in being authorized to acknowledge the service rendered to astronomical science by the re-discovery of ENCKE's comet in 1822 at the observatory at Paramatta; and to present to Dr. P. KARL RUMKER, the superintendant of that observatory, the Society's medal on this account; at the same time that I present, in the Society's name, its medal to Professor ENCKE for the previous investigations relative to that comet, and which led to the re-discovery of it.

On presenting the Silver Medal to M. Pons.

No name has more frequently recurred in the history of comets, than that of M. PONS; and from the very commencement of the present century, he has been in almost exclusive possession of the first discovery of telescopic comets. At Marseilles, while joint director of the observatory at that place, he discovered more than twenty comets; being the first to see the greatest number of them, a very few only having been likewise and independently noticed as early by other observers. His vigilance did not remit, nor has his diligence been unrewarded at Marlia, where he was invited to superintend a new observatory. Previously to his departure from Marseilles he had made the memorable re-discovery of the comet which bears ENCKE's name; and his arrival at Marlia was signalized by the detection of another comet. In a recent period he has been yet more successful, having discovered no less than three comets in the year 1822, with his usual privilege of priority in respect of two of them, under every disadvantage in regard to instruments, joined with other discouraging circumstances, which, it may be feared, have since too much operated, and which the Society most earnestly desire to see removed. The services

which M. Pons has rendered to this branch of astronomical science, have been acknowledged by the Royal Academy of Sciences at Paris adjudging to him a prize for the re-discovery of Encke's comet in 1818, and sharing between him and M. Nicollet a prize for the comet discovered in January 1821.

Had equal diligence been devoted to this research by other observers at remote stations and in various climates, it is highly probable that a greater number of comets might have been detected in the last age; and it may be presumed, that increased vigilance in time to come will be recompensed with enlarged knowledge of comets, and with correspondent advancement in this branch of science.

The Council of the Astronomical Society, desirous of marking their sense of the services rendered by M. Pons, both in acknowledgement to him for his usefulness and with hope that his example may be followed by others, have resolved to present to him the silver medal of the Society; which accordingly I now do, in their name and in that of the Astronomical Society, as a token of the sense entertained of his indefatigable assiduity in that department of astronomy, and especially for the discovery of a comet on 31st May and another on 1st July, 1822.

PRESENTS

RECEIVED BY

THE ASTRONOMICAL SOCIETY OF LONDON.

1822.	PRESENTS.	DONORS.
March 8.	The Chemical Catechism, by Samuel Parkes. London 1822. 10th Edition, 1 vol. 8vo.	S. Parkes, Esq.
	On the Mean Density of the Earth. By Dr. Charles Hutton, F.R.S. 8vo.	Dr. Hutton.
	Rapport fait à l'Académie des Sciences, le Lundi, 4 Juin 1821, sur un Mémoire de M. Fresnel, relatif aux couleurs des lames cristallisées douées de la double réfraction. 8vo.	M. Fresnel.
	Harding's Himmels-charten. folio.	M. Harding.
	Roberti Simson, M.D. Opera quædam Geometrica. Glasguæ 1776. 1 vol. 4to.	J. Bush, Esq.
	Tables Astronomiques, publiées par le Bureau des Longitudes de France, contenant les Tables de Jupiter, de Saturne et d'Uranus, construites d'après la théorie de la Mécanique Céleste. Par M. A. Bouvard. Paris 1821. 4to.	M. A. Bouvard.
	Connaissance des Tems, pour l'An 1824. Paris 1821. 8vo.	Le Bureau des Longitudes de France.
April 12.	Thirty Volumes of the Nautical Almanac. Various dates. 8vo.	Dr. Gregory.
	Histoire de l'Astronomie moderne. Par M. Delambre. Paris 1821. 2 vols. 4to.	M. Delambre.
	Astronomical Observations made at the Royal Observatory at Greenwich from the year 1750 to the year 1762. By the Rev. James Bradley, D.D. Astronomer Royal. Oxford, 1798, 1805. 2 vols. folio.	Rich. Best, Esq.
May 10.	Elements of Captain Hall's Comet. By J. Brinkley, D.D. in a Letter addressed to W. H. Wollaston, M.D. London 1822. 4to.	Dr. Brinkley.

1822.	Presents.	Donors.
May 10.	Remarks on the present defective state of the Nautical Almanac. By Francis Baily, F.R.S. and L.S. London 1822. 8vo.	F. Baily, Esq.
June 14.	Annalen der K. K. Sternwarte in Wien. Von J. J. Littrow, Erster theil. Wien 1821. folio.	Prof. Littrow.
	Der ort des Sterns δ Ursæ Minoris in seiner oberen culmination für jeden Tag der Jahre 1820, 1821, 1822, berechnet aus Bessels Tafeln. Dorpat 1821. 8vo.	———
	A new Theory of the Tides, shewing what is the immediate cause of the phenomenon; and which has hitherto been overlooked by philosophers. By Capt. Forman, R.N. Bath 1822. 8vo.	Capt. Forman.
	A Celestial Atlas, by A. Jamieson. London 1822. 1 vol. 4to.	Dr. Gregory.
	Astronomische Nachrichten. Nos. 2 to 12 inclusive. Altona. 4to.	Pr. Schumacher.
	Transactions of the Cambridge Philosophical Society. Vol. 1. Part II. Cambridge 1822. 4to.	The Cambridge Phil. Society.
	Practical Observations on the Nautical Almanac. By James South, F.R.S. London 1822. 8vo.	J. South, Esq.
Nov. 8.	Asiatic Researches. Vols. 6 to 12 inclusive. Calcutta. 4to.	The Asiatic Soc.
	The Universal Cambist and Commercial Instructor; being a full and accurate treatise on the Exchanges, Monies, Weights and Measures of all trading nations and their colonies. By P. Kelly, L.L.D. London 1821, 2nd Edition, 2 vols. 4to.	Dr. Kelly.
	A Practical Introduction to Spherics and Nautical Astronomy. By P. Kelly, L.L.D. London 1822, 5th Edition, 1 vol. 8vo.	———
	Transactions of the Royal Society of Edinburgh. Edinburgh 1821. Vol. 9, Part I. 4to.	Roy. Soc. of Edinburgh.
	F. G. W. Struve Observationes Astronomicas institutas in specula Universitatis Cæsareæ Dorpatensis, publici juris facit Senatus Universitatis. Volumen 1: Observationes annorum 1814 et 1815, una cum reductionibus. Volumen 2: Observationes annorum 1818 et 1819. Dorpati 1817 & 1820. 4to	Prof. Struve.
	Catalogus 795 Stellarum duplicium, ex diversorum astronomorum observationibus congestus in specula Dorpatensi. (Ex volumine 3 Observationum Dorpatensium.) Dorpati 1822. 4to.	———
	A Letter to Sir Humphrey Davy, Bart. Pres. Roy. Soc. etc. on the application of Machinery to the purpose of calculating	C. Babbage, Esq.

1822.	Presents.	Donors.
Nov. 8.	and printing Mathematical Tables; from Charles Babbage, Esq. M.A. F.R.S. Sec. Ast. Soc. &c. &c. 4to.	
	Ephemerides Astronomicas calculadas para o meridiano do observatorio real da Universidade de Coimbra: para o uso do mesmo observatorio, e para o da navegaçaõ Portugueza. Vol. 14. Para os annos de 1821 e 1822. Coimbra 1818. 8vo.	Dr. Gregory.
	Astronomisches Jahrbuch für des Jahr 1822, 1823, 1824, nebst einer Sammlung, der neuesten in die astronomischen Wissenschaften einschlagenden Abhandlungen, Beobachtungen und Nachrichten. von Dr. J. E. Bode. Berlin 1819, 1820, 1821. 3 vol. 8vo.	Dr. Bode.
	Annalen der K. K. Sternwarte in Wien. von J J. Littrow. Zweyter Theil. Wien 1822. folio.	Prof. Littrow.
	Agli Amatori della lingua e letteratura Tedesca gli Editori della Biblioteca Germanica. 8vo.	———
	Astronomische Beobachtungen auf der Königlichen Universitäts-Sternwarte in Königsberg von F. W. Bessel. Achte Abtheilung. Vom 1. Januar bis 31. December 1822. Königsberg 1823. folio.	Prof. Bessel.
	De observationibus astronomicis a Flamsteedio institutis dissertatio quam scripsit et auctoritate amplissimi philosophorum ordinis pro venia legendi in auditorio maximo d. 12 Aprilis 1822, hora 10, publice defendet Frid. Wilh. Aug. Argelander, Phil. Dr. A.A.L.L.M. contra opponentes Ottonem Augustum Rosenberger Curonem et Henricum Ferdinandum Scherk Posnaniensem. 4to.	———
	Untersuchungen über die Bahn des Grossen Cometen vom Jahre 1811 von Dr. Fried. Wilh. Aug. Argelander. mit einem Kupfer. Königsberg 1822. 4to.	———
	Note sur quelques observations astronomiques, faites en 1821 et 1822, à l'observatoire de Genène. Par Alfred Gautier, Professeur d'Astronomie dans l'Academie de Genève. Genève 1822. 4to.	Prof. Gautier.
	Astronomische Nachrichten. Nos. 13 to 16 inclusive. Altona. 4to.	Pr. Schumacher.
Dec. 13.	Connaissance des Tems, pour l'an 1825. Paris 1822. 8vo.	Le Bureau des Longitudes de France.

	Presents.	Donors.
1822. Dec. 13.	Astronomische Nachrichten. Nos. 17 to 22 inclusive. Altona. 4to.	Pr. Schumacher.
	The quantity of solar nutation, as affecting the north polar distances of the Fixed Stars, deduced from observation, and the application of this determination to confirm the conclusions relative to the parallaxes of certain Fixed Stars. By the Rev. John Brinkley, D.D. F.R.S. &c. Dublin 1822. 4to.	Dr. Brinkley.
1823. March 14	Asiatic Researches, vol. 14. Calcutta 1822. 4to.	Asiatic Society.
	Astronomical Observations made at the Royal Observatory at Greenwich in the year 1820. Vol. 4, part I. folio.	J. Pond, Esq.
	Notice Nécrologique sur M. Delambre. 8vo.	M. Dupin.
	Tables Astronomiques, publiées par le Bureau des Longitudes de France. Par M. Bouvard et M. Delambre. Paris 1808. 4to.	M. Bouvard.
	Transactions of the American Philosophical Society. Philadelphia, 1786 to 1818. 8 vols. 4to.	American Phil. Soc.
	An Epitome of the elementary principles of Natural and Experimental Philosophy,—part the 1st.: with a copious account of the invention, progress, and present state of the Steam Engine. By John Millington, Esq. London 1823. 8vo.	J. Millington, Esq.
	Schreiben an den Herrn Doctor W. Olbers in Bremen von H. C. Schumacher, Professor in Copenhagen, R. v. D. enthaltend eine Nachricht über den apparat, dessen er sich zur Messung der Basis bei Braack im Jahre 1820, bedient hat; mit 2 von Herrn Ingenieur-Capitain, v. Caroc gestochenen Steindrücken. Altona 1821. 4to.	Pr. Schumacher.
April 11.	Somatopsychonoologia, shewing that the proofs of body, life, and mind, considered as distinct essences cannot be deduced from Physiology, but depend on a distinct sort of evidence; being an examination of the controversy concerning life carried on by M.M. Laurence, Abernethy, Rennell, and others. By Philostratus. 1 vol. 8vo.	Dr. Forster.
	Traité de Mécanique Céleste. Par M. le Marquis de la Place. Paris 1823. Tome cinquième. 4to.	Le Marquis de la Place.
May 9.	Four Propositions shewing that the distance of the sun as attempted to be determined from the theory of gravitation is erroneous, &c. By John Dawson, Esq. of Sudbury. 8vo.	Dr. Gregory.
	Observations on the present state of Nautical Astronomy: with remarks on the expediency of promoting a more general ac-	E. Riddle, Esq.

1823.	Presents.	Donors.
May 9.	quaintance with the modern improvements in the science among the seamen, in the British Merchant Service. By Edward Riddle, Esq. 8vo.	
	A Tabular Series of Decimal Quotients for all the proper Vulgar Fractions, of which, when in their lowest terms, neither the numerator nor the denominator is greater than 1000. By Henry Goodwyn, Esq. 8vo.	H. Goodwyn, Esq.
	A Table of the Circles arising from the division of a unit, or any other whole number, by all the integers from 1 to 1024; being all the pure decimal quotients that can arise from this source. By Henry Goodwyn, Esq. 8vo.	———
	Introductory remarks respecting the Imperial Gallon and Diagonal Table. By Henry Goodwyn, Esq. 4to.	———
	Traité de Mécanique Céleste. Livre XII. Avril 1823. 4to.	Le Marquis de la Place.
	Essai Historique sur le Problème des trois corps, ou Dissertation sur la théorie des mouvemens de la lune, et des planètes, abstraction faite de leur figure. Par Alfred Gautier de Genève. Paris 1817. 1 vol. 4to.	Prof. Gautier.
	The Spots on the Sun, observed by Charles H. Adams, Esq. folio.	C. H. Adams, Esq.
	Méthode facile et générale pour calculer les latitudes d'un lieu par les hauteurs de l'Etoile Polaire, observées à toute heure. Par M. Horner, Conseiller Aulique de S. M. l'Empereur de toutes les Russies; suivie d'une autre méthode pour réduire au méridien les hauteurs circum-méridiennes d'un astre quelconque, observées avec un cercle-répétiteur. Par M. le Docteur Young, Sec. Soc. Roy. des Sci. de Londres, &c. Gênes 1822. 8vo.	Prof. Gautier.
	Mémoire sur la réduction des distances lunaires, contenant une méthode courte et facile, avec des tables nouvelles, destinées à calculer l'effet de la réfraction sur les distances de deux astres quelconques. Par M. Horner. Gênes 1822. 8vo.	———
	A short and easy method for correcting the apparent distances of the moon from the sun or a fixed star, with several new tables by Dr. Horner. Genoa 1822. 8vo.	———

1823.	PRESENTS.	DONORS.
June 13.	Astronomische Abhandlungen herausgegeben von H. C. Schumacher. Erstes & Zweites heft Taffeln zum zweiten Hefte der Astronomischen Abhandlungen. 4to.	Pr. Schumacher.
	Astronomische Nachrichten. No 23 to 28 inclusive. Altona. 4to.	———
	Artronomische Hülfstafeln für 1823, herausgegeben von H. C. Schumacher R. v. D. Copenhagen. 8vo.	———
	The Nautical Almanac and Astronomical Ephemeris for the years 1796, 1797, 1799.	J. Millington, Esq.
	Transactions of the Royal Society of Edinburgh, vol. 9. 4to.	Royal Society of Edinburgh.
Nov. 14.	An elegant Inkstand for the table of the Society, transmitted through Daniel Moore, Esq.	Unknown.
	Observations on the casual and periodical influence of particular states of the atmosphere on human health and diseases, particularly insanity. By Dr. Thos. Forster, F.L.S. &c. 8vo.	Dr. T. Forster.
	Researches about Atmospheric phenomena. By Thomas Forster, F.L.S. 2nd edition, corrected and enlarged; with a series of engravings illustrative of the modifications of the clouds, &c. London 1815. 8vo.	———
	Sketch of the new anatomy and physiology of the brain and nervous system of Drs. Gall and Spurzheim, considered as comprehending a complete system of Zoonomy, &c. By Dr. Thos. Forster, F.L.S. &c. 8vo.	———
	The Nautical Almanac and Astronomical Ephemeris for the years 1821 and 1822. 8vo.	Capt. Beaufort, R.N.
	Astronomische Beobachtungen auf der Königlichen Universitäts -Sternwarte in Köuigsberg von F. W. Bessel. Siebente Abtheilung vom 1. Januar bis 31. December 1821. Mit einer Kupfertafel. Königsberg, 1822. folio.	Prof. Bessel.
	Astronomische Abhandlungen herausgegeben von H. C. Schumacher, R. v. D. Zweites Heft. Taffeln zum zweiten hefte der Astronomischen Abhandlungen. Altona 1823. 2 vols. 4to.	Pr. Schumacher.
	F. G.W. Struve Observationes astronomicas, institutas in specula Universitatis Cæsareæ Dorpatensis, publici juris facit senatus Universitatis. Volumen 3. Observationes annorum 1820 et 1821. Dorpati 1822. 4to.	Prof. Struve.

1823.	Presents.	Donors.
Nov. 14.	Astronomische Nachrichten. Nos. 29 to 38 inclusive. Altona. 4to.	Pr. Schumacher.
	Three pieces of flint glass (for object glasses) from Neufchatel; one of which is 7¼ inches in diameter.	M. Guineand.
Dec. 12.	The Nautical Almanac and Astronomical Ephemeris for the year 1767; to which is added the first edition of the requisite tables. 8vo.	N. Bosworth, Esq.
	The Imperial Almanac, or annual compendium of astronomical, statistical, and scientific information for 1824.	Dr. Gregory.
	The Cœlestial Atlas, or improved Ephemeris for 1824. By Robert White.	———
	Lettre de M. le Professeur Dan. Kmeth à M. le Baron de Zach; sur une nouvelle angosiade, ou sur un délit astronomique, commis à l'Observatoire Royal de Bude en Hongrie. 8vo.	F. Baily, Esq.
	A new, general, and algebraical solution of the higher orders of Equations, to the tenth degree inclusive. By John Buck, Esq. of Queen's College, Cambridge.	John Buck, Esq.
	A lithographic drawing showing the trigonometrical measurement of the distance between the Royal Observatory at Greenwich, and the Observatory of Stephen Groombridge, Esq. on Blackheath, as taken by himself.	S. Groombridge, Esq.
1824. Jan. 9.	Astronomical Observations made at the Royal Observatory at Greenwich for the years 1818, 1819 and 1821, 3 parts. fol.	J. Pond, Esq.

A LIST OF THE MEMBERS

OF

THE ASTRONOMICAL SOCIETY OF LONDON

On February 13, 1824.

A * prefixed to a name indicates that such person is a resident Member, who has compounded for his Annual Contributions; and a † indicates a non-resident Member, who has in like manner compounded.

Should any errors or omissions be found in this list, it is requested that notice thereof be given to the Secretaries.

William	Allen, Esq. F.R.S. and L.S. *Plough Court, Lombard Street.*
† John W.	Anderson, M.D. *Edinburgh.*
CHARLES	BABBAGE, Esq. M.A. F.R.S. L. & E. V. Pres. Ast. Soc. Fell. Camb. Phil. Soc. Corr. Philom. Soc. of Paris, Assoc. of the Acad. of Dijon, No. 5 *Devonshire Street, Portland Place.*
* Arthur	Baily, Esq. *Lavender Hill.*
* FRANCIS	BAILY, Esq. F.R.S. and L.S. V. Pres. Ast. Soc. *Gray's Inn.*
† Rev. L. P.	Baker, B.D. Fellow of St. John's College, *Cambridge.*
† Rev. Thomas	Barber, B.D. *Houghton Conquest, Bedfordshire.*
John	Barton, Esq. Deputy Comp. of His Majesty's Mint, *Tower Hill.*
Captain F.	Beaufort, of the Royal Navy, F.R.S. *Grove, Epping.*
Col. Mark	Beaufoy, F.R.S. and L.S. *Bushey-heath, near Stanmore.*
† Rev. Thomas	Belgrave, M.A. *North Kilworth, Leicestershire.*
Richard	Best, Esq. *Greenwich.*
George	Birkbeck, M.D., No. 50 *Broad Street, Bishopsgate.*
Capt. Benjamin	Blake, *Bengal.*

Major Thomas	Blanchard, of the Royal Engineers, No. 19 *New Ormond Street.*
† Rev. Miles	Bland, M.A. F.R.S. *St. John's College, Cambridge.*
W. H.	Booth, Esq. No. 6 *Lincoln's Inn Old Square.*
John	Bostock, M.D. F.R.S. *Upper Bedford Place.*
† William	Bowles, Esq. F.L.S. *Abingdon, Berks.*
* Timothy	Bramah, Esq. *Pimlico.*
† David	Brewster, LL.D. F.R.S. L. and E. *Edinburgh.*
† Rev. John	Brinkley, D.D. F.R.S. M.R.I.A. and Andrews Professor of Astronomy in the University of Dublin. *Dublin.*
† Maj. Gen. Sir Thos.	Brisbane, K.C.B. F.R.S. L. & E. Governor of New South Wales.
† Sir Edward F.	Bromhead, Bart. M.A. F.R.S. *Thurlby, Lincolnshire.*
† Charles F.	Bromhead, Esq. M.A. *Trinity College, Cambridge.*
George H.	Browne, Esq. *King Street, Covent Garden.*
William	Bullock, Esq. *Prittlewell, Essex.*
* Rev. C. Parr	Burney, D.D. F.R.S. *Greenwich.*
James	Bush, Esq. *Hadley Green, Barnet.*
* Rev. G.	Butler, D.D. F.R.S. & S.A. *Harrow.*
† J. Strode	Butt, Esq. *Pulteney Street, Bath.*
John	Caley, Esq. F.R.S. & L.S. *Gray's Inn.*
William	Cary, Esq. No. 277 *Strand.*
† Rev. Thomas	Catton, B.D. F.R.S. *St. John's College, Cambridge.*
John	Clark, Esq. *Chatham Place, Blackfriars.*
John Alden	Clarke, Esq. *Battersea Rise.*
W. Stanley	Clarke, Esq. F.R.S. *Leatherhead.*
† Henry	Coddington, M.A. Fell. Trin. Coll. *Cambridge.*
† William	Coffin, Esq. *Llandaff.*
* Major Thomas	Colby, of the Royal Engineers, LL.D. F.R.S. L. & E. *Tower.*
* HENRY T.	COLEBROOKE, Esq. F.R.S. L. & E. & L.S. Pres. Ast. Soc. No. 32 *Argyle Street.*
† Richard	Comfield, Esq. *Northampton.*
† Samuel	Cooper, Esq, *Tranby, Yorkshire.*
† Capt. A. L.	Corry, of the Royal Navy, *Enniskillen, Ireland.*
Edmund Robert	Daniell, Esq. *Lincoln's Inn.*

Rev. William	Dealtry, M.A. F.R.S. *Clapham.*
* George	Dollond, Esq. F.R.S. *St. Paul's Churchyard.*
William	Domville, Esq. *Lincoln's Inn.*
Bryan	Donkin, Esq. No. 80 *Great Surry Street, Blackfriars Road.*
† Thomas	Donkin, Esq. *Bishop's Burton, Yorkshire.*
Daniel	Dowling, Esq. *Highgate.*
Sir R. L.	Dundas, No. 19 *Arlington Street, Piccadilly.*
† Capt. William	D'Urban, R.N. LL.D. *Newland, Gloucester.*
† Rev. James C.	Ebden, M.A. *Trinity Hall, Cambridge.*
James	Edwards, Esq. *Putney.*
Thomas F.	Ellis, Esq. M.A. No. 15 *Bedford Place.*
† Rev. Lewis	Evans, *Froxfield, Wilts.*
† Captain George	Everest, *Bengal.*
† Rev. Fearon	Fallows, M.A. F.R.S. *St. John's College, Cambridge,* Astronomer at the Cape of Good Hope.
Michael	Faraday, Esq. F.R.S. *Royal Institution.*
† John	Fardell, Esq. F.S.A. *Lincoln.*
† Rev. William	Farish, B.D. F.R.S. Professor of Experimental Philosophy in the University of Cambridge.
† John	Fletcher, Esq. *Lisbon.*
* Thomas	Forster, Esq. B.M. F.L.S. & M. Acad. Nat. Sci. Philadelphia, *Hartfield, East Grinstead, Surry.*
T. Furley	Forster, Esq. F.L.S. *Clapton.*
* Captain John	Franklin, R. N. F. R. S. No. 55 *Devonshire Street, Portland Place.*
Rev. Stephen	Freeman, *Enfield, Middlesex.*
John	Fuller, Esq. No. 36 *Devonshire Place.*
* Davies	Gilbert, Esq. M.P. Treas. & V.P.R.S. F.L.S. *East Bourne, Sussex.*
John	Goldingham, Esq. Superintendant of the Observatory at Madras.
Isaac Lyon	Goldsmid, Esq. *Camberwell.*
Benjamin	Gompertz, Esq. F.R.S. *Kennington Terrace, Vauxhall.*
Right Hon. Alexander	Duke of Gordon, F.R.S. No. 23 *New Norfolk Street, Park Lane.*

OLINTHUS G.	GREGORY, LL.D. Professor of Mathematics, *Royal Military Academy, Woolwich*, Sec. Ast. Soc., *Woolwich.*
† His Excellency	Alexeis Greig, Vice Admiral of the Imperial Russian Navy, *Odessa.*
† Rev. James	Grooby, M.A. *Cirencester, Gloucestershire.*
* Stephen	Groombridge, Esq. F R.S. S.R.A. Nap. *Blackheath.*
† Rev. Richard	Gwatkin, M.A. *St. John's College, Cambridge.*
† Robert	Haldane, LL.D. Professor of Mathematics, *St. Andrews.*
† Captain Basil	Hall, Royal Navy, F.R.S. L. & E. No. 128 *George Street, Edinburgh.*
Captain F. A.	Halliday, of the Royal Navy, *Ham, Surry.*
† Rev. H. P.	Hamilton, M.A. F.R.S.E. *Trinity College, Cambridge.*
William	Handley, Esq. No. 12 *Winchester Place, Pentonville.*
† George	Harvey, Esq. *Plymouth.*
Anthony	Haviside, Esq. No. 19 *Bucklersbury.*
J. F. W.	HERSCHEL, Esq. M.A. F.R.S. L. & E. Corr. of Philomath. Soc. of Paris, For. Sec. Ast. Soc. *Slough.*
† Rev. J. P.	Higman, M.A. F.R.S. *Trinity College, Cambridge.*
† Rowland	Hill, Esq. *Hazlewood, near Birmingham.*
Chamb.	Hinchliff, Esq. *Park Street, Southwark.*
† John	Hinde, Esq. M.A. Fellow of Sidney College, Cambridge, Tutor and Mathematical Lecturer.
* Sir BENJAMIN	HOBHOUSE, Bart. F.R.S. V. Pres. Ast. Soc. *Whitton, Middlesex.*
Charles	Holford, Esq. *Hampstead.*
† Rev. T. W.	Hornbuckle, *St. John's College, Cambridge.*
James	Horsburgh, Esq. F.R.S. *Herne Hill, Dulwich.*
† James	Hoy, Esq. *Gordon Castle.*
† Thomas	Jackson, LL.D. F.R.S.E. Professor of Natural Philosophy, *St. Andrews.*
† James	Jardine, Esq. F.R.S.E. *Edinburgh.*
Rev. Thomas	Jephson, *St. John's College, Cambridge.*
Thomas	Jones, Esq. No. 62 *Charing Cross.*
Patrick	Kelly, LL.D. *Finsbury Square.*
† Joshua	King, Esq. M.A. Tutor of Queen's College, *Cambridge.*

† Andrew	Lang, Esq. *St. Croix, West Indies.*
Thomas	Leybourn, Esq. *Royal Military College, Sandhurst.*
RIGHT HONOURABLE GEORGE EARL OF MACCLESFIELD, F.R.S. V. Pres. Ast. Soc. No. 9 *Conduit Street.*	
* Rev. John	Maddy, D.D. F.R.S. & S.A. No. 16 *Argyle Street.*
J. S.	Memes, Esq. *Moulsey Park, Surry.*
William	Mickleham, LL.D. Prof. Nat. Phil. in the University, *Glasgow.*
JOHN	MILLINGTON, Esq. F.L.S. Prof. of Mechanics in the Royal Institution, Sec. Ast. Soc. No. 36 *Bloomsbury Square.*
† William P.	Moffat, Esq. *North Wales.*
† Captain John	Molesworth, R.N. No. 15 *Gloucester Place, Brighton.*
Basil	Montagu, Esq. No. 10 *New Square, Lincoln's Inn.*
* Daniel	Moore, Esq. F.R.S. S.A. & L.S. *No.* 8 *New Square, Lincoln's Inn.*
† Joseph	Moore, Esq. *Lincoln.*
W. M.	Mosely, Esq. *Winterdyne House, near Bewdley, Worcestershire.*
William	Mylne, Esq. *New River Head.*
† James Ady	Ogle, M.D. *Trinity College, Oxford.*
† Sir Edward William Campbell Rich Owen, K.C.B. *Deal.*	
† Capt. W. F. W.	Owen, of the Royal Navy, *Deal.*
Samuel	Parkes, Esq. F.L.S. Acad. Scien. Nat. Philadel. & Soc. Phil. Paris. Cor. Soc. Imp. Hist. Nat. Mosc. Socius, No. 30 *Mecklenburg Square.*
Joseph	Parkinson, Esq. No. 3 *Newman Street, Oxford Street.*
† Captain Edward	Parry, F.R.S. of the Royal Navy, *Bath.*
Stephen	Pattisal, Esq. *'Change Alley, Cornhill.*
† William	Paul, Esq. Professor of Natural Philosophy, *Aberdeen.*
† Rev. G.	Peacock, M.A. F.R.S. Lecturer on Mathematics in Trinity College, *Cambridge.*
* Rev. WILLIAM	PEARSON, LL.D. F.R.S. Treas. Ast. Soc. No. 6 *Compton Terrace, Islington.*
H. F.	Pelerin, Esq. No. 11 *New North Street, Red Lion Square.*
John	Penn, Esq. *Greenwich.*
Wm. Hasl.	Pepys, Esq. F.R.S. *Poultry.*

Charles	Perkins, Esq. No. 67 *Mark Lane.*
* Frederick	Perkins, Esq. *Lee, Kent.*
* Henry	Perkins, Esq. *Springfield, Surry.*
* Lewis H.	Petit, Esq. F.R.S. No. 9 *New Square, Lincoln's Inn.*
* Rev. J. H.	Pinckney, D.D. *East Sheen, Surry.*
John	Pond, Esq. F.R.S. Astronomer Royal, *Greenwich.*
Edward	Riddle, Esq. *Royal Naval Asylum, Greenwich.*
† Rev. H.	Robinson, M.A. *St. John's College, Cambridge.*
* Peter Mark	Roget, M.D. F.R.S. No. 39 *Bernard Street, Russell Square.*
† Captain John	Ross, of the Royal Navy, K.S. *Observatory, Stranraer, N.B.*
Maj. Gen. John	Rowley, of the Royal Engineers, F.R.S. No. 11 *Queen Square, Bloomsbury.*
† Charles	Rumker, Esq. *New South Wales.*
Edward	Ryan, Esq. No. 3 *Hare Court, Temple.*
John	Sanders, Esq. *Reigate, Surry.*
Samuel	Scott, Esq. *Lower Grosvenor Street.*
† Captain Hugh	Scott, *Butterley Hall, Derby.*
* Captain W. H.	Shireff, of the Royal Navy, No. 5 *Cumberland St., Portman Sq.*
* Captain W. H.	Smith, of the Royal Navy.
* James	South, Esq. F.R.S. & L.S. Hon. Mem. Camb. Phil. Soc. *Paris.*
John	Stevens, Esq. *Chelsea Hospital.*
† Robert	Stevenson, Esq. F.R.S. E. *Edinburgh.*
Charles	Stokes, Esq. F.R.S. S.A. & L.S. *Gray's Inn.*
Edward	Strachey, Esq. No. 31 *Fitzroy Square.*
† William	Strutt, Esq. F.R.S. *Derby.*
W. H. Fox	Talbott, Esq. B.A. *Trinity College, Cambridge.*
Richard	Taylor, Esq. Sec. Linn. Soc. No. 38 *Shoe Lane, Holborn.*
* Thomas	Telford, Esq. F.R.S.E. *Abingdon Street, Westminster.*
Alexander	Tilloch, LL.D. M.R.I.A. *Picket Place, Temple Bar.*
* Edward	Troughton, Esq. F.R.S. L. & E. No. 136 *Fleet Street.*
Henry	Tucker, Esq. No. 34 *Baker Street.*
Charles	Tulley, Esq. *Territt's Court, Islington.*
Andrew	Ure, M.D. F.R.S. Professor of Chemistry and Natural Philosophy in the Andersonian Institution, *Glasgow.*

* Joseph F.	Vandercom, Esq. *Bush Lane, Cannon Street.*
William	Vaughan, Esq. F.R.S. *Fenchurch Street.*
† Edward	Walker, Esq. *Gloucestershire.*
† William	Wallace, Esq. Prof. of Mathematics, *University of Edinburgh.*
† Rev. M.	Ward, LL.B. *Tamworth, Staffordshire.*
† Sir William	Watson, F.R.S. *Great Pulteney Street, Bath.*
† Richard	Watt, Esq. *Bishops Burton, Yorkshire.*
† William	Whewell, Esq. M.A. F.R.S. *Trinity College, Cambridge.*
† Joseph	Whidby, Esq. F.R.S. & L.S. *Plymouth.*
† Rev. J. W.	Whittaker, B.D. *Blackburn, Lancashire.*
† Rev. G. B.	Wildig, M.A. *Liverpool.*
† Daniel	Wilson, Esq. *Paris.*
Captain John	Wilson, *Grove, Epping.*
† J. W.	Woollgar, Esq. *Lewes, Sussex.*
John	Wrottesley, Esq. No. 10 *New Square, Lincoln's Inn.*
Sir Samuel	Young, Bart. F.R.S. *Formosa, near Maidenhead, Berks.*
Lieut. George	Young, R.N. *Formosa.*

ASSOCIATES.

François	Arago,	*Paris.*
F. W.	Bessel,	*Königsberg.*
Jean Bapt.	Biot,	*Paris.*
John Elert	Bode,	*Berlin.*
M. Alexis	Bouvard,	*Paris.*
Carlo	Brioschi,	*Naples.*
Charles F.	Gauss,	*Göttingen.*
Alfred	Gautier,	*Geneva.*
C. Louis	Harding,	*Göttingen.*
Le Marquis	de Laplace,	*Paris.*
J. J.	Littrow,	*Vienna.*
Gerard	Moll,	*Leyden.*
Jean N.	Nicollet,	*Paris.*
William	Olbers,	*Bremen.*
Barnabas	Oriani,	*Pavia.*
Giuseppe	Piazzi,	*Palermo.*
H. C.	Schumacher,	*Copenhagen.*
Peter	Slawinski,	*Wilna.*
F. G. Wilhelm	Struve,	*Dorpat.*
Frederick	Treschel,	*Berne.*
J. D.	Vallot,	*Dijon.*
Baron	de Zach,	*Genoa.*

LIST OF THE OFFICERS

OF

THE ASTRONOMICAL SOCIETY OF LONDON,

FOR THE YEAR 1824—5,

As elected on the 13th of February, 1824.

The Names, under each Office, are arranged alphabetically.

President.

HENRY THOMAS COLEBROOKE, Esq. F.R.S. and L.S.

Vice-Presidents.

CHARLES BABBAGE, Esq. M.A. F.R.S. L. & E.
FRANCIS BAILY, Esq. F.R.S. and L.S.
Sir BENJAMIN HOBHOUSE, Bart. F.R.S.
GEORGE Earl of MACCLESFIELD, F.R.S.

Treasurer.

Rev. WILLIAM PEARSON, LL.D. F.R.S.

Secretaries.

OLINTHUS G. GREGORY, LL.D. *Prof. Math. Roy. Mil. Acad. Woolwich.*
JOHN MILLINGTON, Esq. F.L.S. *Prof. Mech. Phil. Roy. Inst.*
J. F. W. HERSCHEL, Esq. M.A. F.R.S. L. & E. (*Foreign*)

Council.

Major THOMAS COLBY, *Roy. Eng.* LL.D. F.R.S. L. & E.
GEORGE DOLLOND, Esq. F.R.S.
BRYAN DONKIN, Esq.
Captain JOHN FRANKLIN, R.N. F.R.S.
DAVIES GILBERT, Esq. M.P. V.P.R.S. F.L.S.
BENJAMIN GOMPERTZ, Esq. F.R.S.
STEPHEN GROOMBRIDGE, Esq. F.R.S.
DANIEL MOORE, Esq. F.R.S. S.A. & L.S.

Trustees.

ARTHUR BAILY, Esq.
DANIEL MOORE, Esq. F.R.S. S.A. & L.S.
CHARLES STOKES, Esq. F.R.S. S.A. & L.S.
The Treasurer for the time being.

Collector to the Society.

Mr. HENRY HOLLAND HARRISON, No. 41 *Bedford Street, Covent Garden.*

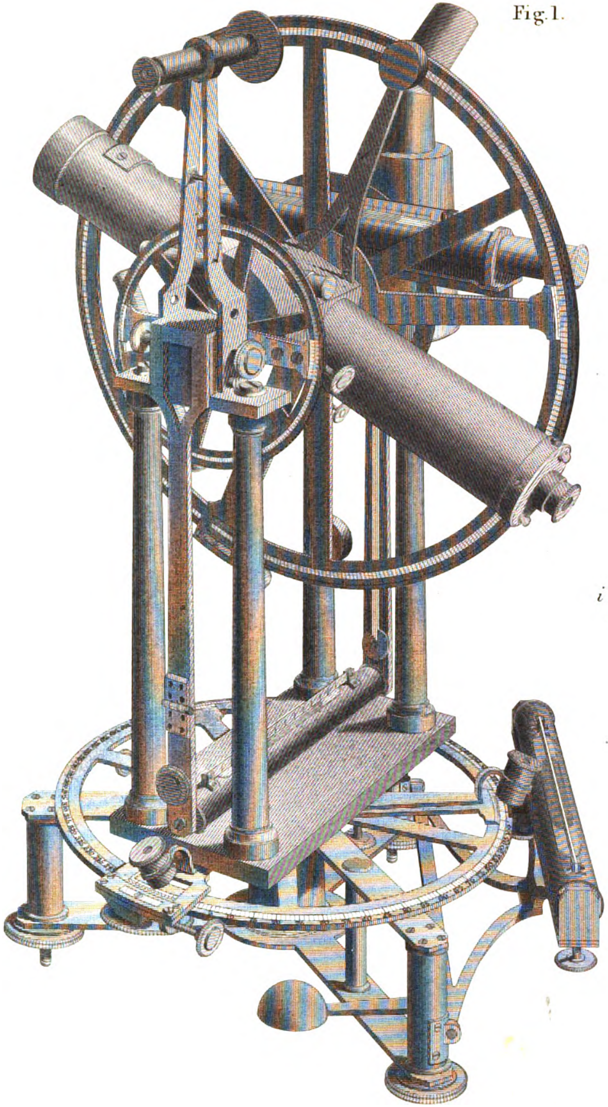
LONDON:
PRINTED BY RICHARD TAYLOR,
SHOE-LANE.

ALERE FLAMMAM.

Minutes.	h ′ ° ′ 0.0	h ′ ° ′ 0.5	h ′ ° ′ 0.10	h ′ ° ′ 0.15
0.0		1.3802	1.0792	9031
0.1	3.0792	1.3716	1.0749	9002
0.2	2.7782	1.3632	1.0706	8973
0.3	2.6021	1.3549	1.0663	8945
0.4	2.4771	1.3468	1.0621	8917
0.5	2.3802	1.3388	1.0580	8888
0.6	2.3010	1.3310	1.0539	8861
0.7	2.2341	1.3233	1.0498	8833
0.8	2.1761	1.3158	1.0458	8805
0.9	2.1249	1.3083	1.0418	8778
1.0	2.0792	1.3010	1.0378	8751
1.1	2.0378	1.2939	1.0339	8724
1.2	2.0000	1.2868	1.0300	8697
1.3	1.9652	1.2798	1.0261	8670
1.4	1.9331	1.2730	1.0223	8643
1.5	1.9031	1.2663	1.0185	8617
1.6	1.8751	1.2596	1.0147	8591
1.7	1.8487	1.2531	1.0110	8565
1.8	1.8239	1.2467	1.0073	8539
1.9	1.8004	1.2403	1.0036	8513
2.0	1.7782	1.2341	1.0000	8487
2.1	1.7570	1.2279	9964	8462
2.2	1.7368	1.2218	9928	8437
2.3	1.7175	1.2159	9893	8411
2,4	1.6990	1.2099	9858	8386
2.5	1.6812	1.2041	9823	8361
2.6	1.6642	1.1984	9788	8337
2.7	1.6478	1.1927	9754	8312
2.8	1.6320	1.1871	9720	8288
2.9	1.6168	1.1816	9686	8263
3.0	1.6021	1.1761	9652	8239
3.1	1.5878	1.1707	9619	8215
3.2	1.5740	1.1654	9586	8191
3.3	1.5607	1.1601	9553	8167
3.4	1.5477	1.1549	9521	8144
3.5	1.5351	1.1498	9488	8120
3.6	1.5229	1.1447	9456	8097
3.7	1.5110	1.1397	9425	8073
3.8	1.4994	1.1347	9393	8050
3.9	1.4881	1.1298	9362	8027
4.0	1.4771	1.1249	9331	8004
4.1	1.4664	1.1201	9300	7981
4.2	1.4559	1.1154	9269	7959
4.3	1.4457	1.1107	9238	7936
4.4	1.4357	1.1061	9208	7914
4.5	1.4260	1.1015	9178	7891
4.6	1.4164	1.0969	9148	7869
4.7	1.4071	1.0924	9119	7847
4.8	1.3979	1.0880	9089	7825
4.9	1.3890	1.0835	9060	7803
5.0	1.3802	1.0792	9031	7782

Mr. Dollond's new Repeating circle.

Fig. 1.



Dr. Pearsons Crystal Micrometers

Fig. 2.

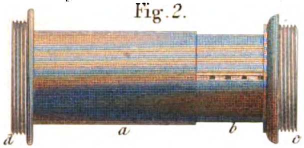


Fig. 3.

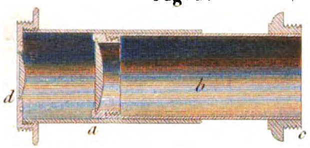


Fig. 4.

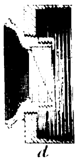


Fig. 5.

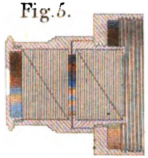
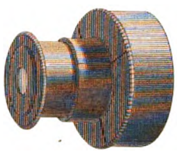
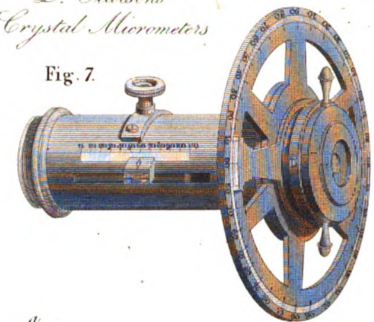


Fig. 6.



Dr. Pearsons Crystal Micrometers

Fig. 7.



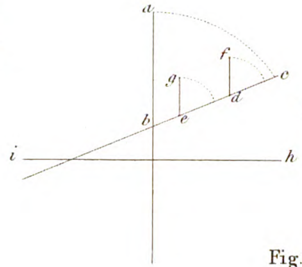
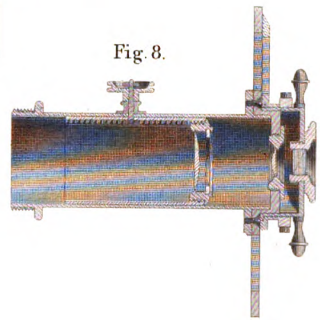


Fig. 9.

Fig. 8.



Drawn by J. Farey. Engraved for the Astronomical Society of London, by Edmd. Turrell.

Printed by R. Fenner.

Solar Eclipse. Sept.r 7th 1820.

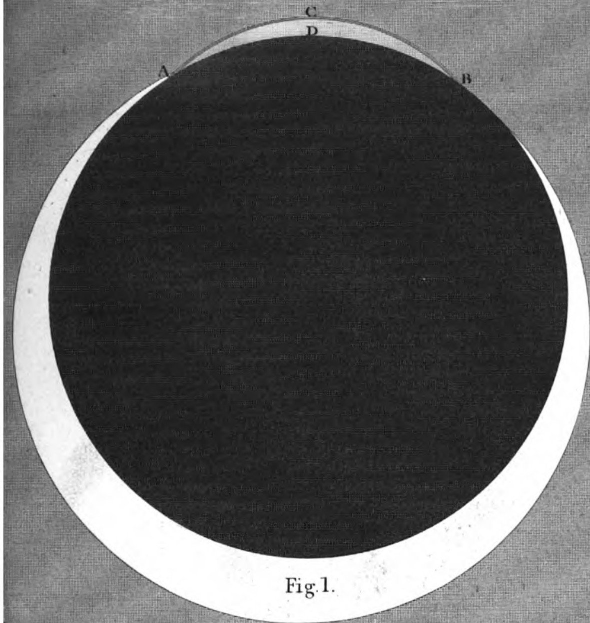


Fig. 1.

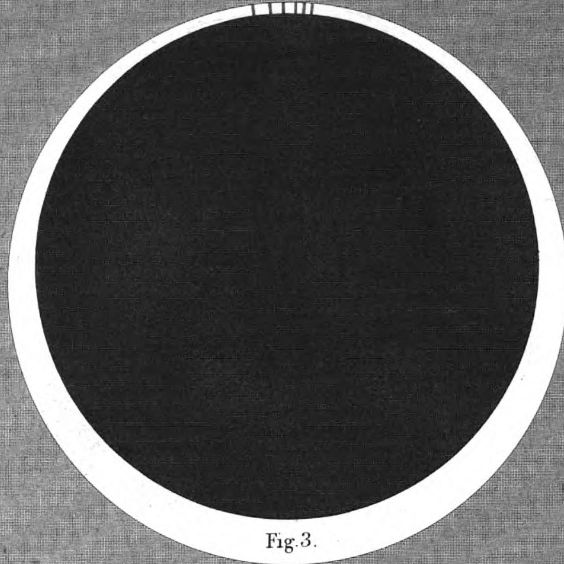


Fig. 3.

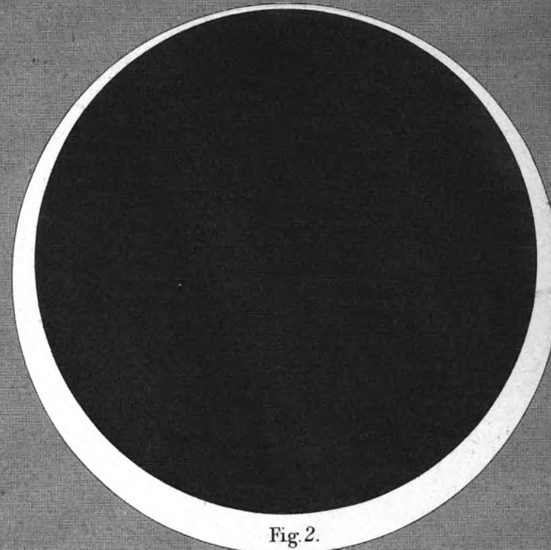


Fig. 2.

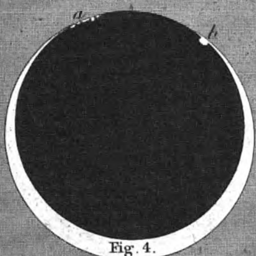


Fig. 4.

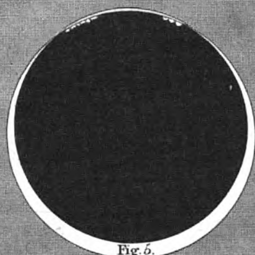


Fig. 5.

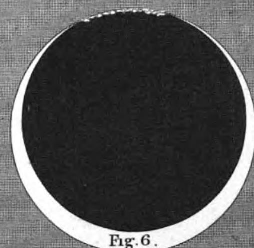


Fig. 6.

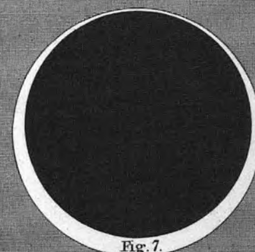
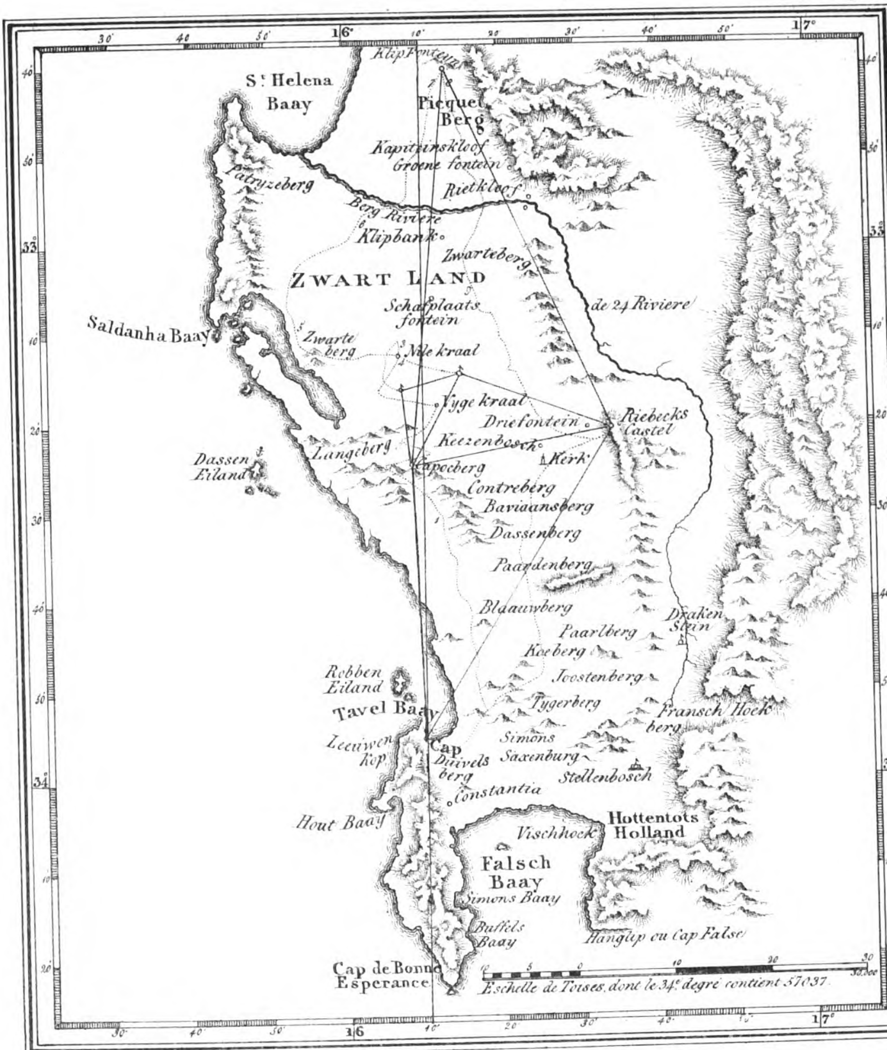


Fig. 7.

Engraved for the Astronomical Society of London, by Edm.d Turrell.

Printed by R. Fen



CARTE
DU CAP DE BONNE ESPERANCE,
et de ses Environs.
par M. l'Abbé de la Caille.
1752.

Lithog.d by C. Hullmandel. For the Memoirs of the Astronomical Society of London

Fig. 1.

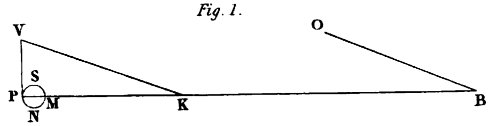


Fig. 2.

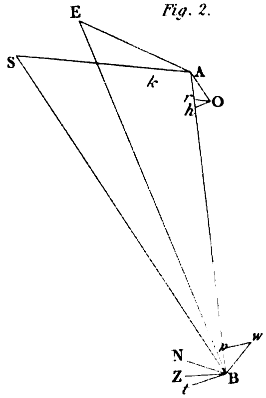


Fig. 3.

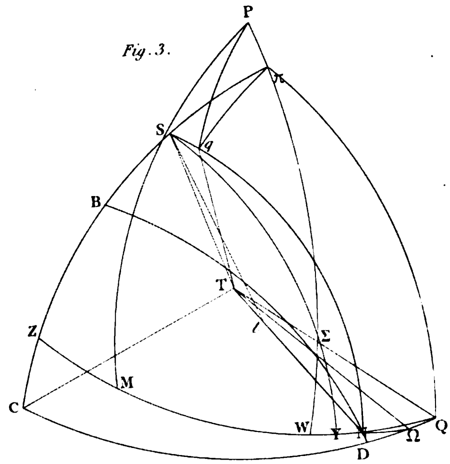


Fig. 4.

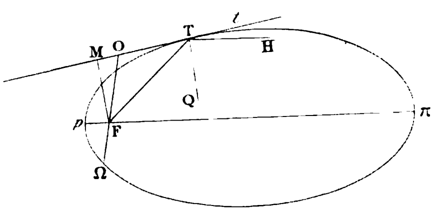
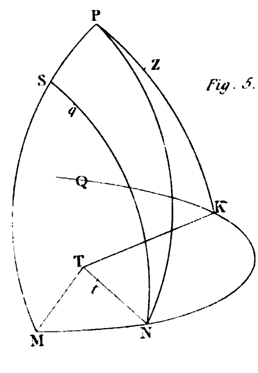


Fig. 5.



Printed by R. Fenner

Engraved for the Astronomical Society of London by S. Porter

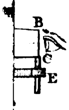


Fig. 2.

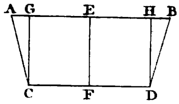
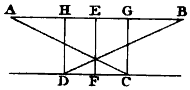
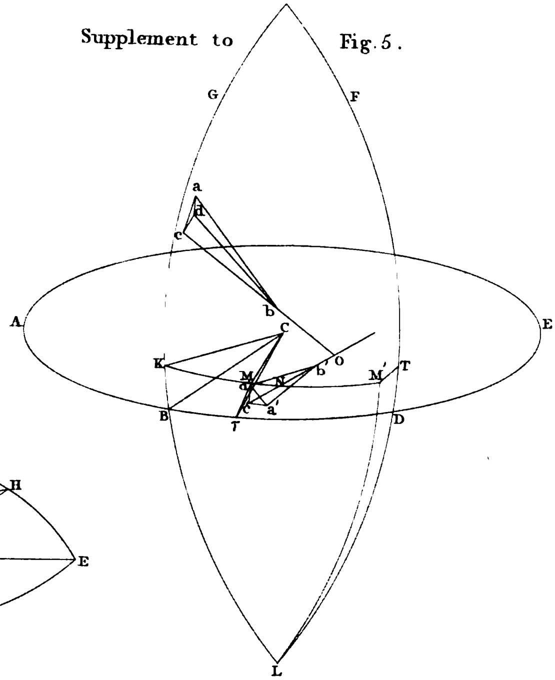


Fig 3.



Supplement to Fig. 5.



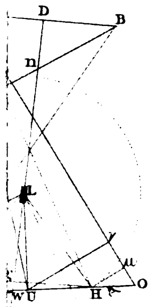
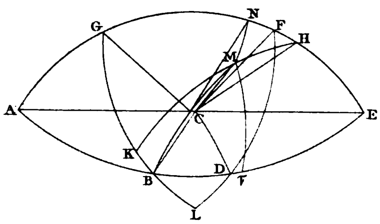


Fig. 5.



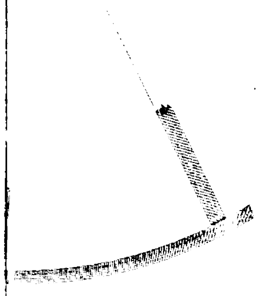
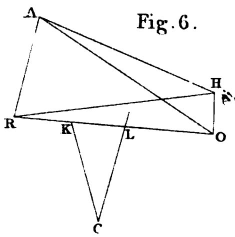


Fig. 6.



Supplement to Fig. 9.

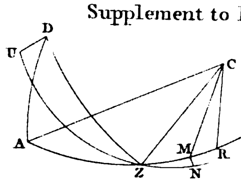


Fig. 9.

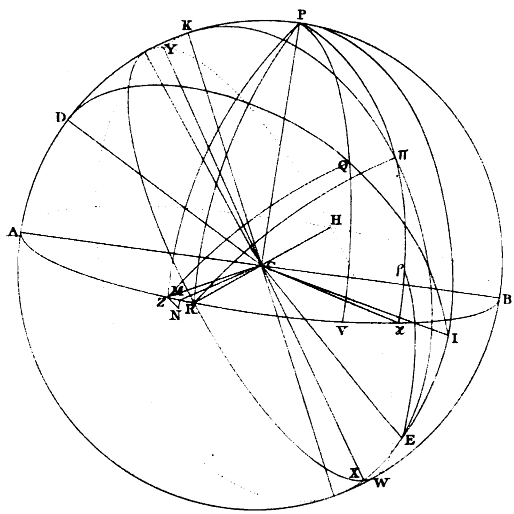
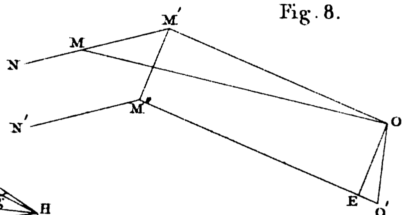
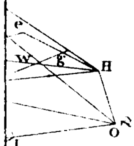


Fig. 8.





Engraved for the Astronomical Society of London by E. Turrell.

Plate VI.

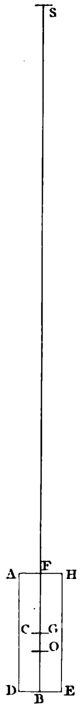


Fig. 1.

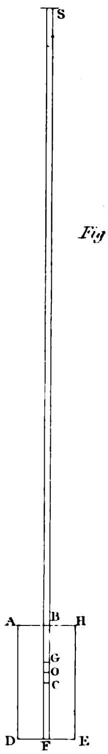


Fig. 2.

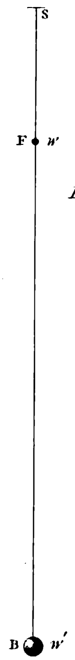


Fig. 3.

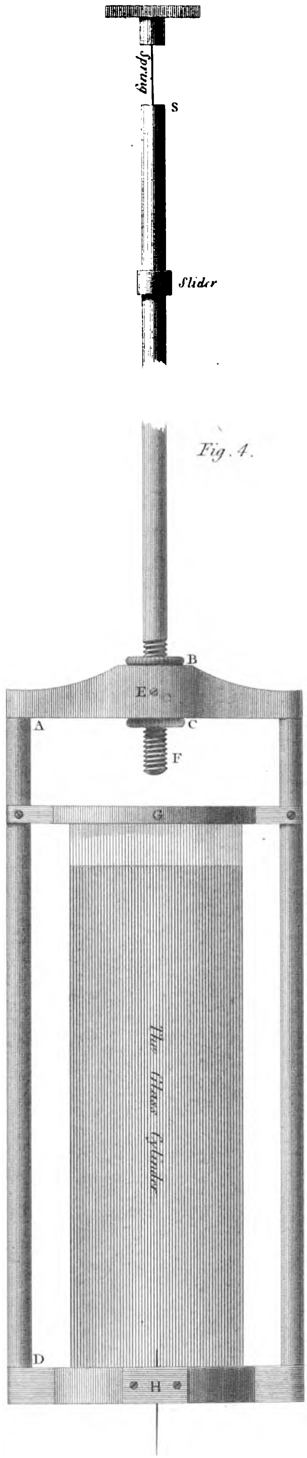


Fig. 4.

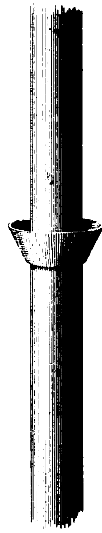


Fig. 6.

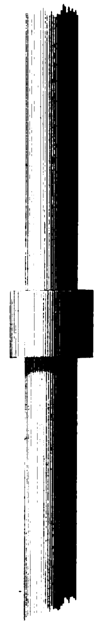


Fig. 5.

Printed by R. Fenner

Engraved for the Astronomical Society of London, by E. Turrell.